OCR
Oxford Cambridge and RSA

Practice Paper – Set 2

A Level Further Mathematics A

Y540/01 Pure Core 1

MARK SCHEME

Duration: 1 hour 30 minutes

MAXIMUM MARK 75

FINAL

This document consists of 16 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Further Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows thatthe method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.

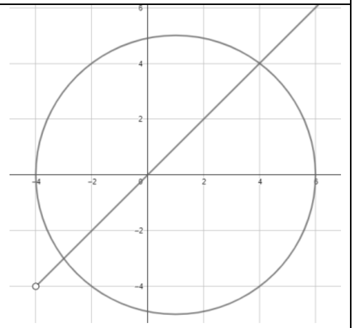
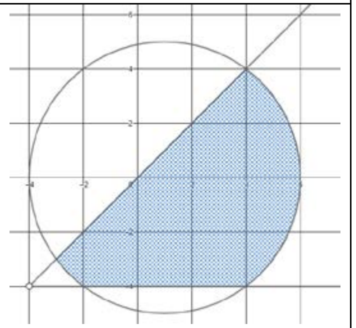
g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(i)		**DR** $z = 2\cos\frac{\pi}{6} + 2\text{i}\sin\frac{\pi}{6}$ $= \sqrt{3} + \text{i}$	**M1** **A1** **[2]**	1.1a 1.1		
	(ii)		**DR** $z^2 = \left(\sqrt{3} + \text{i}\right)^2 = 3 + 2\sqrt{3}\text{i} - 1$ $= 2 + 2\sqrt{3}\text{i}$	**M1** **A1ft**	1.1a 1.1	for $(\text{their } z)^2$ **FT** their z	
			Or $z = \left(2, \frac{\pi}{6}\right) \Rightarrow z^2 = \left(4, \frac{\pi}{3}\right) = 4\left(\cos\frac{\pi}{3} + \text{i}\sin\frac{\pi}{3}\right)$ $= 2 + 2\sqrt{3}\text{i}$	**M1** **A1ft** **[2]**		**FT** their z	
	(iii)		**DR** $z^* = \sqrt{3} - \text{i}$ $\frac{z}{z^*} = \frac{\sqrt{3}+\text{i}}{\sqrt{3}-\text{i}} = \frac{\sqrt{3}+\text{i}}{\sqrt{3}-\text{i}} \times \frac{\sqrt{3}+\text{i}}{\sqrt{3}+\text{i}} = \frac{1}{2} + \frac{1}{2}\sqrt{3}\text{i}$	**B1ft** **M1** **A1**	1.1a 1.1 1.1	**FT** their z	Or $zz^* = \|z\|^2 = 4$ (**B1**) and $\frac{z^2}{\|z\|^2} = \frac{2+2\sqrt{3}\text{i}}{4} = \frac{1}{2} + \frac{1}{2}\sqrt{3}\text{i}$
			Or $z = \left(2, \frac{\pi}{6}\right) \Rightarrow z^* = \left(2, -\frac{\pi}{6}\right)$ $\Rightarrow \frac{z}{z^*} = \left(\frac{2}{2}, \frac{\pi}{6} - -\frac{\pi}{6}\right) = \left(1, \frac{\pi}{3}\right) = \left(\cos\frac{\pi}{3} + \text{i}\sin\frac{\pi}{3}\right)$ $\frac{1}{2} + \frac{\sqrt{3}}{2}\text{i}$	**B1ft** **M1** **A1** **[3]**		**FT** their z	

Question			Answer	Marks	AO	Guidance	
2	(i)		C_1: Circle, centre (1,0) radius 5 C_2: Half line starting at (but not including) (–4, –4) Gradient 1 (i.e. through (0, 0))	B1 B1 B1 B1 [4]	1.2 1.1 2.2a 1.1	Soi eg by a circle through points (6,0) and (-4, 0)	
	(ii)		Shading: Inside circle Under line but above horizontal through (–4, –4). But nothing extra	 B1 B1 [2]	 2.2a 2.2a	Withhold one mark for anything extra or omissions	

Question			Answer	Marks	AO	Guidance	
3			Base case, $n=1$, $a_1=\frac{11\times5^0+1}{2}=6$	**B1**	2.1		
			Assume true for $n=k$ $\Rightarrow a_k=\frac{11\times5^{k-1}+1}{2}$	**M1**	2.1	Statement of inductive hypothesis	
			Then $a_{k+1}=5a_k-2=5\left(\frac{11\times5^{k-1}+1}{2}\right)-2$ $=\left(\frac{11\times5^k+5}{2}\right)-2=\left(\frac{11\times5^k+1+4}{2}\right)-2$	**M1**	2.2a	Use of both recurrence relation for a_{k+1} in terms of a_k and also inductive hypothesis in attempt to derive required form for a_{k+1}	
			$=\left(\frac{11\times5^k+1}{2}\right)+2-2=\frac{11\times5^{(k+1)-1}+1}{2}$	**A1**	2.5		
			So if true for $n=k$ then true also for $n=k+1$. But since it is true for $n=1$ then it must be true for all integers $n\geqslant1$.	**E1**	2.4		
				[5]			

