



Oxford Cambridge and RSA

A Level Further Mathematics A

Y541/01 Pure Core 2

Practice Paper – Set 1

Time allowed: 1 hour 30 minutes

You must have:

- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You may use:

- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

- 1 Plane Π has equation $3x - y + 2z = 33$. Line l has the following vector equation.

$$l: \quad \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$$

- (i) Find the acute angle between Π and l . [3]
- (ii) Find the coordinates of the point of intersection of Π and l . [3]
- (iii) S is the point $(4, 5, -5)$. Find the shortest distance from S to Π . [2]

- 2 The complex number $2 + i$ is denoted by z .

- (i) Show that $z^2 = 3 + 4i$. [2]
- (ii) Plot the following on the Argand diagram in the Printed Answer Booklet.
- z
 - z^2 [1]
- (iii) State the relationship between $|z^2|$ and $|z|$. [1]
- (iv) State the relationship between $\arg(z^2)$ and $\arg(z)$. [1]

- 3 **In this question you must show detailed reasoning.**

Use the formula $\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1)$ to evaluate $121^2 + 122^2 + 123^2 + \dots + 300^2$. [3]

- 4 You are given that the cubic equation $2x^3 - 3x^2 + x + 4 = 0$ has three roots, α , β and γ .

By making a suitable substitution to obtain a related cubic equation, determine the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$. [4]

5 In this question you must show detailed reasoning.

An ant starts from a fixed point O and walks in a straight line for 1.5 s. Its velocity, $v \text{ cm s}^{-1}$, can be modelled by $v = \frac{1}{\sqrt{9-t^2}}$.

By finding the mean value of v in $0 \leq t \leq 1.5$, deduce the average velocity of the ant. [5]

6 In this question you must show detailed reasoning.

(i) Find the coordinates of all stationary points on the graph of $y = 6\sinh^2 x - 13\cosh x$, giving your answers in an exact, simplified form. [7]

(ii) By finding the second derivative, classify the stationary points found in part (i). [3]

7 In the following set of simultaneous equations, a and b are constants.

$$\begin{aligned} 3x + 2y - z &= 5 \\ 2x - 4y + 7z &= 60 \\ ax + 20y - 25z &= b \end{aligned}$$

(i) In the case where $a = 10$, solve the simultaneous equations, giving your solution in terms of b . [3]

(ii) Determine the value of a for which there is **no** unique solution for x , y and z . [3]

(iii) (a) Find the values of α and β for which $\alpha(2y - z) + \beta(-4y + 7z) = 20y - 25z$ for any y and z . [3]

(b) Hence, for the case where there is **no** unique solution for x , y and z , determine the value of b for which there is an infinite number of solutions. [2]

(c) When a takes the value in part (ii) and b takes the value in part (iii)(b) describe the geometrical arrangement of the planes represented by the three equations. [1]

8 In this question you must show detailed reasoning.

Show that $\int_0^2 \frac{2x^2 + 3x - 1}{x^3 - 3x^2 + 4x - 12} dx = \frac{3}{8}\pi - \ln 9$. [12]

9 In this question you must show detailed reasoning.

(i) Show that $e^{i\theta} - e^{-i\theta} = 2i \sin \theta$. [1]

(ii) Hence, show that $\frac{2}{e^{2i\theta} - 1} = -(1 + i \cot \theta)$. [3]

(iii) Two series, C and S , are defined as follows.

$$C = 2 + 2 \cos \frac{\pi}{10} + 2 \cos \frac{\pi}{5} + 2 \cos \frac{3\pi}{10} + 2 \cos \frac{2\pi}{5}$$

$$S = 2 \sin \frac{\pi}{10} + 2 \sin \frac{\pi}{5} + 2 \sin \frac{3\pi}{10} + 2 \sin \frac{2\pi}{5}$$

By considering $C + iS$, find a simplified expression for C in terms of only integers and $\cot \frac{\pi}{20}$. [8]

(iv) Verify that $S = C - 2$ and, by considering the series in their original form, explain why this is so. [2]

END OF QUESTION PAPER

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