OCR
Oxford Cambridge and RSA

Practice Paper – Set 3

A Level Further Mathematics B (MEI)
Y421 Mechanics Major

Practice Paper - Set 3

MARK SCHEME

Duration: 2 hours 15 minutes

MAXIMUM MARK 120

This document consists of 17 pages

Text Instructions

1. **Annotations and abbreviations**

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs	Guidance	
1	(a)		$\begin{pmatrix}-30\\20\\15\end{pmatrix}+\begin{pmatrix}10\\-10\\-18\end{pmatrix}+5\begin{pmatrix}0\\0\\-9.8\end{pmatrix}=5\mathbf{a}$	M1	3.3	Use of $\mathbf{F}=m\mathbf{a}$ with at least two terms of **F** correct	
				A1	1.1	Correct use of $\mathbf{F}=m\mathbf{a}$	
			$\mathbf{a}=\begin{pmatrix}-4\\2\\-10.4\end{pmatrix}$	A1	1.1		
				[3]			
1	(b)		$\mathbf{s}=\frac{1}{2}\begin{pmatrix}-4\\2\\-10.4\end{pmatrix}(5)^2$	M1	3.3	Use of $\mathbf{s}=\mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$ with $\mathbf{u}=0$	
				A1ft	1.1	Correct displacement following through candidate's **a** from (a)	
			Position vector $=\begin{pmatrix}2\\-5\\3\end{pmatrix}+\begin{pmatrix}-50\\25\\-130\end{pmatrix}=\begin{pmatrix}-48\\20\\-127\end{pmatrix}$	A1	1.1		
				[3]			
2	(a)		CLM: $4(2)+3(1.5)=3.5v_{AB}$	M1	3.3	Conservation of linear momentum (correct number of terms)	
			$v_{AB}=\frac{25}{7}$ m s^{-1}	A1	1.1		3.571428…
				[2]			

Question			Answer	Marks	AOs	Guidance	
2	(b)		$\frac{1}{2}(2)(4)^2+\frac{1}{2}\left(\frac{3}{2}\right)(3)^2-\frac{1}{2}\left(\frac{7}{2}\right)\left(\frac{25}{7}\right)^2$	M1	1.1a	Change in kinetic energy (correct masses and use of their v_{AB})	
			KE lost = $\frac{3}{7}$ J	A1	1.1	Do not accept negative answer, must be in context of the energy lost	0.4285714…
				[2]			
2	(c)		$J=3.5\left(0.84\left(\frac{25}{7}\right)-\left(-\frac{25}{7}\right)\right)$	M1 A1ft	3.3 1.1	Use of $J=mv-mu$ Correct application of Impulse = change in momentum with correct m and candidate's value for v_{AB}	
			$J=23\,\text{N s}$	A1	1.1		
				[3]			
3	(a)		$[F]=\text{MLT}^{-2}$	B1	1.2		
			$[G]=\frac{[F][r^2]}{[m_1][m_2]}=\frac{\text{MLT}^{-2}\text{L}^2}{\text{M}^2}$	M1	2.1	Re-arrange to mark G the subject and substitute correctly	
			$[G]=\text{M}^{-1}\text{L}^3\text{T}^{-2}$	A1	2.2a	**AG** – sufficient working must be shown as answer given	
				[3]			
3	(b)		$t=kG^\alpha M^\beta R^\gamma \Rightarrow \text{T}=\left(\text{M}^{-1}\text{L}^3\text{T}^{-2}\right)^\alpha \text{M}^\beta\text{L}^\gamma$	M1	1.1	Using $[G]$ to obtain an equation in T, M and L	
			T: $1=-2\alpha\Rightarrow\alpha=-\frac{1}{2}$	A1	1.1		
			M: $0=-\alpha+\beta\Rightarrow\beta=-\frac{1}{2}$	A1	1.1		
			L: $0=3\alpha+\gamma\Rightarrow\gamma=\frac{3}{2}$	A1	1.1		
				[4]			

