OCR
Oxford Cambridge and RSA

A Level Further Mathematics B (MEI)

Y421/01 Mechanics Major

Practice Paper – Set 2

Time allowed: 2 hours 15 minutes

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **120**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **12** pages.

Section A (25 marks)

Answer **all** the questions.

1 A child drags a sledge in a straight line along horizontal ground by means of a rope attached to the sledge. The rope makes an angle of 20° with the horizontal and the tension in the rope, T N, is constant. The work done by the tension in moving the sledge a distance of 125 m is 5250 J.

Calculate the value of T. **[3]**

2

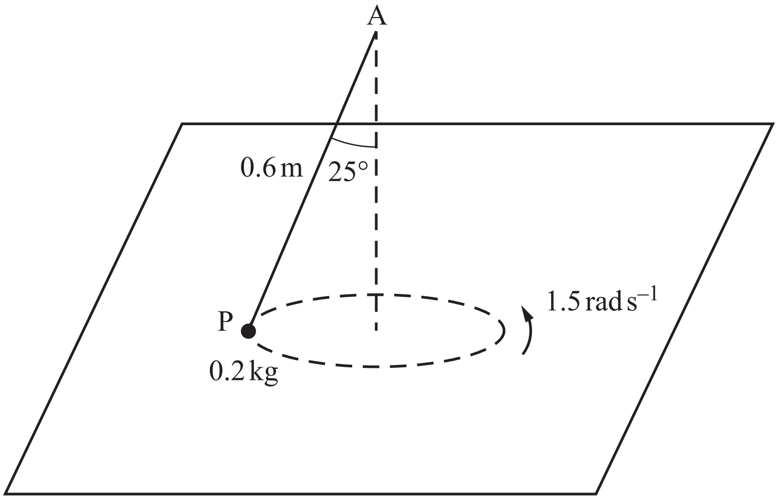

Fig. 2

One end of a light inextensible string of length 0.6 m is attached to a fixed point A which lies above a smooth horizontal table. The other end of the string is attached to a particle P, of mass 0.2 kg, which moves in a horizontal circle on the table with constant angular speed 1.5 rad s^{-1}. AP makes an angle of 25° with the vertical, as shown in Fig. 2.

(i) Calculate the tension in the string. **[3]**

(ii) Calculate the normal contact force between P and the table. **[2]**

3 A particle P of mass m has position vector $\mathbf{r}$ at time t given by

$$\mathbf{r} = (4t^3 - 5t)\mathbf{i} + 2t^2\mathbf{j} \quad \text{for } t \geqslant 0.$$

The motion of P is due to a variable force $\mathbf{F}$ acting in the x-y plane.

(i) Show that, when $t = 0$, the direction of $\mathbf{F}$ is parallel to the y-axis. **[5]**

(ii) Show that, as t increases, the direction of $\mathbf{F}$ approaches that of the x-axis. **[1]**

(iii) (A) Find the value of t for which the direction of $\mathbf{F}$ is equally inclined to the axes. **[2]**

(B) Find, in terms of m, the magnitude of $\mathbf{F}$ at that instant. **[2]**

4 A particle P moves in a straight line through a fixed point O. At time t seconds its displacement from O is x metres and its equation of motion is

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -16x.$$

(i) Write down the general solution of this differential equation. **[1]**

It is given that when $t = \frac{\pi}{16}$, $x = 3\sqrt{2}$ and P is instantaneously at rest.

(ii) Find the total distance moved by P in the interval $t = \frac{\pi}{16}$ to $t = \frac{\pi}{4}$. **[6]**

Section B (95 marks)

Answer **all** the questions.

5 A light elastic string L with natural length 1.2 m and modulus of elasticity 15 N, has its ends attached a distance 1.5 m apart on the same horizontal level. A particle of mass 0.5 kg is attached to the midpoint of the string.

A student investigates the period of the particle when it makes small oscillations in a vertical line about its equilibrium position. The student records the time for one complete oscillation of the particle to be 0.3 seconds.

