OCR

Oxford Cambridge and RSA

A Level Further Mathematics B (MEI)

Y421/01 Mechanics Major

Practice Paper – Set 3

Time allowed: 2 hours 15 minutes

OCR supplied materials:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You must have:
- Scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **120**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **8** pages.

Section A (35 marks)

Answer **all** the questions.

1 In this question the acceleration due to gravity is $\mathbf{g} = \begin{pmatrix} 0 \\ 0 \\ -9.8 \end{pmatrix} \mathrm{m\,s^{-2}}$.

A particle P of mass 5 kg, initially at rest at the point with position vector $\begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}$ m, is acted on by gravity and also by two other forces, $\begin{pmatrix} -30 \\ 20 \\ 15 \end{pmatrix}$ N and $\begin{pmatrix} 10 \\ -10 \\ -18 \end{pmatrix}$ N.

(a) Find the acceleration vector of P. **[3]**

(b) Find the position vector of P after 5 seconds. **[3]**

2 The masses of two particles A and B are 2 kg and 1.5 kg respectively. The particles are moving in the same direction along a straight line on a smooth horizontal surface. Particle A is moving with speed $4\,\mathrm{m\,s^{-1}}$ and particle B with speed $3\,\mathrm{m\,s^{-1}}$. The two particles collide and coalesce to form particle C. Calculate

(a) the speed of C after the collision, **[2]**

(b) the kinetic energy lost during the collision. **[2]**

Particle C subsequently collides with a vertical wall which is perpendicular to its direction of motion. The coefficient of restitution between C and the wall is 0.84.

(c) Find the magnitude of the impulse that the wall exerts on particle C. **[3]**

3 The force F of gravitational attraction between two objects with masses m_1 and m_2, at a distance r apart, is given by

$$F = \frac{Gm_1 m_2}{r^2},$$

where G is the universal gravitational constant.

(a) Show that the dimensions of G are $\mathrm{M}^{-1}\mathrm{L}^{3}\mathrm{T}^{-2}$. **[3]**

For a satellite in circular orbit of radius R around a planet of mass M, the time, t, taken to complete one orbit is given by

$$t = kG^{\alpha}M^{\beta}R^{\gamma},$$

where k is a dimensionless constant.

(b) Use dimensional analysis to find α, β and γ. **[4]**

4

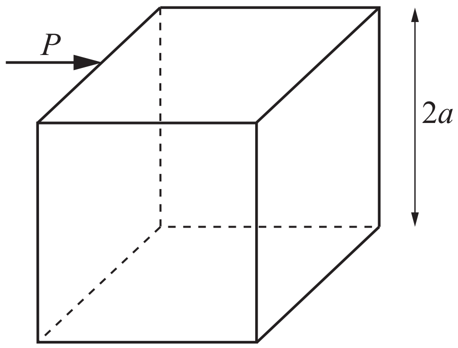

Fig. 4

A uniform solid cube of edge $2a$ rests in contact with a rough horizontal plane. The coefficient of friction between the cube and the plane is 0.3. A horizontal force P is applied to the midpoint of one of the top edges of the cube, perpendicular to that edge, as shown in Fig. 4. The magnitude of P is slowly increased.

Determine whether the equilibrium of the cube will be broken by sliding or toppling. **[6]**

5 The region bounded by the x-axis, the line $x = 1$, and the curve $y = kx^3$, where k is a positive constant, is occupied by a uniform lamina.

(a) Find exactly the x-coordinate of the centre of mass of the lamina. **[5]**

(b) Given that the x- and y-coordinates of the centre of mass of the lamina are equal, find the value of k. **[4]**

Section B (85 marks)

Answer **all** the questions.

6 The total mass of a motorcyclist and his motorcycle is 250 kg.

- The maximum power developed by the motorcycle is P kW.
- When the speed of the motorcycle is $v\,\mathrm{m\,s^{-1}}$, the resistance force to the motion of the motorcycle is kv N, where k is a constant.
- At maximum power the motorcycle has a steady speed of $20\,\mathrm{m\,s^{-1}}$ when ascending a hill inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{20}$.

(a) Show that $20P = 8k + 49$. **[4]**

The maximum steady speed of the motorcycle down the same hill is $25\,\mathrm{m\,s^{-1}}$.

(b) Obtain a second equation relating k and P. **[3]**

(c) Hence find the values of k and P. **[1]**

(d) Calculate the maximum steady speed of the motorcycle along a horizontal road. **[2]**

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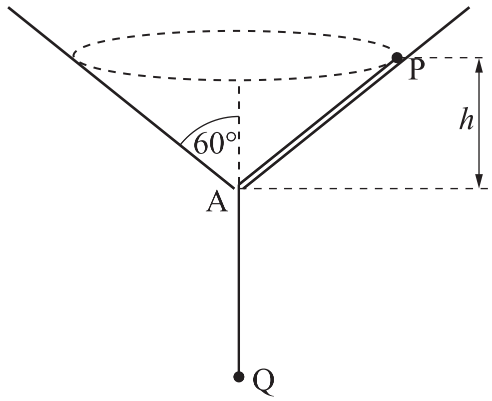

Fig. 7

A smooth hollow cone of semi-vertical angle 60° is fixed with its axis vertical and its vertex A at its lowest point. A light inextensible string passes through a small smooth hole at A and particles of equal mass are attached to the two ends of the string. The lower particle Q hangs at rest in equilibrium while the upper particle P moves with constant speed v in a horizontal circle on the inner surface of the cone. The circle is at a height h above A, as shown in Fig. 7.