Question			Answer	Marks	AOs	Guidance	
4			Sliding: resolving horizontally gives $P = 0.3R$	M1	3.1b	Allow this mark for equivalent statement e.g. $P = F$ and use of $F = \mu R$	
			resolving vertically gives $R = W$	B1	1.1		
			$\Rightarrow P = 0.3W$	A1	1.1		
			Toppling: taking moments about bottom edge	M1	3.1b	Two terms both must of the correct form (e.g. force × distance)	
			$2aP = aW \Rightarrow P = 0.5W$	A1	1.1		
			$0.3W < 0.5W$ so sliding occurs first	A1	2.2a		
				[6]			
5	(a)		$A = k\int_0^1 x^3 \mathrm{d}x = ...$	M1	1.1a	Correct integral with power increased by 1	Limits not required for M marks in (a)
			$= k\left[\frac{x^4}{4}\right]_0^1 = \frac{k}{4}$	A1	1.1		
			$A\bar{x} = k\int_0^1 x^4 \mathrm{d}x = ...$	M1	1.1a	Correct integral with power increased by 1	
			$= k\left[\frac{x^5}{5}\right]_0^1 = \frac{k}{5}$	A1	1.1		
			$\bar{x} = \frac{k/5}{k/4} = \frac{4}{5}$	A1	1.1		
				[5]			

Question			Answer	Marks	AOs	Guidance	
5	(b)		$A\bar{y} = \frac{k^2}{2}\int_0^1 x^6 \mathrm{d}x = \ldots$	M1*	1.1	Correct integral $\left(\frac{1}{2}\int y^2 \mathrm{d}x\right)$ with power increased by 1	
			$A\bar{y} = \frac{k^2}{2}\left[\frac{x^7}{7}\right]_0^1 = \frac{k^2}{14}$	A1	1.1		
			$\frac{k}{4}\left(\frac{4}{5}\right) = \frac{k^2}{14}$	Dep*M1	3.1a	Substiuting A to form an equation in k only (this mark is dependent on all previous method marks in (a) and (b))	
			$k = \frac{14}{5}$	A1	1.1		
				[4]			
6	(a)		Driving force = $\frac{1000P}{v}$	B1	1.2	Or for $50P$ seen	
				M1	3.3	Resolve forces parallel to the plane (correct number of terms) – allow D (oe) for driving force – weight must be resolved	
			$\frac{1000P}{20} = 20k + 250g\sin\theta$	A1	3.4	Correct substitution of driving force	
			$50P = 20k + \frac{245}{2} \Rightarrow 20P = 8k + 49$	A1	1.1	**AG –** as given result sufficient working must be shown	
				[4]			

Question			Answer	Marks	AOs	Guidance	
6	(b)		Driving force = $\frac{1000P}{25}$ and resitive force = $25k$	B1	1.1		
			$\frac{1000P}{25} + 250g\sin\theta = 25k$	M1	3.3	Resolving forces parallel to the plane (correct number of terms)	
			$16P + 49 = 10k$	A1	1.1	Or equivalent	
				[3]			
6	(c)		$k = 24.5, P = 12.25$	B1	1.1	**BC**	
				[1]			
6	(d)		$kv = \frac{1000P}{v}$	M1	3.1b	Resistive force equal to driving force	
			$v = 22.4\,\text{m s}^{-1}$	A1	1.1	$\sqrt{500}$	22.36067…
				[2]			

Question			Answer	Marks	AOs	Guidance	
7			Resolving vertically for Q: $T = mg$	**B1**	**1.1**	Where T is the tension in the string and m is the mass of Q	
			Resolving vertically for P	**M1**	**3.3**	Correct number of terms with components of both the tension and normal contact forces	
			$R\sin 60 = T\cos 60 + mg$	**A1**	**1.1**	Where R is the normal contact force	Note that $R = \sqrt{3}mg$
			Resolving horizontally for P	**M1**	**3.3**	Correct number of terms – allow a for acceleration	
			$R\cos 60 + T\sin 60 = \frac{mv^2}{r}$	**A1**	**1.1**	Allow $r\omega^2$ for a	
			$r = h\tan 60$	**B1**	**1.1**		
			$\frac{\sqrt{3}}{2}mg + \frac{\sqrt{3}}{2}mg = \frac{mv^2}{h\tan 60}$	**M1**	**3.4**	Eliminate R and T to form an equation in the required variables (allow m throughout)	
			$\sqrt{3}g = \frac{v^2}{h\sqrt{3}} \Rightarrow v^2 = 3gh$	**A1**	**2.2a**	$k = 3$	
				[8]			
8	**(a)**			**M1**	**3.1b**	Attempt to differentiate **r** with respect to t	
			$\mathbf{v} = -a\sin t\mathbf{i} + b\cos t\mathbf{j}$	**A1**	**1.1**		
			$t = 0, \mathbf{r} = \begin{pmatrix} a \\ 0 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 0 \\ b \end{pmatrix}$	**M1**	**3.4**	Substitute $t = 0$ into both **r** and their **v**	
			$\mathbf{r}\cdot\mathbf{v} = \begin{pmatrix} a \\ 0 \end{pmatrix}\cdot\begin{pmatrix} 0 \\ b \end{pmatrix} = a(0) + 0(b) = 0 \Rightarrow \mathbf{v}$ is perp. to **r**	**A1**	**2.2a**		
				[4]			