Based on this investigation the student suggests the following formula for the period of oscillations T for a particle of mass m, attached to the midpoint of an elastic string of natural length a, modulus of elasticity λ and k is a positive constant.

$$T = km^{\alpha}a^{\beta}\lambda^{\gamma}$$

Find the period when a particle of mass 0.7 kg makes small oscillations in a vertical line centred on the position of equilibrium when attached to the midpoint of L. **[6]**

6 **In this question you must show detailed reasoning.**

The region bounded by the x-axis, the line $x = \frac{\pi}{4}$, the line $x = \frac{\pi}{2}$, and the curve $y = \sin x$ for $\frac{\pi}{4} \leqslant x \leqslant \frac{\pi}{2}$, is occupied by a uniform lamina. Find, in an exact form, the coordinates of the centre of mass of this lamina. **[9]**

7

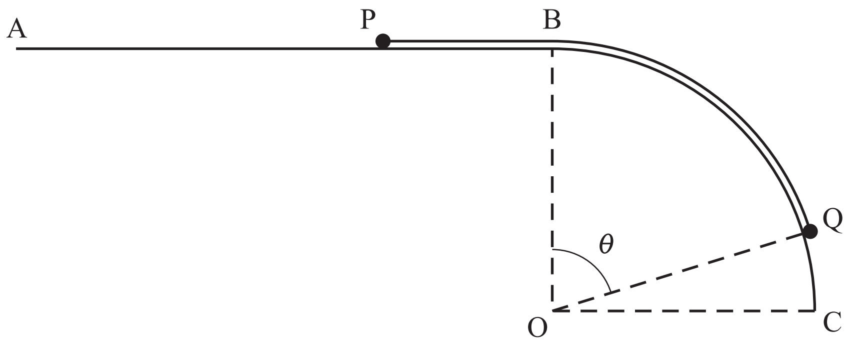

Fig. 7

Fig. 7 shows a smooth surface ABC which is horizontal between A and B and in the form of a quarter circle between B and C. The radius of the quarter circle is a and its centre O is vertically below B. The distance from A to B is greater than the length of the arc BC.

Two particles P and Q, each of mass m, are connected by a light inextensible string. Initially P and Q are at A and B respectively, with the string taut. The particle Q is then slightly disturbed so that it slides down the circular part of the surface with P travelling along AB. While Q remains in contact with the circular part of the surface, OQ makes an angle θ with the upward vertical.

(i) Determine the value of θ when Q loses contact with the circular part of the surface. **[6]**

(ii) Hence find, in terms of m and g, the tension in the string at the instant when Q loses contact with the circular part of the surface. **[5]**

(iii) State whether you would expect the value of θ found in part **(i)** to be larger, smaller or the same if the surface was rough. Give a reason for your answer. **[1]**

8

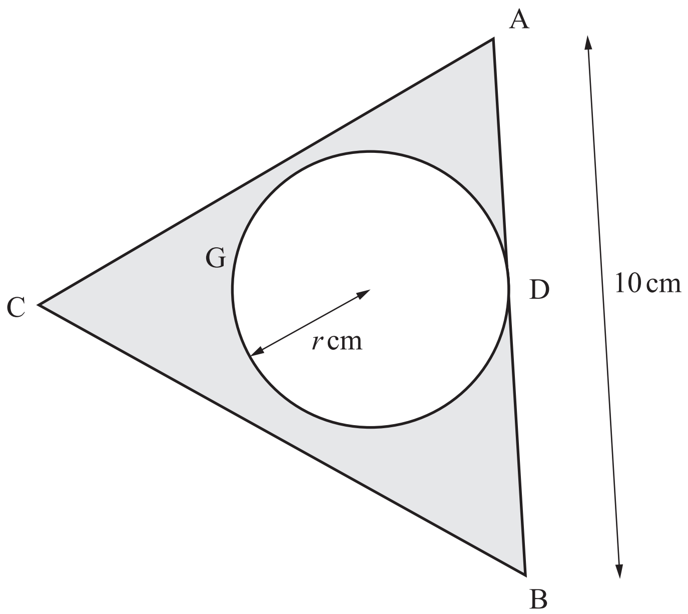

Fig. 8

A uniform equilateral triangular lamina ABC, of side 10 cm, has a disc of radius r cm removed from it leaving the lamina shown in Fig. 8. Point D is the midpoint of AB and G is the centre of mass of the resulting lamina; DG is a diameter of the circular edge of the resulting lamina when the disc has been removed.

(i) Show that r satisfies the equation

$$\pi r^3 - 50\sqrt{3}r + 125 = 0.$$ **[5]**

(ii) Find the roots of this equation and hence determine the value of r. **[4]**

The lamina is freely suspended from A and hangs in equilibrium with AB inclined at an angle θ to the downward vertical through A.

