Oxford Cambridge and RSA

Practice Paper – Set 2

A Level Further Mathematics B (MEI)

Y421 Mechanics Major

MARK SCHEME

Duration:
2 hours 15 minutes

MAXIMUM MARK 120

This document consists of 17 pages

Text Instructions

1. **Annotations and abbreviations**

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs		Guidance
1			$T\cos 20$ $(T\cos 20)(125) = 5250$ $T = 44.7$	**B1** **M1** **A1** **[3]**	**1.1** **3.3** **1.1**	Use of work done = force $\times$ distance with component of T (not just T)	44.6954664…
2	**(i)**		$T\sin 25 =$ $(0.2)(0.6\sin 25)\omega^2$ $T = 0.27$ N	**M1** **M1** **A1** **[3]**	**1.1** **1.2** **1.1**	Use of component of T in N2L Use of $mr\omega^2$ with $r \neq 0.6$	
2	**(ii)**		$R + T\cos 25 = 0.2g$ $R = 1.72\text{ N}$	**M1** **A1** **[2]**	**3.3** **1.1**	Resolving vertically – three terms with component of T	1.715296…
3	**(i)**		$\dot{\mathbf{r}} = (12t^2 - 5)\mathbf{i} + 4t\mathbf{j}$ $\ddot{\mathbf{r}} = 24t\mathbf{i} + 4\mathbf{j}$ $\mathbf{F} = m(24t\mathbf{i} + 4\mathbf{j}), t = 0$ $\mathbf{F} = 4m\mathbf{j} \Rightarrow \mathbf{F}$ is parallel to the y-axis	**M1 A1** **A1** **M1** **A1** **[5]**	**3.1a** **1.1** **1.1** **3.3** **2.2a**	Attempt to differentiate **r** (all powers reduced by 1), A1 correct first derivative Use of N2L and substitute $t = 0$	
3	**(ii)**		As $t \to \infty, 24t$? 4 and so the direction of **F** approached that of the x-axis	**B1** **[1]**	**2.4**		
3	**(iii)**	(*A*)	$24t = 4$ $t = \frac{1}{6}$	**M1** **A1** **[2]**	**3.1a** **1.1**	Equating **i** and **j** components and solving for t	

Question			Answer	Marks	AOs		Guidance
3	(iii)	(*B*)	$\mathbf{F} = m(4\mathbf{i}+4\mathbf{j}) \Rightarrow \lvert\mathbf{F}\rvert = \ldots$	M1	1.2	Substitute their *t* from part (a) and attempt magnitude	
			$\lvert\mathbf{F}\rvert = m\sqrt{4^2+4^2} = 4m\sqrt{2}$	A1	1.1	oe e.g. $\sqrt{32m^2}$	
				[2]			
4	(i)		$x = A\cos 4t + B\sin 4t$	B1 [1]	1.2	oe	
4	(ii)		$\dot{x} = -4A\sin 4t + 4B\cos 4t$	M1	2.1	Attempt to differentiate their *x*	
			$t = \frac{\pi}{16}, -4A\left(\frac{\sqrt{2}}{2}\right) + 4B\left(\frac{\sqrt{2}}{2}\right) = 0$	M1	1.1	Substituting $t = \frac{\pi}{16}$ and setting $\dot{x} = 0$	$A = B$
			$t = \frac{\pi}{16}, A\left(\frac{\sqrt{2}}{2}\right) + B\left(\frac{\sqrt{2}}{2}\right) = 3\sqrt{2}$	M1	1.1	Substituting $t = \frac{\pi}{16}$ and setting $x = 3\sqrt{2}$	$A + B = 6$
			$x = 3\cos 4t + 3\sin 4t$	A1	1.1		
			$t = \frac{\pi}{16}, x = 3\sqrt{2}$ and $t = \frac{\pi}{4}, x = -3$	M1	3.4	Substituting both $t = \frac{\pi}{16}$ and $t = \frac{5\pi}{16}$	
			Total distance travelled is $3 + 3\sqrt{2}$	A1	1.1		7.24264…
				[6]			
5			$\lambda = \text{MLT}^{-2}$	B1	1.2		
			$\text{T} = \text{M}^{\alpha}\text{L}^{\beta}\left(\text{MLT}^{-2}\right)^{\gamma}$	M1	2.1	Using their λ to obtain an equation in T, M and L	
			$\alpha + \gamma = 0, \beta + \gamma = 0, -2\gamma = 1$	M1	1.1a	Setting up all three equations	
			$\alpha = \frac{1}{2}, \beta = \frac{1}{2}, \gamma = -\frac{1}{2}$	A1	1.1		
			$k\sqrt{\frac{0.5(1.2)}{15}} = 0.3 \Rightarrow k = \ldots$	M1	3.4	Attempt to find *k*	$k = 1.5$
			$T = 1.5\sqrt{\frac{0.7(1.2)}{15}} = 0.355\,\text{s}$	A1	2.2b		0.354964…
				[6]			