Question			Answer	Marks	AOs	Guidance	
8	(b)		$\mathbf{r}\cdot\mathbf{v} = -a^2\sin t\cos t + b^2\sin t\cos t$	M1	3.4	Calculate scalar product of **r** and **v**	
			$\mathbf{r}\cdot\mathbf{v} = 0 \Rightarrow \sin t\cos t = 0$ Q $a^2 \neq b^2$	M1	2.1	Set scalar product equal to zero and obtain equation in t only	
			$\frac{1}{2}\sin 2t = 0 \Rightarrow t = \frac{\pi}{2}$	A1	1.1		
				[3]			
8	(c)		$\mathbf{a} = -a\cos t\mathbf{i} - b\sin t\mathbf{j}$	M1	1.1	Attempt to differentiate their **v** with respect to t	
			$\mathbf{a} = -\mathbf{r}$	A1	1.1		
			Therefore the direction is always towards the origin	B1	2.4		
				[3]			
9	(a)		$T_0 = mg$ and $T_0 = \frac{2mge}{a}$	M1	3.3	Resolving vertically and attempt at Hooke's law	e is the extension of the string at the equilibrium position
			$\frac{2mge}{a} = mg \Rightarrow e = \frac{1}{2}a$	A1	1.1	Correct expression for the extension of the string at its equilibrium position	
			$mg - T = m\ddot{x}$	M1	3.3	N,II applied at distance x from equilibrium position – allow T for tension	
			$mg - \frac{2mg}{a}\left(\frac{1}{2}a + x\right) = m\ddot{x}$	A1ft	3.4	Correct equation with their extension	
			$mg - mg - \frac{2mgx}{a} = m\ddot{x} \Rightarrow \ddot{x} = -\frac{2gx}{a}$	A1	2.2a	**AG** – sufficient working must be shown as answer given	
				[5]			

Question			Answer	Marks	AOs	Guidance	
9	(b)		The differential equation in part **(a)** is valid provided that the string is still taut (so the distance OP cannot be less than a)	**B1** **[1]**	**3.5b**	oe e.g. DE is valid for $-0.5a < x < 0.5a$	
9	(c)		At a distance b below A: GPE = 0, KE = 0 and EPE = $\frac{2mg}{2a}\left(b+\frac{a}{2}\right)^2$	**M1**	**3.3**	Attempt to apply $\frac{\lambda e^2}{2a}$ with their e	Reference level of GPE is taken to be at a distance b below A
			At O, KE = EPE = 0, GPE = $mg\left(b+\frac{3}{2}a\right)$	**M1**	**1.1**	Attempt to find GPE at O	
			$\frac{mg}{a}\left(b+\frac{a}{2}\right)^2 = mg\left(b+\frac{3}{2}a\right)$	**M1**	**2.1**	Attempt to apply principle of conservation of energy – dependent on both previous M marks	
			$b^2+ba+\frac{a^2}{4} = ba+\frac{3}{2}a^2 \Rightarrow 4b^2 = 5a^2$	**A1**	**1.1**	A correct simplified equation relating a and b	
			$\ddot{x} = -\frac{2g}{a}x \Rightarrow x = A\cos\sqrt{\frac{2g}{a}}t + B\sin\sqrt{\frac{2g}{a}}t$	**B1**	**1.2**		
			$t=0, x=b \Rightarrow A=b$	**B1**	**3.4**		
			$\ddot{x} = 0, t=0 \Rightarrow B=0$	**M1**	**3.4**	Attempt to find B by using initial conditions	$x = b\cos\sqrt{\frac{2g}{a}}t$
			$x = -\frac{1}{2}a \Rightarrow -\frac{1}{2}a = b\cos\sqrt{\frac{2g}{a}}t_1 \Rightarrow t_1 = \ldots$	**M1**	**3.4**	Attempt to find the time while the string is taut	$\cos^2\sqrt{\frac{2g}{a}}t_1 = \frac{1}{5}$ $\Rightarrow t_1 = \sqrt{\frac{a}{2g}}\arccos\left(-\frac{1}{\sqrt{5}}\right)$

