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Examiners' report

FURTHER MATHEMATICS A

H245

For first teaching in 2017

Y543/01 Summer 2022 series

Version 1

ocr.org.uk/maths

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers are also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Advance Information for Summer 2022 assessments

To support student revision, advance information was published about the focus of exams for Summer 2022 assessments. Advance information was available for most GCSE, AS and A Level subjects, Core Maths, FSMQ, and Cambridge Nationals Information Technologies. You can find more information on our website.

Paper Y543/01 series overview

This proved to be a good standard of paper to test candidates' understanding of the course. Questions 7 (b) and 8 (c) were less accessible to candidates with 8 (c) being the most challenging. A number of candidates are still showing a lack of understanding of the command words which is either costing them time from being too detailed with their solutions or not being credited for missing justification on 'show that' or 'determine' questions.

Candidates who did well on this paper generally did the following:	Candidates who did less well on this paper generally did the following:
• showed clear labelled diagrams • set out work clearly and logically • considered real life application of concepts.	• omitted multiple questions • confused key formulae • missed steps in detailed reasoning questions.

OCR support

A poster detailing the different command words and what they mean is available on the OCR website.

Question 1 (a)

1 A car has mass 1200 kg. The total resistance to the car's motion is constant and equal to 250 N.

(a) The car is driven along a straight horizontal road with its engine working at 10 kW.

Find the acceleration of the car at the instant that its speed is $5\,\mathrm{m\,s^{-1}}$. **[3]**

This question was answered well by the majority of candidates. Less than a fifth of candidates were not credited full marks, the most common errors were incorrectly applying $P = Fv$ or for applying both forces in the same direction.

Question 1 (b)

The maximum power that the car's engine can generate is 20 kW.

(b) Find the greatest constant speed at which the car can be driven along a straight horizontal road. **[2]**

Candidates answered this question well. The most common error was using 10000W instead of 20000 which was only credited the method mark.

Question 1 (c)

The car is driven up a straight road which is inclined at an angle θ above the horizontal where $\sin\theta = 0.05$.

(c) Find the greatest constant speed at which the car can be driven up this road. **[2]**

The majority of candidates answered this question well, although not as great a proportion as the first 2 parts of this question. Again, some candidates used 10000W which was only credited 1 mark. The majority of candidates understood that the question required the acceleration to be equal to 0.

Misconception

Common errors were confusing mass and weight or finding the incorrect weight component.

Question 2 (a)

2 The coordinates of two points, A and B, are $(-1, 6)$ and $(5, 12)$ respectively, where the units of the coordinate axes are metres. A particle P moves from A to B under the action of several forces. The force $\mathbf{F} = 7\mathbf{i} - 2\mathbf{j}$ N is one of the forces acting on P.

(a) Calculate the work done by $\mathbf{F}$ on P as P moves from A to B. **[2]**

The method mark was awarded here for finding the vector AB which the majority of candidates did correctly. About a fifth of candidates were only awarded this mark with the most common error being finding the scalar values of F and s and multiplying these together incorrectly.

Question 2 (b)

At the instant when P reaches B its velocity is $-\mathbf{i} - 5\mathbf{j}\,\mathrm{m\,s^{-1}}$.

(b) Find the power generated by $\mathbf{F}$ at the instant that P reaches B. **[2]**

Candidates who answered the first part of the question generally were able to answer this question as well. Of the fifth of candidates who were credited 0 marks the most common error again was considering scalars.

Misconception

Confusion of methods for scalar and vectors. Candidates need to make sure that they understand which method is appropriate and shown explicitly why they cannot consider the scalar method when using vectors.

Question 2 (c)

One end of a light elastic string was attached to the origin of the coordinate system and the other to P when P was at A, before it moved to B. The natural length of the string is 8 m and its modulus of elasticity is 24 N.

(c) At the instant that P reaches B, find the following.

- The tension in the string
- The elastic potential energy stored in the string **[3]**

Many candidates answered this question well. A number of candidates found the distance AB (instead of OB) but correctly subtracted 8 to find the extension and used both formulae correctly; this was credited with 2 marks. A considerable number of candidates did not find the extension and use the full length in the formula, this was not given any credit.