Question			Answer	Marks	AO	Guidance	
4	(i)		**DR**				
			$x^2-2x+10=(x-1)^2+9$	**M1**	3.1a	Completing the square	
			$\Rightarrow=\int_1^4 \frac{1}{(x-1)^2+9}\mathrm{d}x=\frac{1}{3}\left[\tan^{-1}\left(\frac{x-1}{3}\right)\right]_1^4$	**A1**	1.1a	Ignore limits	
			$=\frac{1}{3}\left(\frac{\pi}{4}-0\right)=\frac{\pi}{12}$	**A1**	1.1		
				[3]			
	(ii)		**DR**				
			Mean value $=\frac{1}{\left(\frac{1}{2}-0\right)}\int_0^{\frac{1}{2}}\frac{1}{\sqrt{1-x^2}}\mathrm{d}x$	**M1**	1.1a	Uses formula for mean value	
			$=2\left[\sin^{-1}x\right]_0^{\frac{1}{2}}=2\left(\frac{1}{6}\pi-0\right)$	**M1**	1.1	Uses formula for integral and attempts substitution of limits	
			$=\frac{1}{3}\pi$	**A1**	1.1		
				[3]			

Question			Answer	Marks	AO	Guidance	
5	(i)		$\mathbf{n_1}.\mathbf{n_2} = 9 = \lvert\mathbf{n_1}\rvert\lvert\mathbf{n}_2\rvert\cos\theta = \sqrt{14}\sqrt{6}\cos\theta$	**M1**	1.1a		$\mathbf{n_1} = \begin{pmatrix}3\\2\\1\end{pmatrix}$ and $\mathbf{n_2} = \begin{pmatrix}2\\1\\1\end{pmatrix}$
			$\Rightarrow \cos\theta = \dfrac{9}{\sqrt{84}}$	**A1**	1.1	soi	
			$\Rightarrow \theta \approx 10.9°$ or 0.190 rads	**A1**	1.1		
				[3]			
	(ii)		Direction of line is $\begin{pmatrix}1\\-1\\-1\end{pmatrix}$	**B1**	2.2a	direction	
			$\mathbf{n}_1.\begin{pmatrix}1\\-1\\-1\end{pmatrix} = 0$ and $\mathbf{n}_2.\begin{pmatrix}1\\-1\\-1\end{pmatrix} = 0$	**M1**	3.1a	Scalar product with both normals (shown, not just stated)	**OR** **M1** for attempt to find vector product of the normals
				A1	1.1	finding zero twice	**A1** for showing it to be a multiple of the direction vector
			Point (0, 1, 2) is on *L*: Π_1 : $3\times0+2\times1+2=0+2+2=4$ ✓	**M1**	3.1a	A point on the line into both equations	
			Π_2: $2\times0+1+2=3$ ✓ So a point on *L* lies in both planes. But *L* is parallel to both planes so *L* lies in both planes.	**A1**	3.2a	showing it is in both planes convincingly and conclusion	

			Or Equation of line is $\mathbf{r} = \begin{pmatrix}0\\1\\2\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\-1\end{pmatrix}$ or $\begin{pmatrix}\lambda\\1-\lambda\\2-\lambda\end{pmatrix}$	**B1**			
			$\Pi_1 : \mathbf{r}.\begin{pmatrix}3\\2\\1\end{pmatrix} = \begin{pmatrix}\lambda\\1-\lambda\\2-\lambda\end{pmatrix}.\begin{pmatrix}3\\2\\1\end{pmatrix}$	**M1**		Substituting into equation of Π_1 or Π_2	
			$= 3\lambda + 2 - 2\lambda + 2 - \lambda = 4$	**A1**			
			$\Pi_2 : \mathbf{r}.\begin{pmatrix}2\\1\\1\end{pmatrix} = \begin{pmatrix}\lambda\\1-\lambda\\2-\lambda\end{pmatrix}.\begin{pmatrix}2\\1\\1\end{pmatrix}$ $= 2\lambda + 1 - \lambda + 2 - \lambda = 3$	**M1**		Substituting into other plane equation	
			So L lies in both planes.	**A1**		Correct values and conclusion	
			Or $y = 1 - x,\ z = 2 - x$	**B1**		Expressing two of x, y and z correctly in terms of the other	
			$\Pi_1 : 3x + 2y + z = 3x + 2(1-x) + 2 - x$	**M1**		Substituting into equation of Π_1	
			$= 3x + 2 - 2x + 2 - x = 4$	**A1**			
			$\Pi_2 : 2x + y + z = 2x + 1 - x + 2 - x$	**M1**		Substituting into equation of Π_2	
			$= 3$ So L lies in both planes.	**A1**		Correct values and conclusion.	
				[5]			

