OCR

Oxford Cambridge and RSA

Practice Paper – Set 1

A Level Further Mathematics A

Y543/01 Mechanics

MARK SCHEME

Duration: 1 hour 30 minutes

MAXIMUM MARK 75

This document consists of 10 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question includes the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Further Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs	Guidance	
1	(i)		$\text{WD} = \int_0^4 (12x - 3x^2)\,\text{d}x$ Work done by the force = 32 J	M1 A1 [2]	1.1a 1.1	 BC	
1	(ii)		Initial KE = $\frac{1}{2} \times 4.2 \times 2^2$ (=8.4) Work done = change in KE $\frac{1}{2} \times 4.2v^2 = 32 + 8.4$ Speed is $4.39\,\text{m}\,\text{s}^{-1}$	B1 M1 A1FT A1 [4]	1.1 3.3 1.1 2.2b	Need not be evaluated here Work-Energy principle used	
2	(i)		1	B1 [1]	2.2a		
2	(ii)		$a\int_0^2 x(2-x)\,\text{d}x$ $\frac{4}{3}a$ $\frac{4}{3}a\bar{y} = a^2\int_0^2 \frac{1}{2}x^2(2-x)^2\,\text{d}x$ $= \frac{8}{15}a^2$ $\bar{y} = \frac{\frac{8}{15}a^2}{\frac{4}{3}a} = \frac{2}{5}a$	M1 A1 M1 A1 A1 [5]	1.1a 1.1 1.2 1.1 2.2a	Attempt to integrate to find area. BC Use of $A\bar{y} = \frac{1}{2}\int y^2\,\text{d}x$ BC Integral correctly evaluated AG	Ignore wrong/missing limits for M mark Ignore wrong/missing limits for M mark
2	(iii)		$3M\bar{Y} = 2M \times \frac{1}{2}a + M \times \left(a + \frac{2}{5}a\right)$ $\bar{Y} = \frac{4}{5}a$ $\tan 20° = \frac{1}{\frac{4}{5}a}$ So value of a is 3.43 (3 sf)	M1 A1 M1 A1 [4]	3.1b 1.1 1.1 3.2a	Moments about base oe (eg $\bar{Y} = -\frac{1}{5}a$ wrt original x-axis) Use of CofM position to find a	or other horizontal axis

Question			Answer	Marks	AOs	Guidance	
3	(i)		$T\sin\theta = 3.5\times 3^2 \times r$ $r = L\sin\theta$ $T\sin\theta = 3.5\times 3^2 \times L\sin\theta$ oe $T = 31.5L$	**M1** **B1** **M1** **A1** **[4]**	**3.3** **1.1** **1.1** **1.1**	Resolving tension and using NII Eliminating r **AG** working must be clear	may use $T\cos\phi$ instead or $r = L\cos\phi$
3	(ii)		$T = \dfrac{75(L-0.8)}{0.8}$	**B1** **[1]**	**3.3**	Correct use of Hooke's law	$(T = 93.75L - 75)$
3	(iii)		$31.5L = 93.75(L - 0.8)$ $T = 38.0$ $L = 1.20$	**M1** **A1** **A1** **[3]**	**1.1a** **1.1** **1.1**	Solution of simultaneous equations	
3	(iv)		$T\cos\theta = mg$ 25.3°	**M1** **A1** **[2]**	**3.4** **1.1**	Resolve vertically	