Question			Answer	Marks	AOs		Guidance
6			**DR**				
			$A=\int_{\pi/4}^{\pi/2}\sin x\,\mathrm{d}x=\frac{\sqrt{2}}{2}$	**B1**	**1.1**		
			$A\bar{x}=\int x\sin x\,\mathrm{d}x=\ldots$	**M1**	**2.1**	Attempt integration by parts for $A\bar{x}$ - must be of the form $\pm x\cos x\pm\int\cos x\,\mathrm{d}x$	
			$=\left[-x\cos x\right]_{\pi/4}^{\pi/2}+\int_{\pi/4}^{\pi/2}\cos x\,\mathrm{d}x$	**A1**	**1.1**		Limits not required for this or the next A mark
			$=\left[-x\cos x+\sin x\right]_{\pi/4}^{\pi/2}$	**A1**	**1.1**		
			$\bar{x}=\frac{A\bar{x}}{A}=\frac{1-\left(-\frac{\pi}{4}\left(\frac{\sqrt{2}}{2}\right)+\frac{\sqrt{2}}{2}\right)}{\frac{\sqrt{2}}{2}}$	**M1**	**1.1**	Use of $\bar{x}=\frac{A\bar{x}}{A}$	
			$A\bar{y}=\frac{1}{2}\int\sin^2 x\,\mathrm{d}x=\frac{1}{4}\int\left(1-\cos 2x\right)\mathrm{d}x$	**M1**	**3.1a**	Use of $A\bar{y}=\frac{1}{2}\int y^2\mathrm{d}x$ and attempt at substitution of double-angle formula – must be of the form $\sin^2 x=\frac{1}{2}\left(\pm 1\pm\cos 2x\right)$	
			$=\frac{1}{4}\left[x-\frac{1}{2}\sin 2x\right]_{\pi/4}^{\pi/2}$	**A1**	**1.1**		
			$\bar{y}=\frac{A\bar{y}}{A}=\frac{\frac{1}{4}\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{1}{2}\right)\right)}{\frac{\sqrt{2}}{2}}$	**M1**	**1.1**	Use of $\bar{y}=\frac{A\bar{y}}{A}$	
			$\left(\bar{x},\bar{y}\right)=\left(\frac{\sqrt{2}}{2}-1+\frac{\pi}{4},\frac{\sqrt{2}}{16}\left(\pi+2\right)\right)$	**A1**	**2.2a**		
				[9]			

Question			Answer	Marks	AOs		Guidance
7	(i)		$\text{PE} = 2mga, \text{KE} = 0$	B1	3.1b	Initial energy of the system	Zero potential energy at C (other alternatives for zero PE are acceptable)
			$\text{PE} = mga + mga\cos\theta, \text{KE} = 2\left(\frac{1}{2}mv^2\right)$	M1	1.1	Energy when Q is at an angle of θ	
			$2mga = mga(1+\cos\theta) + mv^2$	M1	3.3	Use of conservation of energy	
			$mg\cos\theta - R = \frac{m}{a}\left[ag(1-\cos\theta)\right]$	M1	3.3	Use of N2L radially (correct number of terms and component of weight) for Q – condone without R	
			$mg(2\cos\theta - 1) = 0 \Rightarrow \cos\theta = \frac{1}{2}$	M1	3.4	Re-arrange and attempt to find $\cos\theta$ or θ	R must have been set equal to zero for this mark
			$\theta = 60^{\circ}$	A1 [6]	2.2a		
7	(ii)		$v^2 = ag(1-\cos\theta) \Rightarrow 2v\frac{\mathrm{d}v}{\mathrm{d}t} = ag\sin\theta\frac{\mathrm{d}\theta}{\mathrm{d}t}$	M1	3.1a	Differentiate v or v^2 implicitly	
			$2v\frac{\mathrm{d}v}{\mathrm{d}t} = ag\sin\theta\left(\frac{v}{a}\right)$	M1	3.4	Use of $v = r\omega$	
			$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{1}{2}g\sin\theta$	A1	1.1		
			$mg\sin\theta - T = m\left(\frac{1}{2}g\sin\theta\right)$	M1	3.3	Use of N2L tangentially (correct number of terms and component of weight) for Q	
			$T = \frac{1}{2}mg\sin 60 = \frac{\sqrt{3}}{4}mg$	A1 [5]	1.1		

