OCR
Oxford Cambridge and RSA

A Level Further Mathematics A

Y543/01 Mechanics

Practice Paper – Set 1

Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

1 A particle P of mass 4.2 kg is free to move along the x-axis which is horizontal. P is projected from the origin, O, in the positive x direction with a speed of $2\,\mathrm{m\,s^{-1}}$. As P moves between O and the point A where $x = 4$, it is acted upon by a variable force of magnitude $(12x - 3x^2)$ N acting in the direction OA.

(i) Calculate the work done by the force as P moves from O to A. **[2]**

(ii) Hence, assuming that no other force acts on P, calculate the speed of P at A. **[4]**

2 The region bounded by the x-axis and the curve $y = ax(2 - x)$, where a is a constant, is occupied by a uniform lamina L_1 (see Fig. 1). Units on the axes are metres.

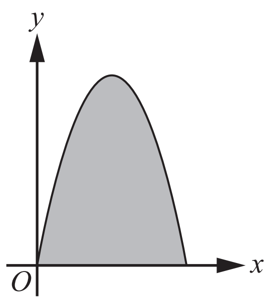


Fig. 1

(i) Write down the value of the x-coordinate of the centre of mass of L_1. **[1]**

(ii) Show that the y-coordinate of the centre of mass of L_1 is $\frac{2}{5}a$. **[5]**

The mass of L_1 is M kg. A uniform rectangular lamina of width 2 m and height a m is made from a different material from that of L_1 and has a mass of $2M$ kg. A new lamina, L_2, is formed by joining the straight edge of L_1 to an edge of the rectangular lamina of length 2 m (see Fig. 2).

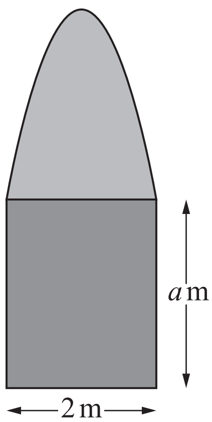
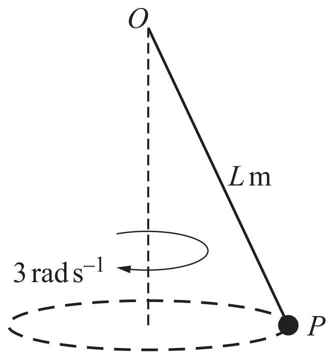


Fig. 2

L_2 is freely suspended from one of its right-angled corners and hangs in equilibrium with its edge of length 2 m making an angle of 20° with the horizontal.

(iii) Find the value of a, giving your answer correct to 3 significant figures. **[4]**

3 A particle P of mass 3.5 kg is attached to one end of a light elastic string of natural length 0.8 m and modulus of elasticity 75 N. The other end of the string is attached to a fixed point O. The particle rotates in a horizontal circle with a constant angular velocity of $3\,\mathrm{rad\,s^{-1}}$. The centre of the circle is vertically below O. The magnitude of the tension in the string is T N and the length of the extended string is L m (see diagram).



(i) By considering the acceleration of P, show that $T = 31.5L$. **[4]**

(ii) Write down another relationship between T and L. **[1]**

(iii) Find the value of T and the value of L. **[3]**

(iv) Find the angle that the string makes with the downwards vertical through O. **[2]**

4 A ball B of mass 1.7 kg is connected to one end of a light elastic spring of natural length 1.2 m. The other end of the spring is attached to a point O on the ceiling of a large room. The modulus of elasticity of the spring is 50 N. The ball is held 3.2 m vertically below O and projected upwards with an initial speed of $0.5\,\mathrm{m\,s^{-1}}$. In order to model the motion of B (before any collision with the ceiling) the following assumptions are made.

- Air resistance is ignored.
- B is small.
- The fully compressed length of the spring is negligible.

(i) Determine whether, according to the model, B reaches O. **[4]**

(ii) Without doing any further calculations, explain whether the answer to part **(i)** could change in each of the following different cases.

(a) A new model is used in which air resistance is taken into account. **[1]**

(b) The spring is replaced by an elastic string with the same natural length and modulus of elasticity. **[1]**

(c) B is initially projected downwards rather than upwards. **[1]**

5 A simple pendulum consists of a small sphere of mass m connected to one end of a light rod of length h. The other end of the rod is freely hinged at a fixed point. When the sphere is pulled a short distance to one side and released from rest the pendulum performs oscillations. The time taken to perform one complete oscillation is called the period and is denoted by P.