Question			Answer	Marks	AO	Guidance	
6	(i)		$\frac{1}{(2r-1)(2r+1)} = \frac{A}{(2r-1)} + \frac{B}{(2r+1)}$	M1	3.1a	Partial fractions	
			$= \frac{1}{2}\left(\frac{1}{(2r-1)} - \frac{1}{(2r+1)}\right)$	A1	1.1		
			$= \frac{1}{2}\left(\frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \ldots + \frac{1}{2n-1} - \frac{1}{2n+1}\right)$	M1	1.1	Use method of differences	
			$= \frac{1}{2}\left(\frac{1}{1} - \frac{1}{2n+1}\right) = \frac{n}{2n+1}$	A1	1.1		
				[4]			
	(ii)		e.g. $\frac{n}{2n+1} = \frac{1}{2} \times \frac{2n+1-1}{2n+1}$ $= \frac{1}{2} \times \left(1 - \frac{1}{2n+1}\right) \to \frac{1}{2}$ as $n \to \infty$	B1ft	2.2a	**FT** from *their* answer providing it is of the form $\frac{an+b}{cn+d}$	
				[1]			

Question			Answer	Marks	AO	Guidance	
7			**DR** $\frac{x+1}{x^3-x^2+x-1}=\frac{x+1}{(x^2+1)(x-1)}$	**M1**	3.1a	Factorise denominator	
			$=\frac{Ax+B}{(x^2+1)}+\frac{C}{(x-1)}$	**M1**	2.1	Split into partial fractions	
			$\Rightarrow (Ax+B)(x-1)+C(x^2+1)=x+1$ $x=1: 2C=2 \Rightarrow C=1$ $x=0: -B+C=1 \Rightarrow B=0$ $x=2: 2A+5=3 \Rightarrow A=-1$	**M1**	3.1a	Valid method for finding *A*, *B* and *C*	
			$\Rightarrow \frac{x+1}{(x^2+1)(x-1)}=\frac{1}{(x-1)}-\frac{x}{(x^2+1)}$	**A1**	1.1		
			$\Rightarrow \int_2^3 \frac{x+1}{x^3-x^2+x-1}\mathrm{d}x=\int_2^3\left(\frac{1}{(x-1)}-\frac{x}{(x^2+1)}\right)\mathrm{d}x$				
			$\left[\ln(x-1)-\frac{1}{2}\ln(x^2+1)\right]_2^3=\ln 2-\frac{1}{2}\ln 10+\frac{1}{2}\ln 5$	**M1**	1.1	Substituting correct limits into their integral of the form $\alpha\ln(x-1)+\beta\ln(x^2+1)$	
			$=\frac{1}{2}\ln 2$	**A1**	2.2a	oe in correct form (not eg ln√2)	
				[6]			

Question			Answer	Marks	AO	Guidance	
8	(i)		$2\sinh x.\cosh x = 2\left(\frac{e^x - e^{-x}}{2}\right)\left(\frac{e^x + e^{-x}}{2}\right)$ $= \frac{1}{2}\left(e^{2x} - e^{-2x} - 1 + 1\right) = \frac{1}{2}\left(e^{2x} - e^{-2x}\right) = \sinh 2x$	M1 A1 [2]	2.1 2.1	Use of correct exponential form for sinh or cosh **AG** so evidence of cancellation or difference between squares required	Proof must be properly completed for **A1**; ie there must be complete reasoning from LHS = … to … = RHS
	(ii)		$y = a\cosh x - \cosh 2x$ $\Rightarrow \frac{dy}{dx} = a\sinh x - 2\sinh 2x$ $= a\sinh 0 - 2\sinh 0 = 0$ when $x = 0$	M1 A1 [2]	1.1a 1.1	Diffn and verify = 0 by substitution (which must be seen) oe	
	(iii)		when $x = 0,\ y = a - 1$ so $(0,\ a-1)$	B1 [1]	1.1		
	(iv)		$\frac{dy}{dx} = a\sinh x - 4\sinh x\cosh x = 0$ when $\sinh x = 0$ or $\cosh x = \frac{a}{4}$ $\cosh x = \frac{a}{4}$ has two values if $\frac{a}{4} > 1$ and no values if $\frac{a}{4} < 1$ $\frac{a}{4} = 1$ gives the same stationary point as previously identified, so maximum value of a is 4	M1 A1 M1 A1 [4]	2.1 2.1 2.2a 2.2a	For considering the number of possible values of cosh$x$ For full justification	

