OCR
Oxford Cambridge and RSA

Practice Paper – Set 2

A Level Further Mathematics B (MEI)

Y422/01 Statistics Major

MARK SCHEME

Duration: 2 hours 15 minutes

MAXIMUM MARK 120

This document consists of 15 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for A Level Further Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

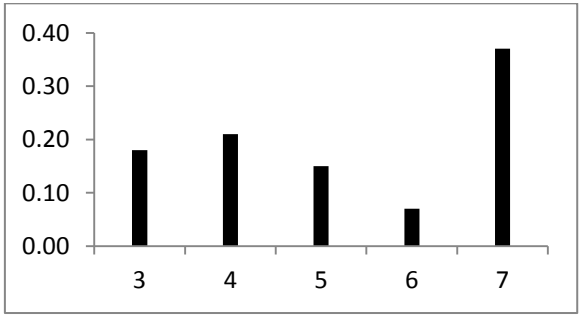
g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs	Guidance	
1	(i)		0.8	B1 [1]	1.1		
1	(ii)		P(at least 3 defects) = 1 – 0.9526 = 0.0474	M1 A1 [2]	3.4 1.1 BC	Or P(at least 3) = 1 – P(at most 2)	
1	(iii)		SD = 0.894, so limits are –0.094 and 1.694 P(0 or 1 defects) = 0.8088	B1 B1 [2]	1.1 1.1 BC		
1	(iv)		Mean = 8 P(fewer than 10 defects) = 0.7166	M1 A1 [2]	3.3 1.1		
1	(v)		Mean = 0.8 + 1.9 = 2.7 P(at most 3 defects) = 0.7141	M1 A1 [2]	3.3 1.1 BC		
1	(vi)		The occurrence of defects in the two carriageways must be independent of each other.	E1 [1]	2.3		

Question			Answer	Marks	AOs	Guidance	
2	(i)			**B1** **B1** **[2]**	**1.1** **1.1**	For heights For axes and labels	
2	(ii)		The distribution is bimodal	**B1** **[1]**	**1.1**	Or other relevant comment	
2	(iii)		E(X) = 5.22 Var(X) = 2.4516	**B1** **B1** **[2]**	**1.1a** **1.1**	**BC** **BC** (2.45 or better)	
2	(iv)	(*A*)	Expected amount = 320 + 80 × (5.22 – 3) £497.60	**M1** **A1** **[2]**	**1.1** **1.1**	Use of E[320 + 80(X – 3)], oe	Or using table of values
2	(iv)	(*B*)	$\text{SD} = \sqrt{80^2 \times 2.45}$, giving £125.26	**B1** **[1]**	**2.2a**		

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AOs</th><th>Guidance</th><th></th></tr>
<tr><td>3</td><td>(i)</td><td></td><td>A random sample enables proper inference about the population to be undertaken</td><td>B2

[2]</td><td>2.4, 2.4</td><td>B2 for correct explanation, as shown</td><td>Allow B1 for partially correct explanation, eg a random sample is less likely to be biased</td></tr>
<tr><td>3</td><td>(ii)</td><td></td><td>DR
H_0: no association between sex and customer loyalty
H_1: some association between sex and customer loyalty

<table><tr><td>Expected frequency</td><td>Bought previously</td><td>New customer</td></tr><tr><td>Female</td><td>23.2</td><td>60.8</td></tr><tr><td>Male</td><td>34.8</td><td>91.2</td></tr></table>
<table><tr><td>Contribution</td><td>Bought previously</td><td>New customer</td></tr><tr><td>Female</td><td>1.1655</td><td>0.4447</td></tr><tr><td>Male</td><td>0.7770</td><td>0.2965</td></tr></table>
$X^2 = 2.68$

Refer to χ^2_1
Critical value at 10% level = 2.71; 2.68 < 2.71
Result is not significant; there is not enough evidence to suggest that there is an association between sex and customer loyalty</td><td>B1

M1

A1

B1

B1
B1

E1
[8]</td><td>3.3

3.4

1.1

1.1

1.1
2.2b

3.5a</td><td>For both hypotheses correct

Values must be seen or calculation method clearly indicated; this mark cannot be implied merely by a correct final value of X^2

For 1 degree of freedom
For comparison with correct c.v.

