OCR
Oxford Cambridge and RSA

Practice Paper – Set 1

A Level Further Mathematics A

Y541/01 Pure Core 2

MARK SCHEME

Duration: 1 hour 30 minutes

MAXIMUM MARK 75

DRAFT

This document consists of 12 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for ALevel Further Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f.Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	**(i)**		$\begin{pmatrix}2\\2\\3\end{pmatrix}\cdot\begin{pmatrix}3\\-1\\2\end{pmatrix}=10$	**M1***	1.1a	Correct process for finding the dot product of normal to the plane and direction vector of line.	
			$\cos(90-\theta)=\dfrac{10}{\sqrt{17}\sqrt{14}}\quad(=\sin\theta)$	**M1dep***	1.1	Correct process for using dot product to find the cosine of the complement or sin of angle	
			(= 90 – 49.6) = awrt 40.4	**A1** **[3]**	1.1	Or awrt 0.705	
1	**(ii)**		$\left(\begin{pmatrix}1\\0\\5\end{pmatrix}+\lambda\begin{pmatrix}2\\2\\3\end{pmatrix}\right)\cdot\begin{pmatrix}3\\-1\\2\end{pmatrix}=33$	**M1**	1.1a	Correct substitution line form for **r** into **r.n** = d	or $3(1+2\lambda) - 2\lambda + 2(5 + 3\lambda) = 33$
			$\lambda = 2$	**A1**	1.1	Correct substitution line form for **r** into **r.n** = d	
			(5, 4, 11)	**A1** **[3]**	1.1	Condone position vector	
1	**(iii)**		$\dfrac{\lvert 3\times 4+(-1)\times 5+2\times(-5)-33\rvert}{\sqrt{3^2+(-1)^2+2^2}}$	**M1**	1.1a	Substitution of *S* and the normal to the plane into the formula.	
			$\dfrac{18\sqrt{14}}{7}$	**A1** **[2]**	1.1	or $\dfrac{36}{\sqrt{14}}$	

Question			Answer	Marks	AO	Guidance	
2	(i)		$2^2+2\times2\times\mathrm{i}+\mathrm{i}^2$ $= 4 + 4\mathrm{i} - 1 = 3+4\mathrm{i}$	**M1** **A1** **[2]**	1.1a 1.1	**AG**	$\mathrm{i}^2 = -1$ must be shown
2	(ii)		Both z and z^2 correctly located	**B1** **[1]**	1.1	If clearly labelled then award mark unless wildly inaccurate	
2	(iii)		$\lvert z^2\rvert = \lvert z\rvert^2$.	**B1** **[1]**	1.2		
2	(iv)		$\arg(z^2) = 2\arg(z)$.	**B1** **[1]**	1.2		

Question			Answer	Marks	AO	Guidance	
3			**DR** $\frac{1}{6}\times300\times(300+1)(2\times300+1)$ $\frac{1}{6}\times120\times(120+1)(2\times120+1)$ (= 9045050 – 583220) = 8461830	 **M1** **M1** **A1** **[3]**	 1.1a 2.1 2.2a	Any correct numerical use of formula Must be 120, not 121 (unless corrected by 121^2 later).	Implied by 9045050 seen Implied by 583220 seen

Question			Answer	Marks	AO	Guidance	
4			**DR**				
			Substitute $y=\frac{1}{x}$	**M1**	3.1a		
			$\Rightarrow \frac{2}{y^3}-\frac{3}{y^2}+\frac{1}{y}+4=0$	**A1**	1.1		
			$\Rightarrow 4y^3+y^2-3y+2=0$				
			has roots $\frac{1}{\alpha},\frac{1}{\beta},\frac{1}{\gamma}$	**M1**	1.1a		
			$\Rightarrow \frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=-\frac{1}{4}$	**A1**	1.1		
				[4]			

Question			Answer	Marks	AO	Guidance	
5			**DR**				
			$\frac{1}{(1.5-0)}\int_0^{1.5}\frac{1}{\sqrt{9-t^2}}\mathrm{d}t$	**B1**	3.4	Using the formula for mean value	May be implied by correct integral and later division by 1.5
			$\left[\sin^{-1}\frac{t}{3}\right]$	**M1**	3.4	Ignore limits	or $\left[\cos^{-1}\frac{t}{3}\right]$
			$\sin^{-1}\frac{1.5}{3}$	**A1**	3.4	Correct use of limits.	or $\cos^{-1}\frac{1.5}{3}-\cos^{-1}0$
			$\frac{\pi}{6}$	**A1**	1.1	Soi by answer	
			So average velocity is 0.35 cms^{-1} along *OA*	**A1**	3.4	correct unit must be present or accept awrt 0.349 or $\frac{\pi}{9}$	
				[5]			