Question			Answer	Marks	AOs	Guidance	
4	(i)		Initial energy = $\frac{1}{2}\times 1.7\times 0.5^2+\frac{50\times(3.2-1.2)^2}{2\times 1.2}$ 83.5 Minimum energy required to reach *O* is $1.7\times 9.8\times 3.2+\frac{50\times 1.2^2}{2\times 1.2}=83.3(12)$ So according to the model *B* reaches *O*	**M1** **A1** **B1** **E1** **[4]**	**3.4** **1.1** **1.1** **3.2a**	May include a gravitational PE term	
4	(ii)	(a)	Yes because work would have to be done against air resistance so more energy would be required to reach *O* and so the conclusion could change	**E1** **[1]**	**2.4**		
	(ii)	(b)	No because a string has no elastic energy after it goes slack so less initial energy would be required to enable *B* to reach *O* and so the conclusion does not change	**E1** **[1]**	**2.4**	Or work has to be done against the thrust as the spring goes into compression	
4	(ii)	(c)	**Either** The direction of projection has no effect on energy calculations. so (if nothing else interferes with the motion) the conclusion does not change **Or** If the downward motion means that the ball hits the floor some energy will be lost, so the conclusion could change **Or** any other sensible discussion about how the model might break down, e.g. the downward motion causing the spring to break or Hooke's law to fail, with an appropriate conclusion stated	**E1** **[1]**	**2.4**		

Question			Answer	Marks	AOs	Guidance	
5	(i)		$[P]$ = T, $[m]$ = M, $[h]$ = L	B1	3.3	All three soi	
			$[g] = \mathrm{LT}^{-2}$	B1	3.3		
			M: $\alpha = 0$	B1	1.1		
			T: $1 = -2\gamma \Rightarrow \gamma = -\frac{1}{2}$	B1	3.4		
			L: $0 = \beta + \gamma \Rightarrow \beta = \frac{1}{2}$	B1	1.1		
				[5]			
5	(ii)		Values of $\frac{P}{\sqrt{h}}$ are: 2.01, 1.37, 2.01	M1	2.1	Or equivalent calculations	(or graph plot/sketch)
			so not consistent	A1	2.1		
				[2]			
5	(iii)		2.17 must be wrong	B1	3.5a	soi	
			$1.27 \times \frac{\sqrt{2.50}}{\sqrt{0.40}}$ or $3.81 \times \frac{\sqrt{2.50}}{\sqrt{3.60}}$	M1	3.4	oe, eg k = 6.286… or $2.008\sqrt{g}$	
			So an estimate of the correct value is 3.175	A1	2.2b	Allow 3sf answer 3.17 or 3.18	
				[3]			

6	(i)		Components of initial vel: $6.5\cos 30° \downarrow$, $6.5\sin 30° \rightarrow$	M1	3.1b	For correct use of either component	
			Upwards component of final vel: $v_V = \frac{2}{3} \times 6.5\cos 30°$	M1	1.1		(3.7527…)
			$-2 = 2.5v_H - 2.5 \times 6.5\sin 30°$	M1	1.1	Use of impulse-momentum; must involve horizontal components only	
			$v_H = 2.45$	A1	1.1		
			$v = \sqrt{v_V^2 + v_H^2}$, $\tan\theta = \frac{v_H}{v_V}$, oe	M1	1.1	For use of either trig or Pythagoras	
			Velocity has magnitude $4.48\,\mathrm{m\,s^{-1}}$	A1	3.2a		
			at 33.1° to the normal	A1	3.2a		Condone 33.2°
				[7]			
6	(ii)		Percentage loss is $\frac{\frac{1}{2} \times 2.5 \times (6.5^2 - 4.482^2)}{\frac{1}{2} \times 2.5 \times 6.5^2} \times 100$	M1	1.1a	Factors of $\frac{1}{2} \times 6.5$ may be omitted	
			= 52.5	A1	1.1	Condone answers 52 and 53	
				[2]			