Assessment for learning

Examiners commented on the number of candidates who omitted the 2 in the denominator of the elastic potential energy formula. Using low stakes quizzes to aid candidates' recall would be beneficial.

Question 3 (a)

3 A particle P of mass 6 kg moves in a straight line under the action of a single force of magnitude F N which acts in the direction of motion of P.

At time t seconds, where $t \geqslant 0$, F is given by $F = \dfrac{1}{5 - 4e^{-t^2}}$.

When $t = 0$, the speed of P is $1.9\,\mathrm{m\,s^{-1}}$.

(a) Find the impulse of the force over the period $0 \leqslant t \leqslant 2$. **[2]**

More than three quarters of candidates knew that they should use their calculator to integrate F between the limits of 0 and 2. Some candidates either did not have a calculator or did not know that the command word 'Find' meant that they could use their calculator to find this integral. This resulted in too much time being given to the question or an incorrect solution.

Question 3 (b)

(b) Find the speed of P at the instant when $t = 2$. **[2]**

Over a third of candidates were not able to use their impulse from part (a) in the impulse-momentum principle to calculate the speed. If candidates had got part (a) incorrect they were still credited with 1 mark for a correct method (with their incorrect value for impulse).

Question 3 (c)

(c) Find the work done by the force on P over the period $0 \leqslant t \leqslant 2$. **[2]**

A number of candidates chose to omit this part, probably due to being less successful in parts (a) and (b). Candidates need to be careful when using answers from previous parts of the question. If the candidate used $2.04\,\mathrm{ms^{-1}}$ rather than 2.0356… it resulted in their value for the work done to be incorrect to 3sf.

Question 4 (a)

4 When two objects are placed a distance apart in outer space each applies a gravitational force to the other. It is suggested that the magnitude of this force depends on the masses of both objects and the distance between them. Assuming that this suggestion is correct, it is further assumed that the magnitude of this force is given by a relationship of the form

$$F = Gm_1^{\alpha} m_2^{\beta} r^{\gamma}$$

where

- F is the magnitude of the force
- m_1 and m_2 are the masses of the two objects
- r is the distance between the two objects
- G is a constant.

(a) Using a dimensional argument based on Newton's third law explain why $\alpha = \beta$. **[1]**

This question was challenging for the majority of candidates. A number quoted Newton's third law or stated dimensions without relating this to the question. About a fifth of candidates were able to give a detailed enough argument with a consideration of both Newton III and dimensional analysis and implying that the masses must be interchangeable.

Question 4 (b)

It is given that the magnitude of the gravitational force **is** given by such a relationship and that $G = 6.67 \times 10^{-11}\,\text{m}^3\text{kg}^{-1}\text{s}^{-2}$.

(b) Write down the dimensions of G. **[1]**

The vast majority of candidates were credited this mark. Some candidates left [ ] around their dimensions which is not the correct notation and was not credited.

Question 4 (c)

(c) By using dimensional analysis, determine the values of α, β and γ. **[3]**

Candidates generally did very well on this part. A common error for the few who did not gain full marks was to neglect the dimensions of G stating that it was dimensionless. It is worth noting that this is a 'determine' question so examiners should see justification for any results found. Examiners did credit any solutions without clear justification on this occasion but would expect to see clear consideration of each dimension on future questions.

Question 4 (d)

You are given that the mass of the Earth is 5.97×10^{24} kg and that the distance of the Moon from the Earth is 3.84×10^{8} m. You may assume that the only force acting on the Moon is the gravitational force due to the Earth.

(d) By modelling the Earth as stationary and assuming that the Moon moves in a circular orbit around the Earth, determine the period of the motion of the Moon. Give your answer to the nearest day. **[3]**