Question			Answer	Marks	AOs	Guidance	
			$-a = -\frac{1}{2}gt_2^{\,2} \Rightarrow t_2 = ...$	**M1**	**3.1b**	Using SUVAT equations to find the time when string is slack	
			$t = t_1 + t_2 = \left(2 + \arccos\left(-\frac{1}{\sqrt{5}}\right)\right)\sqrt{\frac{a}{2g}}$	**A1**	**1.1**	oe e.g. $\left(2 + \frac{\pi}{2} + \arcsin\left(\frac{1}{\sqrt{5}}\right)\right)\sqrt{\frac{a}{2g}}$	
				[10]			
10	**(a)**		Let u_1 and u_2 be the parallel and perpendicular components to the line of centres of the velocity of A before collision. Similarly let v_1 and v_2 be the parallel and perpendicular components of the velocity of A after collision. Let w_1 be the parallel component of the velocity of B after collision.	**B1**	**2.5**	Defining suitable variables – accept if shown on a diagram(s)	
			$mu_1 = mv_1 + mw_1$	**B1**	**3.3**	Correct application of the conservation of linear momentum	Where *m* is the mass of both spheres
			$v_1 - w_1 = -\frac{1}{2}u_1$	**M1**	**3.3**	Attempt at use of Newton's experimental law (applied parallel to the line of centres)	
			$v_1 = \frac{1}{4}u_1$ and $v_2 = u_2$	**A1**	**1.1**	Solve for v_1 and that $v_2 = u_2$ (soi)	
			$\tan\alpha = \frac{u_2}{u_1}$ and let $\tan\gamma = \frac{v_2}{v_1}$	**M1**	**1.1**	Attempt to define α in terms of the velocity components of A and the angle that A is moving at after collision	

Question			Answer	Marks	AOs	Guidance	
			$\tan\beta = \tan(\gamma-\alpha) = \dfrac{\frac{4u_2}{u_1} - \frac{u_2}{u_1}}{1+\frac{4u_2^{\,2}}{u_1^{\,2}}}$	**M1**	**2.1**	Attempt to find $\tan\beta$ using correct compound angle formula for $\tan(A-B)$ in terms of u_1 and u_2	
			$\tan\beta = \dfrac{3\tan\alpha}{1+4\tan^2\alpha}$	**M1**	**3.4**	Deriving an expression for $\tan\beta$ in terms of $\tan\alpha$ - dependent on all previous M marks	
			$\cot\beta = \dfrac{1+4\tan^2\alpha}{3\tan\alpha} = \dfrac{1}{3t}+\dfrac{4t}{3}$	**A1**	**2.2a**	**AG** – as answer given sufficient working must be shown for this mark to be awarded	
				[8]			
10	**(b)**		$\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{1}{3t}+\dfrac{4t}{3}\right)=0$	**M1***	**3.1a**	Attempt to differentiate expression in terms of t (with at least one term correct) and equate to zero	
			$-\dfrac{1}{3t^2}+\dfrac{4}{3}=0 \Rightarrow t=\dfrac{1}{2}$	**Dep*M1**	**1.1**	Solve for t	
			$\cot\beta=\dfrac{4}{3}\Rightarrow\beta=36.869...$ so maximum value of β is 36.9°	**A1**	**2.2b**		
				[3]			

Question			Answer	Marks	AOs	Guidance	
11	(a)		At the bottom of the circle, PE = 0, KE = $\frac{1}{2}mu^2$	B1	1.1		
			Top of the circle, PE = $mg(2a)$, KE = $\frac{1}{2}mk^2u^2$	B1	1.1		
			$u^2 = \frac{4ga}{1-k^2}$	M1*	3.3	Use of conservation of energy to find an expression for u^2 in terms of *a*, *g* and *k*	
			At angle θ: PE = $mg(a - a\cos\theta)$, KE = $\frac{1}{2}mv^2$	B1	1.1		
			$\frac{1}{2}m\left(\frac{4ga}{1-k^2}\right) = \frac{1}{2}mv^2 + mga(1-\cos\theta)$	Dep*M1	2.1	Use of conservation of energy and substitute of their expression for u^2 - dependent on previous M mark	
			$T - mg\cos\theta = \frac{mv^2}{a}$	M1	3.3	Attempt at applying N, II radially – allow $a\omega^2$	
			$T - mg\cos\theta = \frac{m}{a}\left(\frac{4ga}{1-k^2} - 2ga(1-\cos\theta)\right)$	M1	3.4	Substituting of their expression for v^2 into their N,II – dependent on all previous M marks	
			$T = \left(\frac{2(1+k^2)}{1-k^2} + 3\cos\theta\right)mg$	A1	2.2a	**AG** – as answer given sufficient working must be shown	
				[8]			