Show that $v^2 = kgh$, where k is a constant to be determined. **[8]**

8 A particle P moves so that its position vector $\mathbf{r}$ at time t is given by

$$\mathbf{r} = a\cos t\,\mathbf{i} + b\sin t\,\mathbf{j},$$

where a and b are non-zero constants and $a \neq b$.

(a) Show that initially the velocity vector, $\mathbf{v}$, of P is perpendicular to $\mathbf{r}$. **[4]**

(b) Find the time that elapses until $\mathbf{v}$ is next perpendicular to $\mathbf{r}$. **[3]**

(c) What can be deduced about the direction of the acceleration of P? **[3]**

9 A particle P of mass m is attached to one end of a light elastic string of modulus of elasticity $2mg$ and natural length a. The other end of the string is attached to a fixed point O; P rests in equilibrium at a point A with the string vertical.

P is given a velocity vertically downwards from A and moves so that at time t its downward displacement from A is x.

(a) Show that $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{2gx}{a}$. **[5]**

(b) Explain for which parts of the motion of P the differential equation found in part **(a)** is valid. **[1]**

Subsequently P is released from rest from a point B which is at a vertical distance b below A. In the subsequent motion P comes to instantaneous rest at O.

(c) Find, in terms of a and g, the time taken for P to reach O from B. **[10]**

10 Two uniform smooth spheres A and B have equal radii and equal masses. The spheres are on a smooth horizontal surface. Sphere A is moving at an acute angle α to the line of centres, when it collides with B, which is stationary. The coefficient of restitution between A and B is $\frac{1}{2}$.

(a) Show that the direction of motion of A is turned by the impact through an angle β given by

$$\cot\beta = \frac{1}{3t} + \frac{4t}{3},$$

where $t = \tan\alpha$. **[8]**

(b) Find, as α varies, the maximum value of β correct to 3 significant figures. You do not need to justify that this value is a maximum. **[3]**

11 A particle of mass m is attached to one end A of a light inextensible string and the other end of the string is attached to a fixed point O. The particle moves in a vertical circle of centre O, and its speeds at the lowest and highest points of the circle are u and ku respectively, where k is a constant.

(a) Show that, when OA makes an angle θ with the downward vertical, the tension in the string is given by

$$\left(\frac{2(1+k^2)}{1-k^2} + 3\cos\theta\right)mg.$$ **[8]**

The string will break if the tension exceeds $9mg$.

(b) Given that the string does not break and that the particle is moving in a complete vertical circle, find the range of possible values of k. **[6]**

12

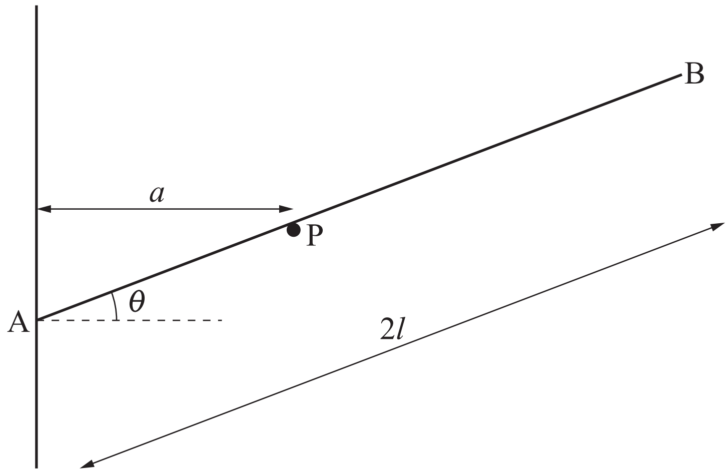

Fig. 12

A uniform rod AB has length $2l$ and weight W. The end A is in contact with a smooth vertical wall which is perpendicular to the vertical plane containing the rod. The rod is held in equilibrium by a rough peg P which is fixed at a horizontal distance a from the wall. The rod is inclined at an angle θ to the horizontal, as shown in Fig. 12.

(a) Find, in terms of W, l, a and θ, expressions for

(i) the normal contact force at A, **[4]**

(ii) the reaction at P normal to AB. **[2]**

(b) Hence deduce that $a \leqslant l\cos\theta$. **[2]**

The coefficient of friction between the rod and the peg is μ.

(c) Deduce that $l\cos^2\theta(\cos\theta - \mu\sin\theta) \leqslant a \leqslant l\cos^2\theta(\cos\theta + \mu\sin\theta)$. **[8]**

END OF QUESTION PAPER