



Oxford Cambridge and RSA

# A Level Further Mathematics A

## Y540/01 Pure Core 1

### Practice Paper – Set 2

Time allowed: 1 hour 30 minutes

**You must have:**

- Printed Answer Booklet
- Formulae A Level Further Mathematics A

**You may use:**

- a scientific or graphical calculator

#### INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

#### INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [ ].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

**1 In this question you must show detailed reasoning.**

For the complex number  $z$  it is given that  $|z| = 2$  and  $\arg z = \frac{1}{6}\pi$ .

Find the following in the form  $a + ib$ , where  $a$  and  $b$  are exact numbers.

(i)  $z$  [2]

(ii)  $z^2$  [2]

(iii)  $\frac{z}{z^*}$  [3]

**2 The loci  $C_1$  and  $C_2$  are given by  $|z - 1| = 5$  and  $\arg(z + 4 + 4i) = \frac{1}{4}\pi$  respectively.**

(i) Sketch on a single Argand diagram the loci  $C_1$  and  $C_2$ . [4]

(ii) Indicate by shading on your Argand diagram the following set of points.

$$\{z: |z - 1| \leq 5\} \cap \{z: 0 \leq \arg(z + 4 + 4i) \leq \frac{1}{4}\pi\}$$
 [2]

**3 A sequence is defined by  $a_1 = 6$  and  $a_{n+1} = 5a_n - 2$  for  $n \geq 1$ .**

Prove by induction that for all integers  $n \geq 1$ ,  $a_n = \frac{11 \times 5^{n-1} + 1}{2}$ . [5]

**4 In this question you must show detailed reasoning.**

Find the exact value of each of the following.

(i)  $\int_1^4 \frac{1}{x^2 - 2x + 10} dx$  [3]

(ii) The mean value of  $\frac{1}{\sqrt{1-x^2}}$  in the interval  $[0, 0.5]$  [3]

**5 Two planes,  $\Pi_1$  and  $\Pi_2$ , have equations  $3x + 2y + z = 4$  and  $2x + y + z = 3$  respectively.**

(i) Find the acute angle between  $\Pi_1$  and  $\Pi_2$ . [3]

The line  $L$  has equation  $x = 1 - y = 2 - z$ .

(ii) Show that  $L$  lies in both planes. [5]

- 6 (i) Find as a single algebraic fraction an expression for  $\sum_{r=1}^n \frac{1}{(2r-1)(2r+1)}$ . [4]
- (ii) Determine the value of  $\sum_{r=1}^{\infty} \frac{1}{(2r-1)(2r+1)}$ . [1]

7 In this question you must show detailed reasoning.

Find  $\int_2^3 \frac{x+1}{x^3-x^2+x-1} dx$ , expressing your answer in the form  $a \ln b$  where  $a$  and  $b$  are rational numbers. [6]

- 8 (i) Using the definitions of  $\cosh x$  and  $\sinh x$  in terms of  $e^x$  and  $e^{-x}$ , show that  $\sinh 2x = 2 \sinh x \cosh x$ . [2]

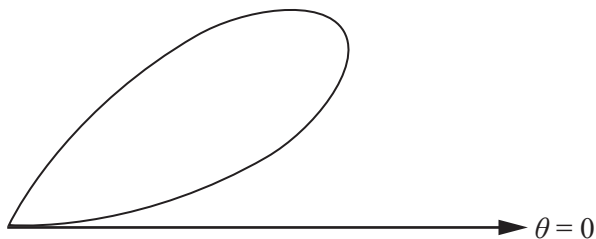
You are given the function  $f(x) = a \cosh x - \cosh 2x$ , where  $a$  is a positive constant.

- (ii) Verify that, for any value of  $a$ , the curve  $y = f(x)$  has a stationary point on the  $y$ -axis. [2]
- (iii) Find the coordinates of the stationary point found in part (ii). [1]
- (iv) Determine the maximum value of  $a$  for which the stationary point found in part (ii) is the only stationary point on the curve  $y = f(x)$ . [4]

You are given that for any value of  $a$  greater than the value found in part (iv) there are three stationary points, the one found in part (ii) and two others, one of which satisfies  $x > 0$ .

- (v) Find the coordinates of this point when  $a = 6$ .  
Give your answer in the form  $(\cosh^{-1} p, q)$ . [4]

- 9 The diagram below shows the curve  $r = 4 \sin 3\theta$  for  $0 \leq \theta \leq \frac{1}{3}\pi$ .



- (i) On the diagram in your Printed Answer Booklet, shade the region  $R$  for which

$$r \leq 4 \sin 3\theta \text{ and } 0 \leq \theta \leq \frac{1}{6}\pi. \quad [1]$$

In this question you must show detailed reasoning.

- (ii) Find the exact area of the region  $R$ . [4]

- 10 (i) Using the Maclaurin series for  $\ln(1+x)$ , find the first four terms in the series expansion for  $\ln(1+3x^2)$ . [2]

- (ii) Find the range of  $x$  for which the expansion is valid. [1]

- (iii) Find the exact value of the series

$$\frac{3^1}{2 \times 2^2} - \frac{3^2}{3 \times 2^4} + \frac{3^3}{4 \times 2^6} - \frac{3^4}{5 \times 2^8} + \dots$$
 [3]

- 11 A particular radioactive substance decays over time.

A scientist models the amount of substance,  $x$  grams, at time  $t$  hours by the differential equation

$$\frac{dx}{dt} + \frac{1}{10}x = e^{-0.1t} \cos t.$$

- (i) Solve the differential equation to find the general solution for  $x$  in terms of  $t$ . [3]

Initially there was 10 g of the substance.

- (ii) Find the particular solution of the differential equation. [2]

- (iii) Find to 6 significant figures the amount of substance that would be predicted by the model at

- (a) 6 hours, [1]

- (b) 6.25 hours. [1]

- (iv) Comment on the appropriateness of the model for predicting the amount of substance over time. [1]

### END OF QUESTION PAPER

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