Question			Answer	Marks	AOs		Guidance
7	(iii)		The value of θ would be larger as friction would reduce the speed of Q and allow Q to remain on the surface BC for longer	B1 **[1]**	3.5a		
8	(i)		Area of triangle = $\frac{1}{2}(10)(10)\sin 60$ Height of triangle $= 10\sin 60$ $\left(25\sqrt{3}\right)\left(\frac{5\sqrt{3}}{3}\right)-\left(\pi r^2\right)r=\left(25\sqrt{3}-\pi r^2\right)(2r)$ $125-\pi r^3=50\sqrt{3}r-2\pi r^3$ $\Rightarrow \pi r^3-50\sqrt{3}r+125=0$	B1 B1 M1 A1 A1 **[5]**	1.1 1.1 2.1 1.1 2.2a	 Table of values idea – correct number of terms **AG –** sufficient working must be shown to establish given result	$25\sqrt{3}$ $5\sqrt{3}$
8	(ii)		$r=4.2724...,1.5888...,–5.8612...$ r cannot be negative so $r\neq –5.861...$ If r = 4.2724 then parts of the removed disc are outside the boundary of the triangle which is not possible $\Rightarrow r=1.59$	B1 B1 B1 B1 **[4]**	1.1 1.1 2.3 2.2a	**BC** With reason for why r cannot equal 4.27… Dependent on all previous B marks	Must state all three values for this mark
8	(iii)		$\tan\theta=\frac{2r}{5}$ $\theta=32.4^{\circ}$	M1 A1 **[2]**	3.4 1.1	With their r – allow reciprocal	$2r$ and θ may be seen in a diagram. 32.4382…

Question			Answer	Marks	AOs		Guidance
9	(i)		$\ddot{x}=0$ $\ddot{y}=-g$	**B1** **B1** **[2]**	**1.2** **1.2**		
9	(ii)		$\dot{x}=A \Rightarrow \dot{x}=V\cos\theta$ $x=Vt\cos\theta$ $\dot{y}=-gt+B$ and when t = 0, $\dot{y}=V\sin\theta$ $y=Vt\sin\theta-\frac{1}{2}gt^2$	**M1** **A1** **M1** **A1** **[4]**	**1.1** **1.1** **3.4** **1.1**	Integrating their $\ddot{x}$ with respect to t Integrating their $\ddot{y}$ with respect to t and use initial conditions	For full marks arbitrary constants need to be considered and, where appropriate, shown to be zero
9	(iii)		$t=\frac{x}{V\cos\theta} \Rightarrow y=...$ $y=V\sin\theta\left(\frac{x}{V\cos\theta}\right)-\frac{1}{2}g\left(\frac{x}{V\cos\theta}\right)^2$ $\Rightarrow y=x\tan\theta-\frac{gx^2}{2V^2}\sec^2\theta$	**M1** **A1** **[2]**	**3.4** **1.1**	Re-arrange and eliminate t **AG -** sufficient working must be shown to establish given result	

Question			Answer	Marks	AOs		Guidance
9	(iv)		$2h < 3h\tan\theta - \frac{g(3h)^2}{2(9gh)}(\sec^2\theta)$	M1	3.4	Substitute given results for x, y, V^2	Allow use of equal for M marks
			$2 < 3\tan\theta - \frac{1}{2}(1+\tan^2\theta)$ $\Rightarrow \tan^2\theta - 6\tan\theta + 5 < 0$	M1	3.1a	Use of $\tan^2\theta + 1 = \sec^2\theta$ and re-arrange to obtain 3-term quaratic in tan	
			$(\tan\theta - 1)(\tan\theta - 5) < 0$	M1	1.1	Correct method for solving 3-term quadratic	
			$1 < \tan\theta < 5$	A1	2.2a	$\alpha = 1, \beta = 5$	If equality used then some written justification must be given for inequalities in answer
				[4]			
9	(v)		$x\tan\theta - \frac{gx^2}{18gh}(1+\tan^2\theta) = 0$	M1	3.4	Set $y = 0$	
			Minimum distance when $\tan\theta = 5 \Rightarrow$ $x = \frac{18h(5)}{1+25}$	M1*	3.1b	Sets their $\tan\theta = 5$ (oe) and re-arranges to make x the subject	May use inequality
			Minimum distance from A is $x - 3h$	Dep*M1	1.1		
			$\frac{6h}{13}$	A1	1.1	$k = \frac{6}{13}$	
				[4]			