Question			Answer	Marks	AO	Guidance	
6	(i)		**DR**			Either term correct	
			$12\cosh x \sinh x$ or $-13\sinh x$	**M1**	3.1a		
			$\dfrac{dy}{dx} = 12\cosh x \sinh x - 13\sinh x$	**A1**	1.1	Both correct	
			$\sinh x(12\cosh x - 13) = 0$	**M1**	3.1a	Setting to zero and attempting to solve	
			$x = 0$, $y = -13$	**A1**	1.1		First solution
			$x = \ln\left(\dfrac{13}{12} + \sqrt{\left(\dfrac{13}{12}\right)^2 - 1}\right)$	**M1**	1.1	Correct numerical use of formula for $\cosh^{-1}$ (could be ±)	
			$x = \ln\left(\dfrac{3}{2}\right)$	**A1**	1.1	or $x = -\ln\left(\dfrac{2}{3}\right)$	Second solution
			$y = -\dfrac{313}{24}$	**A1**	1.1	ft from "their" x	
			or $x = \ln\left(\dfrac{2}{3}\right)$	**A1**	2.2a	From the symmetry of the sinh and cosh functions (could be explicitly calculated)	Third solution
			$y = -\dfrac{313}{24}$	**A1**	2.2a	Or $x = -\ln\left(\dfrac{3}{2}\right)$	
				[9]			
6	(ii)		**DR**				
			$\dfrac{d^2y}{dx^2} = 12\cosh^2 x + 12\sinh^2 x - 13\cosh x$	**M1**	2.5		
			$x = 0 \Rightarrow \dfrac{d^2y}{dx^2} = -1 < 0$ so maximum point	**A1**	3.2a	Correct values only. Must be explicit.	
			$x = \pm\ln\dfrac{3}{2} \Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{25}{12} > 0$ so minimum points	**A1**	3.2a	Correct values only. Must be explicit. Must be both.	
				[3]			

Question			Answer	Marks	AO	Guidance	
7	(i)		$\begin{pmatrix} 3 & 2 & -1 \\ 2 & -4 & 7 \\ 10 & 20 & -25 \end{pmatrix}^{-1} = \frac{1}{40}\begin{pmatrix} -40 & 30 & 10 \\ 120 & -65 & -23 \\ 80 & -40 & -16 \end{pmatrix}$ **oe**	**M1**	1.1a	**BC**	eg $\begin{pmatrix} -1 & \frac{3}{4} & \frac{1}{4} \\ 3 & -\frac{13}{8} & -\frac{23}{40} \\ 2 & -1 & -\frac{2}{5} \end{pmatrix}$
			$\frac{1}{40}\begin{pmatrix} -40 & 30 & 10 \\ 120 & -65 & -23 \\ 80 & -40 & -16 \end{pmatrix}\begin{pmatrix} 5 \\ 60 \\ b \end{pmatrix} = \begin{pmatrix} 40+\frac{1}{4}b \\ -\frac{3300+23b}{40} \\ -50-\frac{2b}{5} \end{pmatrix}$	**A1**	1.1	For any correct component	
			$x = 40+\frac{1}{4}b,\ y = -\frac{3300+23b}{40},\quad z = -50-\frac{2b}{5}$	**A1** **[3]**	1.1	All three correct (**oe**)	
7	(ii)		$\begin{vmatrix} 3 & 2 & -1 \\ 2 & -4 & 7 \\ a & 20 & -25 \end{vmatrix}$ $= 3(100-140)-2(-50-7a)-1(40+4a)$	**M1**	3.1a	Consideration of determinant of correct matrix and genuine attempt to find determinant.	
			$10a-60$	**A1**	1.1		
			$a=6$	**A1** **[3]**	2.2a	Set to 0 and solve	
7	(iii)	(a)	$2\alpha-4\beta=20$ or $-\alpha+7\beta=-25$	**M1**	2.2a		

			$\begin{pmatrix} 2 & -4 \\ -1 & 7 \end{pmatrix}^{-1}\begin{pmatrix} 20 \\ -25 \end{pmatrix} = \frac{1}{10}\begin{pmatrix} 7 & 4 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 20 \\ -25 \end{pmatrix}$	**M1**	1.1	or any valid method to find α and β	
			$\alpha = 4$, $\beta = -3$	**A1** **[3]**	1.1		
7	**(iii)**	**(b)**	$4 \times 5 + (-3) \times 60$	**M1**	2.2a	Their α, β.	
			-160	**A1** **[2]**	1.1		

7	**(iii)**	**(c)**	The three planes form a sheaf with a common line of intersection. oe	**E1** **[1]**	3.2a	Do not ISW (eg if "or two of the planes may be parallel" then **E0**).	