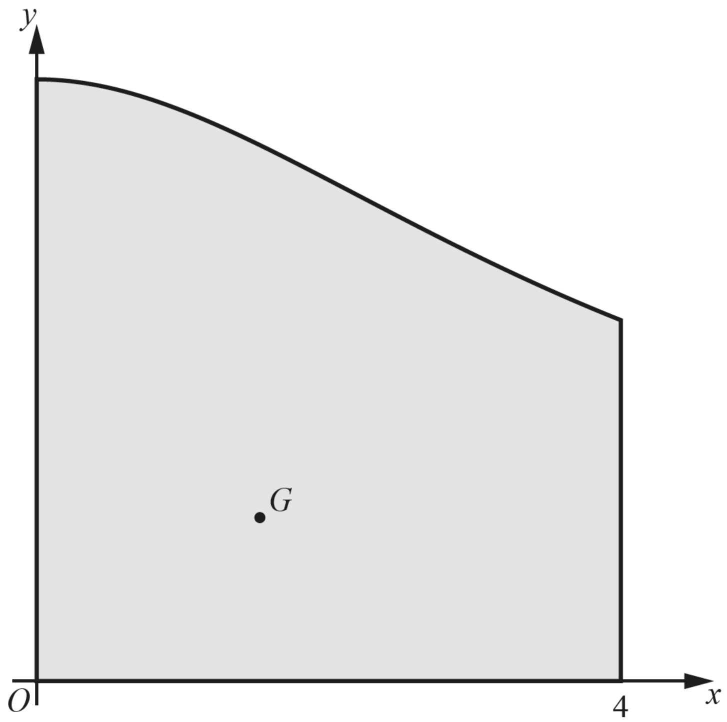
This question was completed variably by candidates. There was generally a lack of clarity from candidates' work of what values were being used. The command word was again 'determine' so this should indicate to candidates that they need to show what values they are using and clearly show justification for each stage of their working.

Question 5 (a)

5 In this question you must show detailed reasoning.

The region bounded by the x-axis, the y-axis, the line $x = 4$ and the curve with equation $y = \dfrac{15}{\sqrt{x^2 + 9}}$ is occupied by a uniform lamina.

The centre of mass of the lamina is at the point $G(\overline{x}, \overline{y})$ (see diagram).



(a) Show that $\overline{x} = \dfrac{2}{\ln 3}$. **[3]**

Most candidates understood the concept here but did not understand that the command 'show that' meant that they needed to show their integration and not rely on their calculator. Over a third of candidates only showed their integral for a result of $15\ln 3$ and were credited 1 mark; less than half of the candidates clearly showed both integrations and a complete method to gain full marks.

Assessment for learning

Candidates should be encouraged to underline command words before commencing the question. In a 'show detailed reasoning' question they must demonstrate that their solution is sufficiently detailed to cover every step of working and show a complete analytical method. Any calculus needs to be shown (although a calculator can be used to calculate their found integral).

Exemplar 1

5(a)

$$A\bar{x} = \int_0^4 xy\,dx$$

$$\text{so } \bar{x} = \frac{\int_0^4 x\left(\frac{15}{\sqrt{x^2+9}}\right)dx}{\int_0^4 \frac{15}{\sqrt{x^2+9}}\,dx}$$

$$\bar{x} = \frac{30}{15\left[\sinh^{-1}\left(\frac{x}{3}\right)\right]_0^4}$$

$$= \frac{2}{\sinh^{-1}\left(\frac{4}{3}\right)}$$

$$= \frac{2}{\ln\left(\frac{4}{3} + \sqrt{\left(\frac{4}{3}\right)^2+1}\right)} = \frac{2}{\ln 3}$$

This is a typical example of the type of solution examiners saw in this question. Examiners were not sure if candidates did not know how to integrate the numerator or assumed that they could rely on their calculator for the integration.

Question 5 (b)

(b) Determine the value of $\overline{y}$. Give your answer correct to **3** significant figures. **[3]**

The candidates who achieved full marks for part (a) generally were credited full marks for this part too.

Question 5 (c)

P is the point on the curved edge of the lamina where $x = 3$. The lamina is freely suspended from P and hangs in equilibrium in a vertical plane.

(c) Determine the acute angle that the longest straight edge of the lamina makes with the vertical. **[3]**

Many candidates effectively found the y coordinate when $x = 3$. Some candidates often stopped at this point or found the angle with the incorrect side. Some candidates used the dot product to calculate the angle which usually resulted in more errors (and took a longer time for the candidate to calculate).

Question 6 (a)

6 A particle P of mass 2.5 kg is free to move along the x-axis. When its displacement from the origin is x m its velocity is $v\,\text{m s}^{-1}$.

At time $t = 0$ seconds, P is at the point where $x = 1$ and is travelling in the negative x-direction with speed $5\,\text{m s}^{-1}$.

