



Oxford Cambridge and RSA

A Level Further Mathematics A

Y545/01 Additional Pure Mathematics

Practice Paper – Set 1

Time allowed: 1 hour 30 minutes

You must have:

- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You may use:

- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

- 1 Determine the solution of the simultaneous linear congruences

$$x \equiv 4 \pmod{7}, \quad x \equiv 25 \pmod{41}. \quad [5]$$

- 2 Four points A, B, C and D have coordinates $(1, 2, 5)$, $(3, 4, -4)$, $(6, 2, 3)$ and $(0, 3, 7)$ respectively. Find the volume of tetrahedron $ABCD$. [5]

- 3 The surface S has equation $z = \frac{x}{y} \sin y + \frac{y}{x} \cos x$ where $0 < x \leq \pi$ and $0 < y \leq \pi$.

(i) Find

- $\frac{\partial z}{\partial x}$,
- $\frac{\partial z}{\partial y}$. [4]

- (ii) Determine the equation of the tangent plane to S at the point A where $x = y = \frac{1}{4}\pi$. Give your answer in the form $ax + by + cz = d$ where a, b, c and d are exact constants. [2]

- (iii) Write down a normal vector to S at A . [1]

- 4 (i) (a) Find all the quadratic residues modulo 11. [2]

- (b) Prove that the equation $y^5 = x^2 + 2017$ has no solution in integers x and y . [5]

(ii) **In this question you must show detailed reasoning.**

The numbers M and N are given by

$$M = 11^{12} - 1 \text{ and } N = 3^2 \times 5 \times 7 \times 13 \times 61.$$

Prove that M is divisible by N . [5]

- 5 (i) (a) Solve the recurrence relation

$$X_{n+2} = 1.3X_{n+1} + 0.3X_n \text{ for } n \geq 0$$

given that $X_0 = 12$ and $X_1 = 1$. [6]

- (b) Show that the sequence $\{X_n\}$ approaches a geometric sequence as n increases. [2]

The recurrence relation in part (i) models the projected annual profit for an investment company, so that X_n represents the profit (in £) at the end of year n .

- (ii) (a) Determine the number of years taken for the projected profit to exceed one million pounds. [2]

- (b) Compare your answer to part (ii)(a) with the corresponding figure given by the geometric sequence of part (i)(b). [2]

- (iii) (a) In a modified model, any non-integer values obtained are rounded down to the nearest integer at each step of the process. Write down the recurrence relation for this model. [1]

- (b) Write down the recurrence relation for the model in which any non-integer values obtained are rounded up to the nearest integer at each step of the process. [1]

- (c) Describe a situation that might arise in the implementation of part (iii)(b) that would result in an incorrect value for the next X_n in the process. [1]

- 6 In this question you must show detailed reasoning.

It is given that $I_n = \int_0^{\sqrt{3}} t^n \sqrt{1+t^2} dt$ for integers $n \geq 0$.

- (i) Show that $I_1 = \frac{7}{3}$. [2]

- (ii) Prove that, for $n \geq 2$, $(n+2)I_n = 8(\sqrt{3})^{n-1} - (n-1)I_{n-2}$. [6]

The curve C is defined parametrically by

$$x = 10t^3, \quad y = 15t^2 \text{ for } 0 \leq t \leq \sqrt{3}.$$

When the curve C is rotated through 2π radians about the x -axis, a surface of revolution is formed with surface area A .

- (iii) Determine

- the values of the integers k and m such that $A = k\pi I_m$,
- the exact value of A . [6]

7 The set M contains all matrices of the form \mathbf{X}^n , where $\mathbf{X} = \frac{1}{\sqrt{3}} \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$ and n is a positive integer.

- (i) Show that M contains exactly 12 elements. [4]
- (ii) Deduce that M , together with the operation of matrix multiplication, form a cyclic group G . [5]
- (iii) Determine all the proper subgroups of G . [5]

END OF QUESTION PAPER

OCR
Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.