Question			Answer	Marks	AO	Guidance	
8			**DR**				
			$f(x) = x^3 - 3x^2 + 4x - 12$ and $f(3) = 0$	**M1**	3.1a	Attempt to find a factor of the denominator	Could be by factorising pairs of terms
			$x - 3$ is a factor	**A1**	2.2a		
			$f(x) = (x-3)(x^2+4)$	**A1**	2.2a	Deduce by inspection but may see symbolic division oe.	
			$\dfrac{2x^2+3x-1}{x^3-3x^2+4x-12} = \dfrac{A}{x-3} + \dfrac{Bx+C}{x^2+4}$	**B1**	3.1a	Correct partial fraction expression using their factorised cubic	
			$2x^2+3x-1 = A(x^2+4) + (Bx+C)(x-3)$	**M1**	3.1a	A correct method for finding A, B or C (no terms omitted)	
			$A = 2$	**A1**	1.1		
			$C = 3$	**A1**	1.1		
			$B = 0$	**A1**	1.1		
			$\int \frac{1}{x-3}\,dx = \ln\lvert x-3 \rvert$	**B1**	1.1a	Ignore multiplicative or additive constant. Condone omitted modulus signs. Allow for (any one of) their linear factor(s)	
			$\int \frac{1}{x^2+4}\,dx = \frac{1}{2}\tan^{-1}\frac{x}{2}$	**B1**	1.1	Ignore additive constant. Allow for their (numerical) a.	

			$2\ln\lvert 2-3\rvert+\frac{3}{2}\tan^{-1}\frac{2}{2}-\left(2\ln\lvert 0-3\rvert+\frac{3}{2}\tan^{-1}\frac{0}{2}\right)$	**M1**	1.1	Correct substitution of limits (and subtraction) into an integral containing ln and $\tan^{-1}$. Condone $\tan^{-1}0$ missing.	
			$-2\ln 3+\frac{3}{2}\times\frac{\pi}{4}=\frac{3}{8}\pi-\ln 9$	**A1** **[12]**	1.1	**AG** Must see evidence of use of laws of logs.	

Question			Answer	Marks	AO	Guidance	
9	**(i)**		**DR** $\mathrm{e}^{\mathrm{i}\theta}-\mathrm{e}^{-\mathrm{i}\theta}=(\cos\theta+\mathrm{i}\sin\theta)-(\cos\theta-\mathrm{i}\sin\theta)$ $=2\mathrm{i}\sin\theta.$	**B1 (AG)** **[1]**	2.1	Must be clear evidence of use of Euler's equation and both $\cos(-\theta)=\cos\theta$ and $\sin(-\theta)=-\sin\theta$ seen or clearly implied.	eg just $(\cos(-\theta)+\mathrm{i}\sin(-\theta))$ is not sufficient.
9	**(ii)**		**DR** $\frac{2\mathrm{e}^{-\mathrm{i}\theta}}{\mathrm{e}^{\mathrm{i}\theta}-\mathrm{e}^{-\mathrm{i}\theta}}$ or $\frac{2}{e^{\mathrm{i}\theta}(e^{\mathrm{i}\theta}-e^{-\mathrm{i}\theta})}$	**B1**	2.1	Condone omission of 2	
			$-\frac{2\mathrm{i}\mathrm{e}^{-\mathrm{i}\theta}}{2\sin\theta}$	**M1**	2.1	Use of **(i)** and 1/i = -i. Condone incorrect 2.	
			$-\frac{\mathrm{i}(\cos\theta-\mathrm{i}\sin\theta)}{\sin\theta}=-(1+\mathrm{i}\cot\theta)$	**A1** **[3]**	2.1	**AG** At least one intermediate step must be shown.	
9	**(iii)**		**DR** $C+\mathrm{i}S=\sum 2\mathrm{e}^{\mathrm{i}\frac{\pi}{10}k}$	**B1**	3.1a	Expressing C + iS as the sum of (multiples of) positive integer powers of $e^{\mathrm{i}\frac{\pi}{10}}$	
			GP with and $r=\mathrm{e}^{\mathrm{i}\frac{\pi}{10}}$ seen or implied	**B1**	2.2a	Could be implied by eg formula even if a and/or n incorrect.	

			$\dfrac{2\left(\left(e^{i\frac{\pi}{10}}\right)^5-1\right)}{e^{i\frac{\pi}{10}}-1}$	M1	2.2a	Use of formula for their series.	
				A1	1.1	All correct	
			$\left(e^{i\frac{\pi}{10}}\right)^5=i$	B1	1.1		
			$\dfrac{2}{e^{i\frac{\pi}{10}}-1}=-(1+i\cot\dfrac{\pi}{20})$	M1	2.2a	Correct use of **(ii)** for their expression.	
			$-(1+i\cot\dfrac{\pi}{20})(i-1)=\cot\dfrac{\pi}{20}+1+i(\cot\dfrac{\pi}{20}-1)$	M1	3.1a	Expanding brackets and collecting	
			$C=\cot\dfrac{\pi}{20}+1$	A1	2.2a		
				[8]			
9	**(iv)**		**DR** $S=\cot\dfrac{\pi}{20}-1=\cot\dfrac{\pi}{20}+1-2=C-2$	B1	2.2a	Must be from fully correct working and solution in **(iii)**.	
			All of the trigonometric terms are, in effect, the same; eg $\cos\dfrac{\pi}{10}=\sin\dfrac{2\pi}{5}$.	B1	2.4	Must be at least one explicit example or an appeal to the statement $\cos\left(\dfrac{\pi}{2}-\theta\right)=\sin\theta$ oe.	
				[2]			