Question			Answer	Marks	AOs		Guidance
10	(i)			M1	3.3	Conservation of linear momentum (applied parallel to the line of centres)	
			$mu\cos\alpha = mv + mw$	A1	1.1	v is the velocity component of A parallel to line of centres, w is the corresponding component for B	
				M1	3.3	Newton's experimental law (applied parallel to the line of centres)	
			$v - w = -eu\cos\alpha$	A1	1.1		
			$v = \frac{1}{2}u\cos\alpha(1-e)$	M1	3.4	Solve for v	
			$\frac{1}{4}u^2 = \frac{1}{4}u^2\cos^2\alpha(1-e)^2 + u^2\sin^2\alpha$	M1	3.3	Use of the fact that the speed of A is halved in the collision	
			$1+\tan^2\alpha = (1-e)^2 + 4\tan^2\alpha \Rightarrow 1+\frac{8}{27} = (1-e)^2 + \frac{32}{27}$	M1	2.1	Re-arrange to obtain an equation in tan and use given result for $\tan^2\alpha$	
			$(1-e)^2 = \frac{1}{9} \Rightarrow e = \frac{2}{3}$	A1 [8]	2.2a	**AG –** sufficient working must be given to establish given result	
10	(ii)		$v = \frac{1}{6}u\cos\alpha$	B1	1.1		
			$\tan\beta = \dfrac{u\sin\alpha}{\frac{1}{6}u\cos\alpha} = 6\tan\alpha$	M1	3.4	Use of tan = $\dfrac{u\sin\alpha}{v}$	Where β is the angle to the line of centres of A after the collision
			$\tan(\beta-\alpha) = \dfrac{\tan\beta - \tan\alpha}{1+\tan\beta\tan\alpha} = \dfrac{\frac{4\sqrt{6}}{3} - \frac{2\sqrt{6}}{9}}{1+\left(\frac{4\sqrt{6}}{3}\right)\left(\frac{2\sqrt{6}}{9}\right)}$	M1	2.1	Use of compound-angle formula for tan with $\tan\alpha = \sqrt{\frac{8}{27}}$ and their $\tan\beta$	
			$(\beta-\alpha) = \tan^{-1}\left(\frac{2\sqrt{6}}{5}\right)$	A1 [4]	2.2a	$k = \frac{2}{5}$	

Question			Answer	Marks	AOs		Guidance
11	(i)		$\dfrac{\frac{2}{3}mg(2a)}{5a}=\dfrac{\lambda a}{3a}$	M1	3.3	Use of Hooke's law $T=\dfrac{\lambda x}{a}$ with $T_{\text{AP}}=T_{\text{BP}}$	
			$\lambda=\dfrac{4}{5}mg$	A1	1.1		
				M1	3.3	Use of N2L (correct number of terms)	
			$T_B-T_A-\dfrac{2}{5}mn\dfrac{\mathrm{d}x}{\mathrm{d}t}=m\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2}$	A1	1.1	Allow in terms of T_A and T_B	
			$\dfrac{\frac{4}{5}mg(a-x)}{3a}-\dfrac{\frac{2}{3}mg(2a+x)}{5a}-\dfrac{2mn}{5}\dfrac{\mathrm{d}x}{\mathrm{d}t}=m\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2}$	A1ft	1.1	Correct expressions for the extension in T_A and T_B with their λ	
				M1	1.1	Expand and re-arrange to 3-term differential equation	
			$5\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2}+2n\dfrac{\mathrm{d}x}{\mathrm{d}t}+2n^2x=0$	A1	2.2a	**AG**	
				[7]			

