OCR
Oxford Cambridge and RSA

A Level Further Mathematics A

Y545/01 Additional Pure Mathematics

Practice Paper – Set 1

Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 Determine the solution of the simultaneous linear congruences

$$x \equiv 4 \pmod{7}, \quad x \equiv 25 \pmod{41}.$$ **[5]**

2 Four points A, B, C and D have coordinates (1, 2, 5), (3, 4, –4), (6, 2, 3) and (0, 3, 7) respectively. Find the volume of tetrahedron $ABCD$. **[5]**

3 The surface S has equation $z = \frac{x}{y}\sin y + \frac{y}{x}\cos x$ where $0 < x \leqslant \pi$ and $0 < y \leqslant \pi$.

(i) Find

- $\frac{\partial z}{\partial x}$,
- $\frac{\partial z}{\partial y}$. **[4]**

(ii) Determine the equation of the tangent plane to S at the point A where $x = y = \frac{1}{4}\pi$. Give your answer in the form $ax + by + cz = d$ where a, b, c and d are exact constants. **[5]**

(iii) Write down a normal vector to S at A. **[1]**

4 **(i)** **(a)** Find all the quadratic residues modulo 11. **[2]**

(b) Prove that the equation $y^5 = x^2 + 2017$ has no solution in integers x and y. **[5]**

(ii) **In this question you must show detailed reasoning.**

The numbers M and N are given by

$$M = 11^{12} - 1 \text{ and } N = 3^2 \times 5 \times 7 \times 13 \times 61.$$

Prove that M is divisible by N. **[5]**

5 **(i)** **(a)** Solve the recurrence relation

$$X_{n+2} = 1.3X_{n+1} + 0.3X_n \text{ for } n \geqslant 0$$

given that $X_0 = 12$ and $X_1 = 1$. **[6]**

(b) Show that the sequence $\{X_n\}$ approaches a geometric sequence as n increases. **[2]**

The recurrence relation in part **(i)** models the projected annual profit for an investment company, so that X_n represents the profit (in £) at the end of year n.

(ii) **(a)** Determine the number of years taken for the projected profit to exceed one million pounds. **[2]**

(b) Compare your answer to part **(ii)(a)** with the corresponding figure given by the geometric sequence of part **(i)(b)**. **[2]**

(iii) **(a)** In a modified model, any non-integer values obtained are rounded down to the nearest integer at each step of the process. Write down the recurrence relation for this model. **[1]**

(b) Write down the recurrence relation for the model in which any non-integer values obtained are rounded up to the nearest integer at each step of the process. **[1]**

(c) Describe a situation that might arise in the implementation of part **(iii)(b)** that would result in an incorrect value for the next X_n in the process. **[1]**

6 **In this question you must show detailed reasoning.**

It is given that $I_n = \int_0^{\sqrt{3}} t^n \sqrt{1+t^2}\, \mathrm{d}t$ for integers $n \geqslant 0$.

(i) Show that $I_1 = \frac{7}{3}$. **[2]**

(ii) Prove that, for $n \geqslant 2$, $(n+2)I_n = 8\left(\sqrt{3}\right)^{n-1} - (n-1)I_{n-2}$. **[6]**

The curve C is defined parametrically by

$$x = 10t^3,\ \ y = 15t^2 \ \text{ for } 0 \leqslant t \leqslant \sqrt{3}.$$

When the curve C is rotated through 2π radians about the x-axis, a surface of revolution is formed with surface area A.

(iii) Determine

- the values of the integers k and m such that $A = k\pi I_m$,
- the exact value of A. **[6]**

7 The set M contains all matrices of the form $\mathbf{X}^n$, where $\mathbf{X} = \frac{1}{\sqrt{3}}\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$ and n is a positive integer.

(i) Show that M contains exactly 12 elements. **[4]**

(ii) Deduce that M, together with the operation of matrix multiplication, form a cyclic group G. **[5]**

(iii) Determine all the proper subgroups of G. **[5]**

END OF QUESTION PAPER