At this time an impulse of I Ns is applied to P in the positive x-direction so that P moves in the positive x-direction with speed $18\,\text{m s}^{-1}$.

(a) Find the value of I. **[1]**

Candidates generally completed this part well. If errors were seen it was usually because of an incorrect sign.

Question 6 (b)

Subsequently, whenever P is in motion, two forces act on it. The first force acts in the positive x-direction and has magnitude $\frac{5v^2}{x}$ N. The second force acts in the negative x-direction and has magnitude $60v$ N.

(b) Show that the motion of P can be modelled by the differential equation $\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{av}{x} + b$ where a and b are constants whose values should be determined. **[2]**

Most candidates showed a good understanding of this question. Some candidates forgot to include mass in Newton II, or did not divide the whole equation by it, or made mistakes when expressing $\frac{dv}{dt}$ in terms of $\frac{dv}{dx}$

Question 6 (c)

(c) By solving the differential equation derived in part **(b)** find an expression for v in terms of x. **[4]**

In general candidates mainly were credited either 0 or 3 marks on this question. The most common error was to substitute in the incorrect initial conditions of $x = 1, v = -5$ with only a fifth of candidates being awarded all 4 marks.

Question 6 (d)

You are given that $x = \frac{4}{3\mathrm{e}^{-24t} + 1}$ when $t \geqslant 0$.

(d) Describe in detail the motion of P when $t \geqslant 0$. **[3]**

The vast majority of candidates were credited 1 mark for this question for understanding that *x* tends to 4 as *t* tends to infinity. Examiners were generous with this mark but would have liked more candidates to use this correct terminology rather than stops at 4. Some candidates appeared to have substituted values of *x* into their calculator and made incorrect statements. Only a very small proportion realised to look at their equation for velocity found in part (c) and use this to comment on how the velocity changes over time, including a statement about when or where the velocity is at a maximum.

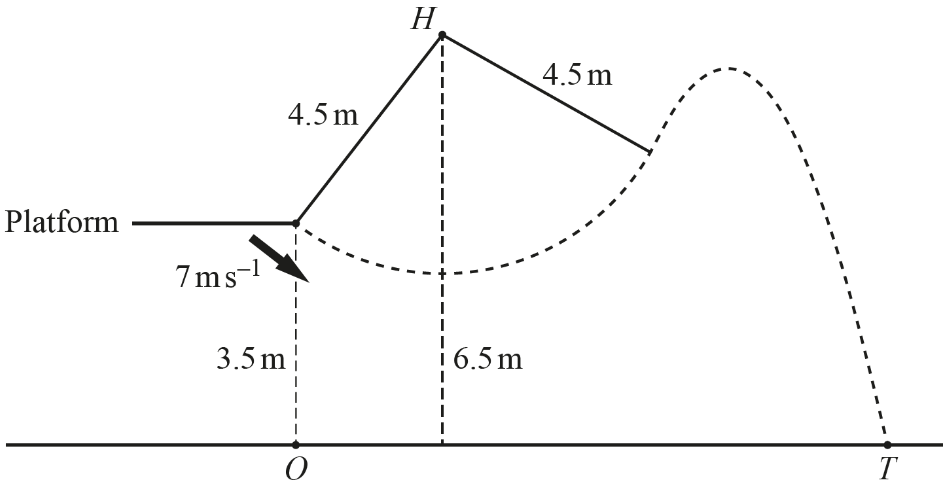
Question 7 (a)

7 The training rig for a parachutist comprises a fixed platform and a fixed hook, H. The platform is 3.5 m above horizontal ground level. The hook, which is not directly above the platform, is 6.5 m above the ground.

One end of a light inextensible cord of length 4.5 m is attached to H and the other is attached to a trainee parachutist of mass 90 kg standing on the edge of the platform with the cord straight and taut.

The trainee is then projected off the platform with a velocity of $7\,\text{m s}^{-1}$ perpendicular to the cord in a downward direction. The motion of the trainee all takes place in a single vertical plane and while the cord is attached to H it remains straight and taut.

When the speed of the trainee reaches $5.5\,\text{m s}^{-1}$ the cord is detached from H and the trainee then moves under the influence of gravity alone until landing on the ground (see diagram).



