OCR
Oxford Cambridge and RSA

A Level Further Mathematics A

Y540/01 Pure Core 1

Practice Paper – Set 3

Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

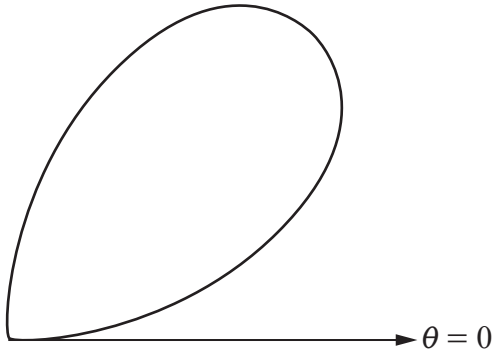
Answer **all** the questions.

1 Points A, B and C have coordinates $(0, 1, -4)$, $(1, 1, -2)$ and $(3, 2, 5)$ respectively.

(a) Find the vector product $\overrightarrow{AB} \times \overrightarrow{AC}$. **[3]**

(b) Hence find the equation of the plane ABC in the form $ax + by + cz = d$. **[2]**

2 The equation of the curve shown on the graph is, in polar coordinates, $r = 3\sin 2\theta$ for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$.



(a) The greatest value of r on the curve occurs at the point P.

(i) Show that $\theta = \frac{1}{4}\pi$ at the point P. **[2]**

(ii) Find the value of r at the point P. **[1]**

(iii) Mark the point P on the copy of the graph in the Printed Answer Booklet. **[1]**

(b) In this question you must show detailed reasoning.

Find the exact area of the region enclosed by the curve. **[5]**

3 You are given that $f(x) = \ln(2+x)$.

(a) Determine the exact value of $f'(0)$. **[2]**

(b) Show that $f''(0) = -\frac{1}{4}$. **[2]**

(c) Hence write down the first three terms of the Maclaurin series for $f(x)$. **[3]**

4 **In this question you must show detailed reasoning.**

You are given that $z = \sqrt{3} + i$.

n is the smallest positive whole number such that z^n is a positive whole number.

(a) Determine the value of n. **[3]**

(b) Find the value of z^n. **[1]**

5 You are given that $\mathbf{A} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 5 & 2 \\ 3 & -2 & -1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 0 & 1 \\ -8 & 4 & 0 \\ 19 & -8 & -1 \end{pmatrix}$.

(a) Find **AB**. **[1]**

(b) Hence write down $\mathbf{A}^{-1}$. **[1]**

(c) You are given three simultaneous equations

$$\begin{aligned} x + 2y + z &= 0 \\ 2x + 5y + 2z &= 1 \\ 3x - 2y - z &= 4 \end{aligned}$$

(i) Explain how you can tell, without solving them, that there is a unique solution to these equations. **[2]**

(ii) Find this unique solution. **[2]**

6 Prove by induction that, for all positive integers n, $7^n + 3^{n-1}$ is a multiple of 4. **[5]**

7 **(a)** Determine an expression for $\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)}$ giving your answer in the form $\frac{1}{4} - \frac{1}{2}f(n)$. **[6]**

(b) Find the value of $\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+2)}$. **[1]**

8 **(a)** Given that $u = \tanh x$, use the definition of $\tanh x$ in terms of exponentials to show that $x = \frac{1}{2}\ln\left(\frac{1+u}{1-u}\right)$. **[4]**

(b) Solve the equation $4\tanh^2 x + \tanh x - 3 = 0$, giving the solution in the form $a\ln b$ where a and b are rational numbers to be determined. **[4]**

(c) Explain why the equation in part **(b)** has only one root. **[1]**

9 **In this question you must show detailed reasoning.**

Find $\int_{-1}^{11} \frac{1}{\sqrt{x^2+6x+13}}\,dx$ giving your answer in the form $\ln\left(p+q\sqrt{2}\right)$ where p and q are integers to be determined. **[7]**

10 In a predator-prey environment the population, at time t years, of predators is x and prey is y. The populations of predators and prey are measured in hundreds.

The populations are modelled by the following simultaneous differential equations.

$$\frac{\mathrm{d}x}{\mathrm{d}t} = y \qquad \frac{\mathrm{d}y}{\mathrm{d}t} = 2y - 5x$$

(a) Show that $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 2\frac{\mathrm{d}x}{\mathrm{d}t} - 5x$. **[2]**

(b) **(i)** Find the general solution for x. **[3]**

(ii) Find the equivalent general solution for y. **[2]**

Initially there are 100 predators and 300 prey.

(c) Find the particular solutions for x and y. **[5]**

(d) Determine whether the model predicts that the predators will die out before the prey. **[4]**

END OF QUESTION PAPER

BLANK PAGE

BLANK PAGE