E0 if hypotheses are reversed</td><td>Allow hypotheses and conclusion in terms of independence, but if ‘correlation’ is mentioned the first B1 and final E1 cannot be earned

Yates’ correction is not expected, but gains full marks if used correctly
<table><tr><td></td><td>Bought previously</td><td>New customer</td></tr><tr><td>F</td><td>0.9522</td><td>0.3633</td></tr><tr><td>M</td><td>0.6348</td><td>0.2422</td></tr></table>$X^2 = 2.19$</td></tr>
</table>

Question			Answer	Marks	AOs	Guidance	
4	(i)		$0.96^3 \times 0.04$ $= 0.0354$	B1 B1 [2]	3.3 1.1		(0.035 389...)
4	(ii)		$0.96^{20} = 0.442$	B1 [1]	1.1		(0.442 200...)
4	(iii)		$\frac{1}{0.04} = 25$	B1 [1]	1.1		
4	(iv)		Require P(at most 2 defective in 99 components) so using B(99, 0.04) gives 0.238	B1 M1 A1 [3]	3.1a 2.2a 1.1	For identifying required probability Use of correct binomial distribution **BC**	(0.238 06...)
4	(v)		Using B(10, 0.238 06...) P(at most 4 weeks) = 0.935	M1 A1 [2]	3.3 1.1	Using their value from **(iv)** **BC**	(0.935 04...)
4	(vi)		Mean = $np = 10 \times 0.238$ $= 2.38$	M1 A1 [2]	1.2 1.1		

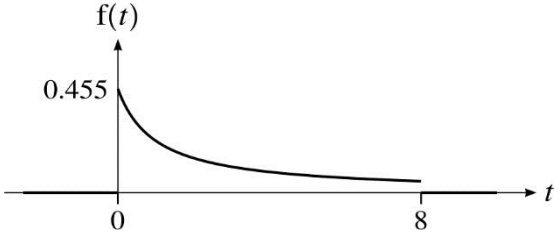
5	(i)		Underlying distribution (of water temperatures) needs to be Normally distributed	E1 [1]	3.3		
5	(ii)		$84.03 < \mu < 85.75$	B1 [1]	1.1		
5	(iii)		μ represents the population mean water temperature	B1 B1 [2]	2.4 2.4	For 'mean temperature' oe For correct reference to 'population'	
5	(iv)		Confidence interval suggests that the mean is different from 84 since the interval does not contain 84	B1 B1 [2]	3.4 2.2b		
5	(v)		By using a higher confidence level	E1 [1]	3.5c		