The trainee is modelled as a particle and air resistance is modelled as being negligible.

(a) Show that at the instant before the cord is detached from H, the tension in the cord has a magnitude of 1005.5 N. **[6]**

This question was completed well by a good number of candidates and showed an improvement on the understanding of this topic than in previous series. The challenge for some candidates was relating their height to θ. Candidates should be encouraged to draw a diagram clearly showing their 0 level of potential energy defined. This will help them to make this connection and also enable examiners to understand their intention when crediting marks. Some candidates broke the journey into two stages, using the minimum point of the swing to break up the motion. This was generally less successful and less efficient. As the answer is given for this question candidates must show each step clearly and values substituted in. Some candidates were not credited full marks due to gaps in their workings or from not explicitly substituting in the variables. There were incidents of candidates using the tension of 1005.5N to work backwards to find the angle which wasn't given any credit.

Assessment for learning

A good practice to develop labelling diagrams is to show candidates the initial information for a question without showing the actual question. Candidates should then label and annotate their diagram with anything relevant before knowing what will be asked of them.

Exemplar 2

7(a)

$F = \frac{mv^2}{r}$ $ke = \frac{1}{2}mv^2$ $gpe = mgh$

initially:

$ke = 0.5 \times 90 \times 7^2 = 2205J$

$gpe = 3.5 \times 9.8 \times 90 = 3087J$

$= 5292J$ total

if $v = 5.5ms^{-1}$

$ke = 1361.25 J$

$gpe = 5292 - 1361.25 = 3930.75J$

$h = \frac{3930.25}{90 \times 9.8} = 4.4566m \rightarrow E$

$h = 6.5 - 4.5\cos\theta$

$\theta = ~~27.0274°~~ \; 62.9941° \rightarrow F$

$T = mg ~~\sin\theta~~ \cos\theta + \frac{mv^2}{r}$

$T = \frac{mv^2}{r} + mg ~~\cos\theta~~$

$= \frac{90(5.5)^2}{4.5} + 90(9.8)(\cos 62.9941)$

$= 605 + 400.5$

$= 1005.5 N$

This candidate has shown a clear logical solution that has been credited with full marks. To improve their solution, it would have been beneficial to have included a labelled diagram. It is important for candidates to be aware that examiners are not given their copy of the question booklet so if they complete workings on this, examiners cannot credit this.

Question 7 (b)

The point on the ground vertically below the edge of the platform is denoted by O. The point on the ground where the trainee lands is denoted by T.

(b) Determine the distance OT. **[5]**

Candidates found this part more challenging and there were a number of errors being seen by examiners. This included confusion of components, sign errors and not considering the three horizontal distances. More successful responses had a clear diagram with labelled components.

Assessment for learning

The B mark for this question was often not awarded because both components needed to be seen and some candidates stopped their method before considering the horizontal components. If they had shown a clear diagram with the components calculated (it was not enough to leave as $5.5\cos\theta$) it would have prevented this.

Question 7 (c)

The ground around T is in fact an elastic mat of thickness $0.5\,\mathrm{m}$ which is angled so that it is perpendicular to the direction of motion of the trainee on landing. The mat, which is very rough, is modelled as an elastic spring of natural length $0.5\,\mathrm{m}$. It is assumed that the trainee strikes the mat at ground level and is brought to rest once the mat has been compressed by $0.3\,\mathrm{m}$.