	(v)		When $a=6, \cosh x=\frac{3}{2} \Rightarrow x=\cosh^{-1}\frac{3}{2}$ $y=6\cosh\left(\cosh^{-1}\frac{3}{2}\right)-\cosh\left(2\cosh^{-1}\frac{3}{2}\right)=\frac{11}{2}$ i.e. $\left(\cosh^{-1}\frac{3}{2}, \frac{11}{2}\right)$	**M1** **A1** **M1** **A1** **[4]**	3.1a 1.1 1.1 1.1	BC. Substituting in *x* value. Can be implied by correct answer. Accept awrt 5.500	

Question			Answer	Marks	AO	Guidance	
9	(i)		Lower half of loop shaded	**B1** **[1]**	1.1	Below axis of symmetry	
	(ii)		$A=\frac{1}{2}\int_0^{\frac{1}{6}\pi} r^2 \mathrm{d}\theta = 8\int_0^{\frac{1}{6}\pi}\sin^2 3\theta \mathrm{d}\theta$ $=4\int_0^{\frac{1}{6}\pi}(1-\cos 6\theta)\mathrm{d}\theta$ $=4\left[\theta-\frac{\sin 6\theta}{6}\right]_0^{\frac{1}{6}\pi}$ $=\frac{2}{3}\pi$	**M1** **M1** **A1** **A1** **[4]**	3.1a 1.1 1.1 1.1	Use of correct formula Use trig identity and integrate	

Question			Answer	Marks	AO	Guidance	
10	(i)		$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\dots$ $\Rightarrow \ln(1+3x^2)=(3x^2)-\frac{(3x^2)^2}{2}+\frac{(3x^2)^3}{3}-\frac{(3x^2)^4}{4}+\dots$ $=3x^2-\frac{9x^4}{2}+9x^6-\frac{81x^8}{4}+\dots.$	**M1** **A1** **[2]**	1.1a 1.1	Substitute $3x^2$ into standard formula or $\frac{27x^6}{3}$	
	(ii)		Valid for $3x^2 \leqslant 1 \Rightarrow \lvert x\rvert \leqslant \frac{1}{\sqrt{3}}$ **oe**	**B1** **[1]**	2.2a		
	(iii)		Substitute $x=\frac{1}{2}$ $\Rightarrow \ln(1+3x^2)=\ln\left(1+\frac{3}{4}\right)$ $=3\left(\frac{1}{2}\right)^2-\frac{9}{2}\left(\frac{1}{2}\right)^4+\frac{27}{3}\left(\frac{1}{2}\right)^6-\frac{81}{4}\left(\frac{1}{2}\right)^8+\dots.$ $\Rightarrow \ln\left(\frac{7}{4}\right)=3\left(\frac{1}{2}\right)^2\left(1-\frac{3}{2}\left(\frac{1}{2}\right)^2+\frac{3^2}{3}\left(\frac{1}{2}\right)^4-\frac{3^3}{4}\left(\frac{1}{2}\right)^6+\dots\right)$ $\Rightarrow \frac{3}{2}\left(\frac{1}{2}\right)^2+\frac{9}{3}\left(\frac{1}{2}\right)^4-\frac{3^3}{4}\left(\frac{1}{2}\right)^6+\dots=1-\frac{4}{3}\ln\left(\frac{7}{4}\right)$	**M1** **M1** **A1** **[3]**	3.1a 1.1 3.2a	Deducing the required value for x Rearranging either series (with either ½ or x) to show the relationship with the other	

Question			Answer	Marks	AO	Guidance	
11	(i)		$\frac{dx}{dt}+0.1x=\cos t e^{-0.1t} \Rightarrow$ I.F = $e^{0.1t}$ $\Rightarrow e^{0.1t}\times\frac{dx}{dt}+0.1e^{0.1t}\times x=\cos t$ $\Rightarrow \frac{d}{dx}\left(e^{0.1t}x\right)=\cos t$ $\Rightarrow e^{0.1t}x=\sin t+c \Rightarrow x=\left(\sin t+c\right)e^{-0.1t}$	**B1** **M1** **A1** **[3]**	3.3 3.4 3.4	Multiplying throughout by IF and writing RHS as an exact derivative of a product	
	(ii)		$x=\left(\sin t+c\right)e^{-0.1t};\ x=10,\ t=0 \Rightarrow c=10$ $x=\left(\sin t+10\right)e^{-0.1t}$	**M1** **A1** **[2]**	3.4 3.4	Substituting x = 10 and t = 0 to find a value for c	
	(iii)	(a)	5.33477	**B1** **[1]**	3.4	to 6sf or better	
	(iii)	(b)	5.33486	**B1** **[1]**	3.4	to 6sf or better	
	(iv)		E.g. the amount at 6.25 hours is higher than at 6 hours, so the amount is not always decreasing, and so model is not appropriate (for predicting decay)	**B1** **[1]**	3.5a	Conclusion required	