Question			Answer	Marks	AOs	Guidance	
11	(b)		Maximum tension is when $\theta = 0$	B1	3.1b		
			$\left(\frac{2\left(1+k^2\right)}{1-k^2}+3\right)mg = 9mg \Rightarrow k^2 = ...$	M1	2.1	Set up an equation/inequality with $9mg$ and attempt to simplify	
			$k \leq \frac{1}{\sqrt{2}}$	A1	2.2a	Allow equality for this mark	
			For particle to complete vetical circle $T \geq 0$ when $\theta = 180$	B1	2.3	Consideration of tension at the highest point	
			$\left(\frac{2\left(1+k^2\right)}{1-k^2}-3\right)mg = 0 \Rightarrow k^2 = ...$	M1	2.1	Set up an equation/inequality with $T = 0$ and attempt to simplify	
			$k \geq \frac{1}{\sqrt{5}}$	A1	3.2a	All correct ranges stated for k (with no extras or inclusion of negative values of k)	Or for $k = \frac{1}{\sqrt{5}}$ if first A mark not awarded
				[6]			
12	(a)	(*i*)	Horizontal distance from P to centre of the rod $= l\cos\theta - a$ or AP $= a\sec\theta$	B1	1.1	oe – could be implied by later working	
				M1	3.1b	Moments about P – correct number of terms	
			$W\left(l\cos\theta - a\right) = R_A\left(a\tan\theta\right)$	A1ft	1.1	Correct equation with their horizontal distance or oe e.g. $\left(l - a\sec\theta\right)W\cos\theta = R_A\sin\theta\left(a\sec\theta\right)$	
			$R_A = \frac{W\left(l\cos\theta - a\right)}{a\tan\theta}$	A1	1.1	oe e.g. $R_A = \frac{W\left(l - a\sec\theta\right)\cos\theta}{a\tan\theta}$	
				[4]			

Question			Answer	Marks	AOs	Guidance	
12	(a)	(ii)	$R_{\text{P}}\left(\frac{a}{\cos\theta}\right) = Wl\cos\theta$ $R_{\text{P}} = \frac{Wl\cos^2\theta}{a}$	M1 A1 [2]	3.1b 1.1	Moments about A – correct number of terms	
12	(b)		$R_{\text{A}} \geq 0$ $l\cos\theta - a \geq 0 \Rightarrow a \leq l\cos\theta$	M1 A1 [2]	3.1b 2.2a	Setting their expression for R_{A} greater than or equal to zero **AG** – as answer given sufficient working must be shown	

Question			Answer	Marks	AOs	Guidance	
12	(c)			M1*	3.1b	Attempt to resolve horizontally	
			Either: $F_{\text{P}}\cos\theta + R_{\text{A}} = R_{\text{P}}\sin\theta$	A1	1.1		In this first case the frictional force is acting in the direction from P to B
			$F_{\text{P}}\cos\theta + \dfrac{W\left(l\cos\theta - a\right)}{a\tan\theta} = \dfrac{lW\cos^2\theta\sin\theta}{a}$	Dep*M1	3.4	Substitute their expressions for R_{A} and R_{P}	
			$F_{\text{P}} = \dfrac{W}{a\sin\theta}\left(a - l\cos^3\theta\right)$	A1	1.1		
			$\dfrac{W}{a\sin\theta}\left(a - l\cos^3\theta\right) \le \mu\left(\dfrac{lW\cos^2\theta}{a}\right)$	M1	3.3	Attempt to use $F_{\text{P}} \le \mu R_{\text{P}}$ - dependent on both previous M marks	
			$a \le l\cos^2\theta\left(\cos\theta + \mu\sin\theta\right)$	A1	1.1	**AG** - fully correct working for either inequality	The first six marks are for deriving either inequality
			Or: $-F_{\text{P}}\cos\theta + R_{\text{A}} = R_{\text{P}}\sin\theta$	A1	1.1		In this second case the frictional force is acting in the direction from P to A
			$\Rightarrow a \ge l\cos^2\theta\left(\cos\theta - \mu\sin\theta\right)$	A1	2.2a	**AG** - either full working or convincing argument for the second inequality	
				[8]			