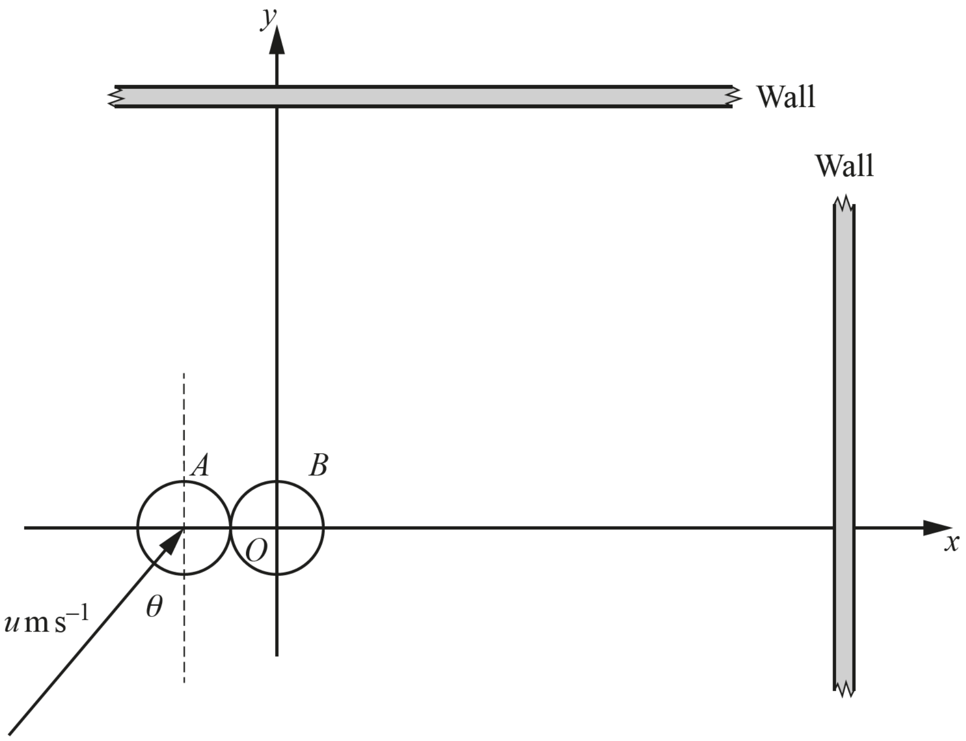
(c) Determine the modulus of elasticity of the mat. Give your answer to the nearest integer. **[4]**

This question was one of the least accessible on the paper. Responses where candidates had calculated an initial energy of 5292J in part (a) and considered that no energy was lost were more successful. The majority of candidates who calculated the KE at the floor only considered the vertical component. A small proportion considered the GPE term and very few then calculated this correctly.

Question 8 (a)

8 Two smooth circular discs, A and B, have equal radii and are free to move on a smooth horizontal plane. The masses of A and B are 1 kg and m kg respectively. B is initially placed at rest with its centre at the origin, O. A is projected towards B with a velocity of $u\,\text{m s}^{-1}$ at an angle of θ to the negative y-axis where $\tan\theta = \frac{5}{2}$. At the instant of collision the line joining their centres lies on the x-axis.

There are two straight vertical walls on the plane. One is perpendicular to the x-axis and the other is perpendicular to the y-axis. The walls are an equal distance from O (see diagram).



After A and B have collided with each other, each of them goes on to collide with a wall. Each then rebounds and they collide again at the same place as their first collision, with disc B again at O.

The coefficient of restitution between A and B is denoted by e. The coefficient of restitution between A and the wall that it collides with is also e while the coefficient of restitution between B and the wall that it collides with is $\frac{5}{9}e$.

It is assumed that any resistance to the motion of A and B may be ignored.

(a) Explain why it must be the case that the collision between A and the wall that it collides with is not inelastic. **[1]**

About two fifths of candidates were credited with this mark. A number of candidates seemed to confuse 'elastic' and 'inelastic', with others stating that all kinetic energy must be retained for the particles to collide again. Some candidates did not reference the collision between A and the wall and instead referred to the collision between the particles.

Question 8 (b)

(b) Show that $e = \frac{1}{m}$. **[4]**

Candidates generally did well on this question with the majority being able to show clear workings for both method marks. Less than half of the candidates spotted that the horizontal final velocity for A must be 0 for A and B to rebound in the same place; those that did, generally went on to be credited with full marks.

Question 8 (c)

(c) Show that $m = \frac{5}{3}$. **[7]**

Candidates found this question very difficult. Some candidates were able to access the first 2 marks by showing their velocity of A before the collision with the wall and either A or B's velocity after the collision with either wall. Candidates with clear labelled diagrams were much more successful here. Many candidates were unable to progress from this point with only the most successful responses equating expressions for time (a common error was equating expressions for velocity).

Question 8 (d)

(d) State **one** limitation of the model used. **[1]**

The majority of candidates were credited with this final mark. A common error was candidates referencing the friction between the ball and the walls which would not affect the solution in this case as the collision is perpendicular so wasn't credited.

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