Question			Answer	Marks	AOs	Guidance	
7	(i)		$\frac{1}{2}\times 0.4v_B^2 = \frac{1}{2}\times 0.4\times 0.8^2 + 0.4g\times 1.4(\cos 30° - \cos 60°)$	**M1**	**3.3**	Conservation of energy from A to B	
			$v_B^2 = 10.68...$	**A1**	**1.1**	soi	
			$R - 0.4g\cos 30° = 0.4\times\frac{v_B^2}{1.4}$	**M1**	**3.3**	Use of Newton II radially, 3 terms	
				A1ft	**1.1**	Correct equation, with their v_B	
			Magnitude of contact force at B is 6.45 N	**A1**	**1.1**	**cao**	
				[5]			
7	(ii)		Vertical velocity component at B is $v_B\sin 30°$	**B1ft**	**3.1b**		(1.634…)
			Height of B above plane is $2.5 + 1.4(1 - \cos 30°)$	**B1**	**1.1**	soi	(2.687…)
			Time of flight given by $-2.687 = 1.634t - \frac{1}{2}gt^2$	**M1**	**3.4**	Use of $s = ut + \frac{1}{2}at^2$ vertically	
				A1	**1.1**	Correct equation	
			$t = 0.9259...$	**A1**	**1.1**	**BC**	
			Horizontal distance covered in flight is $(v_B\cos 30°)t$	**M1**	**3.4**	Using their values of v_B and t	(2.6209…)
			$FH = 1.4\sin 30° + 2.6209... = 3.32$ m	**A1**	**3.2a**		
			Alternative solution				
			Height of B above plane is $2.5 + 1.4(1 - \cos 30°)$	**B1**			(2.687…)
			Projection velocity is v_B at 30° above horizontal	**B1ft**		soi; ft their value of v_B	
			Equation of trajectory is $y = x\tan 30° - \frac{gx^2}{2v_B^2\cos^2 30°}$	**M1**		oe, if origin other than B is used	
				A1		Correct equation, with their v_B	
			$0.61152x^2 - 0.57735x - 2.68756 = 0$	**M1**		Formation of explicit quadratic in x	
			$x = 2.62...$	**A1**		**BC**	
			$FH = 1.4\sin 30° + 2.6209... = 3.32$ m	**A1**			
				[7]			

Question			Answer	Marks	AOs	Guidance	
8	(i)		$F = \frac{6}{t^2} - \frac{3v}{t} = 1.5a = 1.5\frac{\mathrm{d}v}{\mathrm{d}t} \Rightarrow \frac{\mathrm{d}v}{\mathrm{d}t} + \frac{2v}{t} = \frac{4}{t^2}$	**B1** **[1]**	**3.3**	**AG** Using Newton II with $a = \frac{\mathrm{d}v}{\mathrm{d}t}$ and rearranging	Initial $F = ma$ equation must be clear
8	(ii)		Integrating factor is $\mathrm{e}^{\int \frac{2}{t}\mathrm{d}t} = \mathrm{e}^{2\ln t}$ $= t^2$ DE is $\frac{\mathrm{d}}{\mathrm{d}t}(vt^2) = 4$ $vt^2 = 4t + c$ $v = 0$ when $t = 2$ gives $0 = 8 + c$ $vt^2 = 4t - 8$ when $t = 20$, speed of the piston is $0.18\,\mathrm{m\,s^{-1}}$	**M1** **A1** **M1** **A1** **M1** **A1** **A1** **[7]**	**1.1a** **1.1** **3.4** **1.1** **3.4** **1.1** **2.2a**	Using their integrating factor Use of $v = 0$, $t = 2$ in their equation to find a constant of integration	
8	(iii)		$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{4}{t} - \frac{8}{t^2} \Rightarrow \text{distance} = \int_2^{20}\left(\frac{4}{t} - \frac{8}{t^2}\right)\mathrm{d}t$ = 5.6103… so distance is 5.61 m correct to 3sf **Alternative solution** $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{4}{t} - \frac{8}{t^2} \Rightarrow x = 4\ln t + \frac{8}{t} + C$ $x = 0$ when $t = 2$ gives $C = -4\ln 2 - 4$ so $t = 20$ gives $x = 4\ln 20 + 0.4 - 4\ln 2 - 4 = 5.61$ (3sf)	**M1** **A1** **A1** **[3]** **M1** **A1** **A1**	**3.4** **3.4** **1.1** **3.4** **AG**	Use $v = \frac{\mathrm{d}x}{\mathrm{d}t}$ and expression of distance moved as a definite integral Correct integrand and limits **BC; AG** so justification for 3sf accuracy is needed Use $v = \frac{\mathrm{d}x}{\mathrm{d}t}$ and attempt to integrate	 ($C = -6.77\ldots$) (5.61034…)