



Oxford Cambridge and RSA

# A Level Further Mathematics A

## Y540/01 Pure Core 1

### Practice Paper – Set 1

Time allowed: 1 hour 30 minutes

**You must have:**

- Printed Answer Booklet
- Formulae A Level Further Mathematics A

**You may use:**

- a scientific or graphical calculator

#### INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

#### INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [ ].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

**1 In this question you must show detailed reasoning.**

Find the square roots of  $24 + 10i$ , giving your answers in the form  $a + bi$ .

[5]

**2** The matrices **A** and **B** are given by  $\mathbf{A} = \begin{pmatrix} 1 & a \\ 3 & 0 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix}$ .

(i) Find the value of  $a$  such that  $\mathbf{AB} = \mathbf{BA}$ .

[3]

(ii) Prove by counter example that matrix multiplication for  $2 \times 2$  matrices is not commutative.

[2]

(iii) A triangle of area 4 square units is transformed by the matrix **B**. Find the area of the image of the triangle following this transformation.

[1]

(iv) Find the equations of the invariant lines of the form  $y = mx$  for the transformation represented by matrix **B**.

[4]

**3** Prove by mathematical induction that, for all integers  $n \geq 1$ ,  $n^5 - n$  is divisible by 5.

[4]

**4** The lines  $l_1$  and  $l_2$  have equations  $\frac{x-7}{2} = \frac{y-1}{-1} = \frac{z-6}{3}$  and  $\frac{x-2}{1} = \frac{y-6}{2} = \frac{z+2}{1}$  respectively.

(i) Show that  $l_1$  and  $l_2$  intersect.

[4]

(ii) Find the cartesian equation of the plane that contains  $l_1$  and  $l_2$ .

[3]

**5** By using a suitable substitution, which should be stated, show that

$$\int_{\frac{3}{2}}^{\frac{5}{2}} \frac{1}{\sqrt{4x^2 - 12x + 13}} dx = \frac{1}{2} \ln(1 + \sqrt{2}).$$

[6]

- 6 One end of a light inextensible string is attached to a small mass. The other end is attached to a fixed point  $O$ . Initially the mass hangs at rest vertically below  $O$ . The mass is then pulled to one side with the string taut and released from rest.  $\theta$  is the angle, in radians, that the string makes with the vertical through  $O$  at time  $t$  seconds and  $\theta$  may be assumed to be small.

The subsequent motion of the mass can be modelled by the differential equation

$$\frac{d^2\theta}{dt^2} = -4\theta.$$

- (i) Write down the general solution to this differential equation. [1]
- (ii) Initially the pendulum is released from rest at an angle of  $\theta_0$ . Find the particular solution to the equation in this case. [4]
- (iii) State any limitations on the model. [1]

- 7 (i) Using the definition of  $\sinh x$  in terms of  $e^x$  and  $e^{-x}$ , show that

$$4 \sinh^3 x = \sinh 3x - 3 \sinh x. \quad [3]$$

- (ii) **In this question you must show detailed reasoning.**

By making a suitable substitution, find the real root of the equation

$$16u^3 + 12u = 3.$$

Give your answer in the form  $\frac{(a^{\frac{1}{b}} - a^{-\frac{1}{b}})}{c}$  where  $a$ ,  $b$  and  $c$  are integers. [5]

- 8 You are given that  $f(x) = (1 - a \sin x)e^{bx}$  where  $a$  and  $b$  are positive constants. The first three terms in the Maclaurin expansion of  $f(x)$  are  $1 + 2x + \frac{3}{2}x^2$ .

- (i) Find the value of  $a$  and the value of  $b$ . [6]
- (ii) Explain if there is any restriction on the value of  $x$  in order for the expansion to be valid. [1]

- 9 In an experiment, at time  $t$  minutes there is  $Q$  grams of substance present. It is known that the substance decays at a rate that is proportional to  $1 + Q^2$ . Initially there are 100 grams of the substance present and after 100 minutes there are 50 grams present.

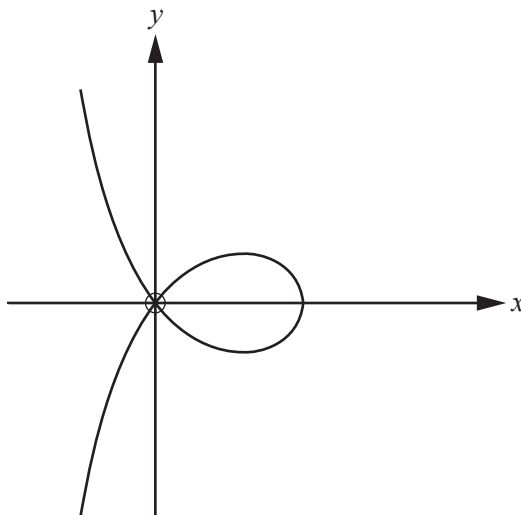
Find the amount of the substance present after 400 minutes. [8]

- 10 (i) (a) A curve has polar equation  $r = 2 - \sec \theta$ . Show that the cartesian equation of the curve can be written in the form

$$y^2 = \left( \frac{2x}{x+1} \right)^2 - x^2.$$

[4]

The figure shows a sketch of part of the curve with equation  $y^2 = \left( \frac{2x}{x+1} \right)^2 - x^2$ .



- (b) Explain why the curve is symmetrical in the  $x$ -axis. [1]
- (c) The line  $x = a$  is an asymptote of the curve. State the value of  $a$ . [1]
- (ii) The enclosed loop shown in the figure is rotated through  $180^\circ$  about the  $x$ -axis. Find the exact volume of the solid formed. [8]

**END OF QUESTION PAPER**

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