



Oxford Cambridge and RSA

# A Level Further Mathematics A

## Y540/01 Pure Core 1

### Practice Paper – Set 3

Time allowed: 1 hour 30 minutes

**You must have:**

- Printed Answer Booklet
- Formulae A Level Further Mathematics A

**You may use:**

- a scientific or graphical calculator

#### INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

#### INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [ ].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

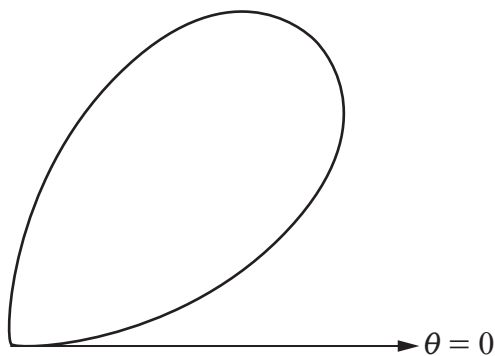
Answer **all** the questions.

- 1 Points  $A$ ,  $B$  and  $C$  have coordinates  $(0, 1, -4)$ ,  $(1, 1, -2)$  and  $(3, 2, 5)$  respectively.

(a) Find the vector product  $\overrightarrow{AB} \times \overrightarrow{AC}$ . [3]

(b) Hence find the equation of the plane  $ABC$  in the form  $ax + by + cz = d$ . [2]

- 2 The equation of the curve shown on the graph is, in polar coordinates,  $r = 3 \sin 2\theta$  for  $0 \leq \theta \leq \frac{1}{2}\pi$ .



- (a) The greatest value of  $r$  on the curve occurs at the point  $P$ .

(i) Show that  $\theta = \frac{1}{4}\pi$  at the point  $P$ . [2]

(ii) Find the value of  $r$  at the point  $P$ . [1]

(iii) Mark the point  $P$  on the copy of the graph in the Printed Answer Booklet. [1]

- (b) **In this question you must show detailed reasoning.**

Find the exact area of the region enclosed by the curve. [5]

3 You are given that  $f(x) = \ln(2+x)$ .

(a) Determine the exact value of  $f'(0)$ . [2]

(b) Show that  $f''(0) = -\frac{1}{4}$ . [2]

(c) Hence write down the first three terms of the Maclaurin series for  $f(x)$ . [3]

4 In this question you must show detailed reasoning.

You are given that  $z = \sqrt{3} + i$ .

$n$  is the smallest positive whole number such that  $z^n$  is a positive whole number.

(a) Determine the value of  $n$ . [3]

(b) Find the value of  $z^n$ . [1]

5 You are given that  $\mathbf{A} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 5 & 2 \\ 3 & -2 & -1 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} 1 & 0 & 1 \\ -8 & 4 & 0 \\ 19 & -8 & -1 \end{pmatrix}$ .

(a) Find  $\mathbf{AB}$ . [1]

(b) Hence write down  $\mathbf{A}^{-1}$ . [1]

(c) You are given three simultaneous equations

$$x + 2y + z = 0$$

$$2x + 5y + 2z = 1$$

$$3x - 2y - z = 4$$

(i) Explain how you can tell, without solving them, that there is a unique solution to these equations. [2]

(ii) Find this unique solution. [2]

- 6 Prove by induction that, for all positive integers  $n$ ,  $7^n + 3^{n-1}$  is a multiple of 4. [5]
- 7 (a) Determine an expression for  $\sum_{r=1}^n \frac{1}{r(r+1)(r+2)}$  giving your answer in the form  $\frac{1}{4} - \frac{1}{2}f(n)$ . [6]
- (b) Find the value of  $\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+2)}$ . [1]
- 8 (a) Given that  $u = \tanh x$ , use the definition of  $\tanh x$  in terms of exponentials to show that
- $$x = \frac{1}{2} \ln \left( \frac{1+u}{1-u} \right). \quad [4]$$
- (b) Solve the equation  $4 \tanh^2 x + \tanh x - 3 = 0$ , giving the solution in the form  $a \ln b$  where  $a$  and  $b$  are rational numbers to be determined. [4]
- (c) Explain why the equation in part (b) has only one root. [1]
- 9 In this question you must show detailed reasoning.

Find  $\int_{-1}^{11} \frac{1}{\sqrt{x^2 + 6x + 13}} dx$  giving your answer in the form  $\ln(p + q\sqrt{2})$  where  $p$  and  $q$  are integers to be determined. [7]

- 10** In a predator-prey environment the population, at time  $t$  years, of predators is  $x$  and prey is  $y$ . The populations of predators and prey are measured in hundreds.

The populations are modelled by the following simultaneous differential equations.

$$\frac{dx}{dt} = y \quad \frac{dy}{dt} = 2y - 5x$$

**(a)** Show that  $\frac{d^2x}{dt^2} = 2\frac{dx}{dt} - 5x$ . **[2]**

**(b) (i)** Find the general solution for  $x$ . **[3]**

**(ii)** Find the equivalent general solution for  $y$ . **[2]**

Initially there are 100 predators and 300 prey.

**(c)** Find the particular solutions for  $x$  and  $y$ . **[5]**

**(d)** Determine whether the model predicts that the predators will die out before the prey. **[4]**

**END OF QUESTION PAPER**

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