Question			Answer	Marks	AOs		Guidance
11	(ii)		$x = ae^{-\frac{nt}{5}}\cos\left(\frac{3nt}{5}\right)$				
			$t = 0 \Rightarrow x = a$	**B1**	**1.1**	Verify that when $t = 0$, $x = a$	
				M1	**1.1**	Attempt to differentiate using the product rule	
			$\frac{\mathrm{d}x}{\mathrm{d}t} = ae^{-\frac{nt}{5}}\left(-\frac{3n}{5}\sin\left(\frac{3nt}{5}\right)\right) - \frac{an}{5}e^{-\frac{nt}{5}}\cos\left(\frac{3nt}{5}\right)$	**A1**	**1.1**		
			$t = 0, \frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{an}{5}$ so speed is $\frac{an}{5}$	**A1**	**1.1**	Verify given speed of P when $t = 0$	
				M1	**1.1**	Attempt to differentiate to find $\ddot{x}$	
			$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \frac{an}{5}e^{-\frac{nt}{5}}\left(\frac{6}{5}n\sin\left(\frac{3nt}{5}\right) - \frac{8}{5}n\cos\left(\frac{3nt}{5}\right)\right)$	**A1**	**1.1**		
			$5\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2n\frac{\mathrm{d}x}{\mathrm{d}t} + 2n^2x$ $= ane^{-\frac{nt}{5}}\left(\frac{6}{5}n\sin\left(\frac{3nt}{5}\right) - \frac{8}{5}n\cos\left(\frac{3nt}{5}\right)\right)$ $-\frac{2an^2}{5}e^{-\frac{nt}{5}}\left(3\sin\left(\frac{3nt}{5}\right) + \cos\left(\frac{3nt}{5}\right)\right)$ $+2n^2e^{-\frac{nt}{5}}\left(a\cos\left(\frac{3nt}{5}\right)\right) = 0$	**A1**	**2.2a**	**AG –** verifying that the given particular solution satisfies the derived differential equation	
				[7]			

Question			Answer	Marks	AOs		Guidance
12	(i)		Distance from line of action of W to the point of contact of the sphere with the plane is $3a\tan\theta$	**B1**	**3.4**		
			Distance of A to point of contact of the sphere with the plane is $4a$	**B1**	**3.4**		
				M1	**3.3**	Taking moments about A	
			$4aR = W(4a + 3a\tan\theta)\cos\theta$	**A1ft**	**1.1**	Correct equation with their $4a + 3a\tan\theta$ oe e.g. $R \times 4a = W(4a\cos\theta + 3a\sin\theta)$ (this would imply the first two B marks)	R is the normal contact force exerted by the plane on the sphere
			$R = \frac{W}{4}(4\cos\theta + 3\sin\theta)$	**A1**	**1.1**	oe	
				[5]			
12	(ii)		$R\sin\theta = T\cos(\alpha - \theta)$	***M1**	**3.3**	Resolving horizontally (T is the tension in the string)	α is the angle between the plane and the string
			$R\sin\theta = T(\cos\alpha\cos\theta + \sin\alpha\sin\theta)$	***M1**	**3.1a**	Expand using compound angle formula	
			$R\sin\theta = \frac{4T}{5}\cos\theta + \frac{3T}{5}\sin\theta$	**A1**	**1.1**		
			$5\sin\theta\left(W\cos\theta + \frac{3}{4}W\sin\theta\right) = T(4\cos\theta + 3\sin\theta)$	***M1**	**3.4**	Substituting their R form part **(i)**	
			$T = \frac{5W\sin\theta}{4}$	**A1**	**1.1**	Correct expression for T (accept unsimplified)	
			$R:T = 4\cos\theta + 3\sin\theta : 5\sin\theta$	**Dep*M1**	**2.5**	Dependent on all previous M marks	
			$R:T = 4\cot\theta + 3 : 5$	**A1**	**2.2a**	**AG** – sufficient working must be shown to establish given result	

Question			Answer	Marks	AOs		Guidance
				[7]			
12	**(iii)**		$4\cos\theta + 3\sin\theta = 0$	**M1**	**3.4**	Setting R = 0	
			$\cos^2\theta - \frac{9}{16}\left(1-\cos^2\theta\right) = 0 \Rightarrow \cos^2\theta = \frac{9}{25}$	**M1**	**3.1a**	Obtaining an equation in cos only	
			$\cos\theta = -\frac{3}{5}$	**A1** **[3]**	**3.2a**	Cao	Must reject positive square root