(iii) Find the value of θ correct to three significant figures. **[2]**

9 A particle P moves freely under gravity in the plane of a fixed horizontal axis Ox and a fixed upward vertical axis Oy. At time t the coordinates of P are (x, y).

(i) Write down the values of $\ddot{x}$ and $\ddot{y}$. **[2]**

The particle is projected from O, at time $t = 0$, with speed V at an angle θ to Ox.

(ii) Find by integration the values of x and y at time t. **[4]**

(iii) Deduce that $y = x\tan\theta - \frac{gx^2}{2V^2}\sec^2\theta$. **[2]**

The point O is on horizontal ground and AB is a vertical wall of height $2h$ whose foot A is at the point $(3h, 0)$. P moves towards the wall and $V^2 = 9gh$.

(iv) Show that P will pass over the wall provided

$$\alpha < \tan\theta < \beta,$$

where α and β are integers to be determined. **[4]**

(v) Given that the inequality from part **(iv)** holds, show that P hits the ground at a minimum distance of kh from A, where k is a rational number to be determined. **[4]**

10 Two uniform smooth spheres A and B have equal radii and equal masses. The spheres are on a smooth horizontal surface. Sphere A is moving with speed u at an angle α to the line of centres, where $\tan^2\alpha = \frac{8}{27}$, when it collides with B, which is stationary. The collision causes the speed of A to be halved.

(i) Show that the coefficient of restitution between A and B is $\frac{2}{3}$. **[8]**

(ii) Show that the direction of motion of A is turned through the angle $\tan^{-1}\left(k\sqrt{6}\right)$ where k is a rational number to be determined. **[4]**

11

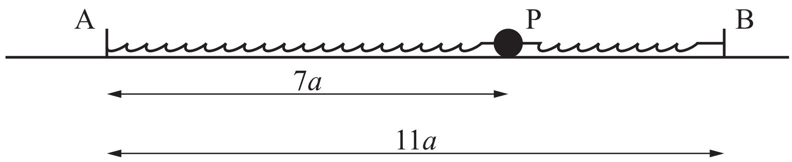

Fig. 11

A and B are two fixed points that are a distance $11a$ apart on a smooth horizontal plane. A light elastic spring of natural length $5a$ and modulus of elasticity $\frac{2}{3}mg$ has one end fixed to A and the other end fixed to a particle P of mass m. Another light elastic spring of natural length $3a$ has one end fixed to B and the other end fixed to P. P is in equilibrium at a distance $7a$ from A, as shown in Fig. 11.

At time $t = 0$, P is moved a distance a from its equilibrium position towards B and released with a speed of $\frac{na}{5}$, where n is a positive constant. The subsequent motion of P is subject to a resistive force of magnitude $\frac{2}{5}mn$ times its speed, where $n^2 = \frac{g}{a}$. In the subsequent motion the displacement of P towards B from its equilibrium position at time t is denoted by x.

(i) Show that the motion of P satisfies the differential equation

$$5\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2n\frac{\mathrm{d}x}{\mathrm{d}t} + 2n^2x = 0.$$

[7]

(ii) Verify that $x = a\mathrm{e}^{-\frac{nt}{5}}\cos\left(\frac{3nt}{5}\right)$. **[7]**

12

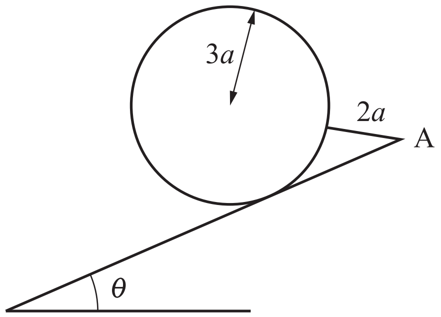

Fig. 12

A uniform smooth sphere, of weight W and radius $3a$, rests on a smooth plane inclined at an angle θ to the horizontal. The sphere is held in equilibrium by a light inextensible string of length $2a$ joining a point of its surface to a point A on the plane, as shown in Fig. 12.

(i) Show that the normal contact force exerted by the plane on the sphere is $\frac{W}{4}(4\cos\theta + 3\sin\theta)$. **[5]**

(ii) Show that the ratio of the normal contact force exerted by the plane on the sphere to the tension in the string is $(4\cot\theta + 3) : 5$. **[7]**

The angle θ is slowly increased.

(iii) Find the exact value of $\cos\theta$ for which the sphere loses contact with the plane. **[3]**

END OF QUESTION PAPER

BLANK PAGE

BLANK PAGE