(i) Assuming that $P = km^{\alpha}h^{\beta}g^{\gamma}$, where g is the acceleration due to gravity and k is a dimensionless constant, find the values of α, β and γ. **[5]**

A student conducts an experiment to investigate how P varies as h varies. She measures the value of P for various values of h, ensuring that all other conditions remain constant. Her results are summarised in the table below.

h (m)	0.40	2.50	3.60
P (s)	1.27	2.17	3.81

(ii) Show that these results are not consistent with the answers to part **(i)**. **[2]**

(iii) The student later realises that she has recorded one of her values of P incorrectly.

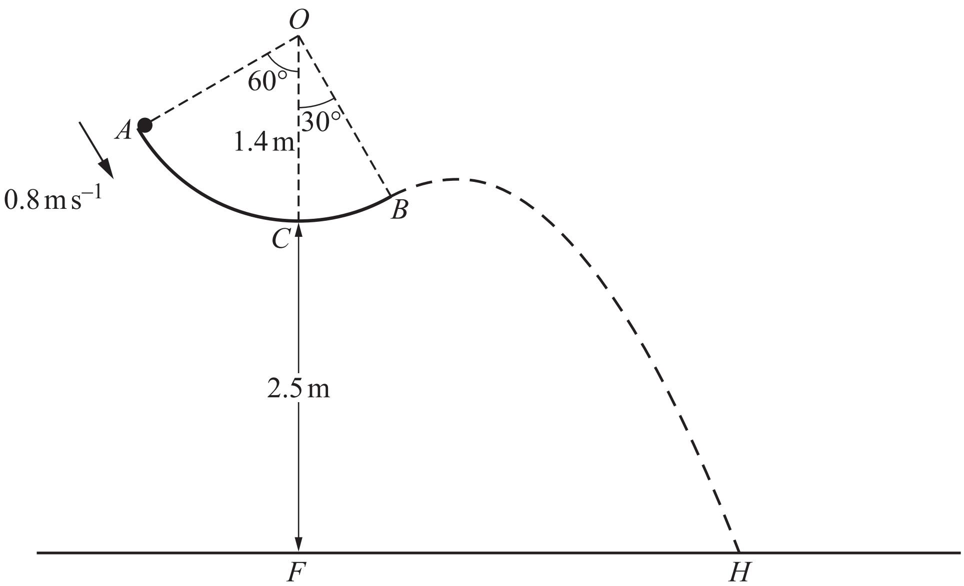
- Identify the incorrect value.
- Estimate the correct value that she should have recorded. **[3]**

6 A particle P of mass 2.5 kg strikes a rough horizontal plane. Immediately before P strikes the plane it has a speed of $6.5\,\text{m}\,\text{s}^{-1}$ and its direction of motion makes an angle of 30° with the normal to the plane at the point of impact. The impact may be assumed to occur instantaneously. The coefficient of restitution between P and the plane is $\frac{2}{3}$. The friction causes a horizontal impulse of magnitude 2 N s to be applied to P in the plane in which it is moving.

(i) Calculate the velocity of P immediately after the impact with the plane. **[7]**

(ii) P loses about x% of its kinetic energy as a result of the impact. Find the value of x. **[2]**

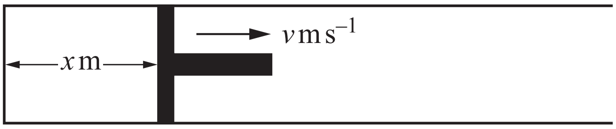
7 A smooth track AB is in the shape of an arc of a circle with centre O and radius 1.4 m. The track is fixed in a vertical plane with A above the level of B and a point C on the track vertically below O. Angle AOC is 60° and angle COB is 30°. Point C is 2.5 m vertically above the point F, which lies in a horizontal plane. A particle of mass 0.4 kg is placed at A and projected down the track with an initial velocity of $0.8\,\text{m}\,\text{s}^{-1}$. The particle first hits the plane at point H (see diagram).



(i) Find the magnitude of the contact force between the particle and the track when the particle is at B. **[5]**

(ii) Find the distance FH. **[7]**

8 A piston of mass 1.5 kg moves in a straight line inside a long straight horizontal cylinder. At time t s the displacement of the piston from its initial position at one end of the cylinder is x m and its velocity is $v\,\text{m}\,\text{s}^{-1}$ (see diagram).



The piston starts moving when $t = 2$ and is brought to rest when it reaches the other end of the cylinder. While the piston is in motion it is acted on by a force of magnitude $\frac{6}{t^2}$ N in the positive x direction, and also by a force of magnitude $\frac{3v}{t}$ N resisting the motion.

(i) Show that, while the piston is in motion, $\frac{dv}{dt} + \frac{2v}{t} = \frac{4}{t^2}$. **[1]**

The piston reaches the other end of the cylinder when $t = 20$.

(ii) Find the speed of the piston immediately before it is brought to rest. **[7]**

(iii) Show that the piston travels a distance of 5.61 m, correct to 3 significant figures. **[3]**

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