Oxford Cambridge and RSA

A Level Further Mathematics A

Y540/01 Pure Core 1

Practice Paper – Set 2

Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 **In this question you must show detailed reasoning.**

For the complex number z it is given that $|z| = 2$ and $\arg z = \frac{1}{6}\pi$.

Find the following in the form $a + \mathrm{i}b$, where a and b are exact numbers.

(i) z **[2]**

(ii) z^2 **[2]**

(iii) $\frac{z}{z^*}$ **[3]**

2 The loci C_1 and C_2 are given by $|z-1| = 5$ and $\arg(z+4+4\mathrm{i}) = \frac{1}{4}\pi$ respectively.

(i) Sketch on a single Argand diagram the loci C_1 and C_2. **[4]**

(ii) Indicate by shading on your Argand diagram the following set of points.

$$\{z : |z-1| \leqslant 5\} \cap \{z : 0 \leqslant \arg(z+4+4\mathrm{i}) \leqslant \tfrac{1}{4}\pi\}$$ **[2]**

3 A sequence is defined by $a_1 = 6$ and $a_{n+1} = 5a_n - 2$ for $n \geqslant 1$.

Prove by induction that for all integers $n \geqslant 1$, $a_n = \dfrac{11 \times 5^{n-1} + 1}{2}$. **[5]**

4 **In this question you must show detailed reasoning.**

Find the exact value of each of the following.

(i) $\displaystyle\int_1^4 \frac{1}{x^2 - 2x + 10}\,\mathrm{d}x$ **[3]**

(ii) The mean value of $\dfrac{1}{\sqrt{1-x^2}}$ in the interval $[0, 0.5]$ **[3]**

5 Two planes, Π_1 and Π_2, have equations $3x + 2y + z = 4$ and $2x + y + z = 3$ respectively.

(i) Find the acute angle between Π_1 and Π_2. **[3]**

The line L has equation $x = 1 - y = 2 - z$.

(ii) Show that L lies in both planes. **[5]**

6 **(i)** Find as a single algebraic fraction an expression for $\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)}$. **[4]**

(ii) Determine the value of $\sum_{r=1}^{\infty} \frac{1}{(2r-1)(2r+1)}$. **[1]**

7 **In this question you must show detailed reasoning.**

Find $\int_2^3 \frac{x+1}{x^3-x^2+x-1}\,dx$, expressing your answer in the form $a\ln b$ where a and b are rational numbers. **[6]**

8 **(i)** Using the definitions of $\cosh x$ and $\sinh x$ in terms of e^x and e^{-x}, show that $\sinh 2x = 2\sinh x\cosh x$. **[2]**

You are given the function $f(x) = a\cosh x - \cosh 2x$, where a is a positive constant.

(ii) Verify that, for any value of a, the curve $y = f(x)$ has a stationary point on the y-axis. **[2]**

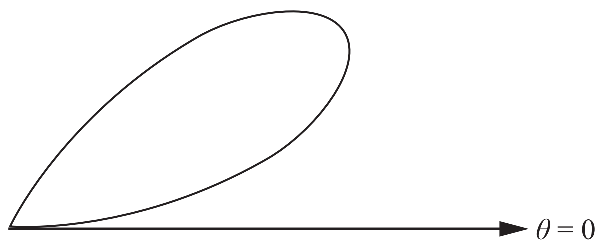
(iii) Find the coordinates of the stationary point found in part **(ii)**. **[1]**

(iv) Determine the maximum value of a for which the stationary point found in part **(ii)** is the only stationary point on the curve $y = f(x)$. **[4]**

You are given that for any value of a greater than the value found in part **(iv)** there are three stationary points, the one found in part **(ii)** and two others, one of which satisfies $x > 0$.

(v) Find the coordinates of this point when $a = 6$.
Give your answer in the form $(\cosh^{-1}p, q)$. **[4]**

9 The diagram below shows the curve $r = 4\sin 3\theta$ for $0 \leqslant \theta \leqslant \frac{1}{3}\pi$.



(i) On the diagram in your Printed Answer Booklet, shade the region R for which

$$r \leqslant 4\sin 3\theta \text{ and } 0 \leqslant \theta \leqslant \tfrac{1}{6}\pi.$$ **[1]**

In this question you must show detailed reasoning.

(ii) Find the exact area of the region R. **[4]**

10 **(i)** Using the Maclaurin series for $\ln(1+x)$, find the first four terms in the series expansion for $\ln(1+3x^2)$. **[2]**

(ii) Find the range of x for which the expansion is valid. **[1]**

(iii) Find the exact value of the series

$$\frac{3^1}{2\times 2^2}-\frac{3^2}{3\times 2^4}+\frac{3^3}{4\times 2^6}-\frac{3^4}{5\times 2^8}+\dots .$$

[3]

11 A particular radioactive substance decays over time.

A scientist models the amount of substance, x grams, at time t hours by the differential equation

$$\frac{dx}{dt}+\frac{1}{10}x=e^{-0.1t}\cos t.$$

(i) Solve the differential equation to find the general solution for x in terms of t. **[3]**

Initially there was 10 g of the substance.

(ii) Find the particular solution of the differential equation. **[2]**

(iii) Find to 6 significant figures the amount of substance that would be predicted by the model at

(a) 6 hours, **[1]**

(b) 6.25 hours. **[1]**

(iv) Comment on the appropriateness of the model for predicting the amount of substance over time. **[1]**

END OF QUESTION PAPER