Question			Answer	Marks	AOs	Guidance	
6	(i)		The scatter diagram appears to be roughly elliptical so the distribution may be bivariate Normal	**E1** **E1** **[2]**	**3.5a** **2.4**		
6	(ii)		$S_{xy} = 115\,430 - \frac{1}{10} \times 1615 \times 709.6$ (= 829.6) $S_{xx} = 312\,647 - \frac{1}{10} \times 1615^2$ (= 51 824.5) $S_{yy} = 50\,395.3 - \frac{1}{10} \times 709.6^2$ (= 42.084) $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{829.6}{\sqrt{51824.5 \times 42.084}}$ = 0.562	**M1** **M1** **M1** **A1** **[4]**	**1.1a** **1.1** **3.3** **1.1**	For either S_{xx} or S_{yy} For general form including sq. root	Numerical evaluations are not required at this stage The three **M1** marks can be implied by a correct numerical answer
6	(iii)		H_0: $\rho = 0$, H_1: $\rho \neq 0$ where ρ is the population pmcc between x and y For n = 10, the 5% critical value is 0.6319 Since 0.562 < 0.6319 the result is not significant, so there is insufficient evidence to reject H_0 There is insufficient evidence at the 5% level to suggest that there is correlation between carbon dioxide emissions and noise level	**B1** **B1** **B1** **M1** **A1** **[5]**	**2.5** **2.5** **3.4** **1.1** **2.2b**	For both hypotheses For defining ρ For correct critical value For comparison and conclusion **FT** for conclusion in words	
6	(iv)		Because as the sample size increases, the random variation in the sample tends to decrease, and so the sample correlation coefficient tends to get closer to the population correlation coefficient	**E1** **E1** **[2]**	**2.2b** **2.2b**		
6	(v)		v = 0.8697 × 55 + 20.621 = 68.45	**M1** **A1** **[2]**	**3.4** **1.1**	For use of regression line of v on u	
6	(vi)		The result is (fairly) reliable as the points lie close to a straight line (r^2 = 0.8596 indicates a very good fit) and u = 55 is interpolation	**E1** **E1** **[2]**	**2.2a** **2.4**	For any two of: points close to line, high value of r^2, interpolation	

Question			Answer	Marks	AOs	Guidance	
6	(vii)		The regression line of v on u is used to predict a value for v given a value of u, and the line of u on v is used to predict a value for u given a value of v	**E1** **[1]**	**2.4**		
6	(viii)		$u = 42.45$, $v = 57.54$	**B1** **[1]**	**1.1**	**BC**	
6	(ix)		This point is the mean values of u and v, i.e. $(\bar{u}, \bar{v})$	**B1** **[1]**	**2.2a**		
7	(i)		Est. population mean $= \frac{2497}{40} = 62.425$ Est. population variance $= \frac{1}{39}\left(155\,970 - \frac{2497^2}{40}\right)$ $= 2.43$	**B1** **M1** **A1** **[3]**	**1.1** **1.1** **1.1**		$(2.430\,128\ldots)$
7	(ii)		**DR** Test is based on a Normal distribution H_0: $\mu = 62$, H_1: $\mu \neq 62$, where μ is the population mean volume of ethyl alcohol Test statistic is $\frac{62.425 - 62}{\sqrt{2.430\,13/40}}$ $=1.724$ Critical value (2-tailed) at 5% level is 1.96 1.724 < 1.96 so not significant (do not reject H_0) Insufficient evidence to suggest that the average volume of ethyl alcohol is not 62 ml	**B1** **B1** **B1** **M1** **A1** **B1** **M1** **E1** **[8]**	**3.4** **1.1a** **1.2** **3.3** **1.1** **1.1** **2.2b** **3.2a**	soi Both correct; if stated in words only, must include 'population' For definition of μ in context Allow wrong mean or sd here FT wrong (sensible) c.v. and test statistic if calculation is of right form	
7	(iii)		By the CLT, for large samples the distribution of the sample mean is approximately Normal Also for large samples the sample variance provides a good approximation to the population variance	**B1** **B1** **B1**	**2.2b** **2.4** **2.2b**	For mention of central limit theorem For full statement (including CLT)	

Question			Answer	Marks	AOs	Guidance	
				[3]			

Question			Answer	Marks	AOs	Guidance	
8	(i)		5 sheets thickness ~ N(5×1.01, 5×0.015^2) i.e. N(5.05, 0.001 125) P(thickness < 5.0) = 0.0680	**M1** **A1** **B1** **[3]**	**3.3** **1.1** **3.4**	For Normal and mean For correct variance **BC**	
8	(ii)		Stack thickness ~ N(5×1.01, $5^2 \times 0.015^2$) P(thickness < 5.0) = 0.2525	**B1** **B1** **[2]**	**3.3** **1.1**	For correct distribution **BC**	N(5.05, 0.005 625)
8	(iii)		Distribution of difference (two 1mm – one 2mm) has mean = $2 \times 1.01 - 2.008$ variance = $2 \times 0.015^2 + 0.019^2$ so distribution is N(0.012, 0.000 811) P(difference < 0) = 0.3367	 **M1** **M1** **A1** **A1** **[4]**	 **3.3** **1.1** **1.1** **3.4**	oe Method for mean Method for variance Correct distribution **BC**	

Question			Answer	Marks	AOs	Guidance	
9	(i)	(*A*)	$k\ln 9 = 1$	B1	3.4		
			$\frac{\ln(m+1)}{\ln 9} = \frac{1}{2} \Rightarrow \ln(m+1) = \frac{1}{2}\ln 9$	M1	1.1	For use of $F(m) = \frac{1}{2}$ with their k	
			$m+1 = 3 \Rightarrow m = 2$	A1 [3]	1.1	**AG**	
9	(i)	(*B*)	$\frac{\ln(q+1)}{\ln 9} = \frac{3}{4}$	M1	1.1a	For use of $F(q) = \frac{3}{4}$	
			upper quartile is $3\sqrt{3} - 1$ or 4.20	A1 [2]	1.1		(4.196…)
9	(ii)		$f(t) = \begin{cases} \frac{1}{(t+1)\ln 9} & 0 \le t \le 8 \\ 0 & \text{otherwise} \end{cases}$	M1	1.1a	For attempt at differentiation	
				A1 [2]	1.1	oe, e.g $\frac{0.455}{t+1}$	
9	(iii)			M1	1.1	For correct curve shape	**FT** their pdf for **M1**
				A1 [2]	1.1	Correct curve correctly located, and 8 marked (do not insist on 0.455 oe shown)	Curve must not extend beyond [0, 8]; condone missing 'zero' sections of graph
9	(iv)		$E(T) = k\int_0^8 \frac{t}{t+1}\,dt$	M1	3.1a		
			$= 2.64\ldots$	A1	1.1	**BC**	(2.640 95…)
			$E(T^2) = k\int_0^8 \frac{t^2}{t+1}\,dt$	M1	1.1		
			$= 11.92\ldots$	A1	1.1	**BC**	(11.922 87…)
			$\text{Var}(T) = 11.922\,87... - (2.640\,95...)^2 = 4.948...$	A1	1.1		(4.948 21…)
			$P(T - \mu > \sigma) = P(T > 2.641 + 2.224)$	M1	3.3		
			$= 1 - F(4.865) = 0.195$	A1 [7]	1.1		

Question			Answer	Marks	AOs	Guidance	
10	(i)		$E(X - Y) = 0$	B1 [1]	1.1		
10	(ii)		$\text{Var}(X) = \frac{1}{12} \times (11^2 - 1)$ $= 10$ $\text{Var}(Y) = \frac{1}{12} \times (10 - 0)^2 \quad \left(= \frac{25}{3}\right)$ $\text{Var}(X - Y) = 10 + \frac{25}{3} = \frac{55}{3}$ or 18.3	M1 A1 B1 B1 [4]	3.3 1.1 1.1 3.4		
10	(iii)		Estimate of $P(X - Y > 0)$ is 0.48 Estimate of $P(X - Y > 1)$ is 0.40	B1 B1 [2]	1.1 1.1		
10	(iv)		By using more rows in the spreadsheet	E1 [1]	2.4		
10	(v)		$P(Z > 0) = 0.5$	B1 [1]	3.3		
10	(vi)		Distribution is $N\left(0, \frac{55}{300}\right)$ $P(Z > 1) = 0.00976$	M1 M1 B1 [3]	3.3 1.1 1.1	For Normal For parameters **BC**	
10	(vii)		Distribution is $N\left(0, \frac{20}{100}\right)$ Required probability is $P(W > 1.005)$ $= 0.0123$	M1 B1 A1 [3]	3.3 3.5c 1.1	For correct distribution For continuity correction If no continuity correction applied allow **SC2** for $P(W > 1) = 0.0127$	

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