OCR
Oxford Cambridge and RSA

Practice Paper – Set 2

A Level Further Mathematics A

Y541/01 Pure Core 2

MARK SCHEME

Duration: 1 hour 30 minutes

MAXIMUM MARK 75

FINAL

This document consists of 16 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Further Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(i)		$\frac{x(+0)}{-2}=\frac{y-5}{2}=\frac{z+(-6)}{-7}$ seen or implied by **a** or **d** correct	M1	1.1a	Expressing in form from which **a** and **d** from **r** = **a** + λ**d** can be read off.	Or writing $\frac{-x}{2}=\lambda$ etc and attempting to solve for *x*, *y* and *z*
			$\mathbf{r}=\begin{pmatrix}0\\5\\-6\end{pmatrix}+\lambda\begin{pmatrix}-2\\2\\-7\end{pmatrix}$ oe	A1 **[2]**	1.1	Must be **r** =. Must have sensible parameter name (not eg *x*, *y*, *z* or *r*).	
	(ii)		Any two of $-2\lambda=2+\mu$, $5+2\lambda=7-2\mu$ and $-6-7\lambda=-1+4\mu$ and attempt to solve	M1	1.1a		
			$\lambda=-3$ or $\mu=4$	A1	1.1		
			(6, –1, 15)	A1 **[3]**	1.1		
	(iii)		$\begin{pmatrix}-2\\2\\-7\end{pmatrix}\cdot\begin{pmatrix}1\\-2\\4\end{pmatrix}$ (= –34)	M1*	1.1a	Correct method for finding (modulus of) dot product of $\begin{pmatrix}1\\-2\\4\end{pmatrix}$ and their direction vector	
			$\cos^{-1}\frac{\pm 34}{\sqrt{57}\sqrt{21}}$	dep*M1	1.1	Correct process for finding moduli and correct use (ie inverse cos of ± their dot product divided by the product of their moduli).	
			awrt 10.7° or 0.186 rads www	A1 **[3]**	1.1	Acute angle only	

Question			Answer	Marks	AO	Guidance	
2	(i)		$\int 2\tan x\,\mathrm{d}x = -2\ln(\cos x)$ or $2\ln(\sec x)$	**B1**	1.1a		Limits can be ignored for **M1A1**
			$-2[\ln(\cos x)]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = -2(\ln\cos\frac{\pi}{3} - \ln\cos\frac{\pi}{4})$	**M1**	1.1	Correct use of limits in integral of the form $a\ln f(x)$ where f(x) is trigonometric	
			$= -2\left(\ln\frac{1}{2} - \ln\frac{1}{\sqrt{2}}\right) = -2\ln 2^{-\frac{1}{2}} = \ln 2$ cao	**A1**	1.1		
				[3]			
	(ii)		$\int_0^z 2\tan x\,\mathrm{d}x = -2\ln\cos z$	**M1**	2.1	Using a variable to denote the upper limit and finding the integral. Could be their incorrect integral from **(i)**	Or $\int_0^z 2\tan x\,\mathrm{d}x = 2\ln\sec z$
			So $\int_0^{\frac{\pi}{2}} 2\tan x\,\mathrm{d}x = \lim_{z\to\frac{\pi}{2}}(-2\ln\cos z)$ but $z \to \frac{\pi}{2} \Rightarrow \cos z \to 0 \Rightarrow \ln\cos z \to -\infty$ so the integral is undefined.	**A1**	2.2a	or ln0 is undefined but argument must be complete	Or $\int_0^{\frac{\pi}{2}} 2\tan x\,\mathrm{d}x = \lim_{z\to\frac{\pi}{2}}(2\ln\sec z)$ but $z \to \frac{\pi}{2} \Rightarrow \sec z \to \infty$ $\Rightarrow \ln\sec z \to \infty$ so the integral is undefined.
				[2]			

Question			Answer	Marks	AO	Guidance	
3	(i)		$\begin{pmatrix}1\\1\\3\end{pmatrix}\times\begin{pmatrix}-1\\2\\1\end{pmatrix}=\ldots$	**M1**	1.1a	Attempt to find cross product (can be implied by one correct component (or all 3 with wrong sign)	
			$\ldots\begin{pmatrix}-5\\-4\\3\end{pmatrix}$	**A1**	1.1	Or any non-zero multiple.	
				[2]			
	(ii)		$\mathbf{r}.\begin{pmatrix}-5\\-4\\3\end{pmatrix}=\begin{pmatrix}2\\-3\\5\end{pmatrix}.\begin{pmatrix}-5\\-4\\3\end{pmatrix}=2\times-5+-3\times-4+5\times3$	**M1**	1.1a	Use their **n** from part **(i)** in **r.n** = **a.n** and attempt to find dot product. "Dot product" must be a scalar.	
			$\mathbf{r}.\begin{pmatrix}-5\\-4\\3\end{pmatrix}=17$	**A1**	1.1	Or any non-zero multiple of entire equation	
				[2]			
	(iii)		$\left\|\begin{pmatrix}-5\\-4\\3\end{pmatrix}\right\|=\sqrt{25+16+9}=\sqrt{50}$	**M1**	1.1	Correct method for finding modulus of their normal vector	
			$d=\dfrac{17}{\sqrt{50}}$ cao	**A1**	2.2a	Must be positive	
				[2]			

Question			Answer	Marks	AO	Guidance	
4	(i)		$a = 1$	B1	2.2a	soi	
			$\frac{1}{-24}\begin{pmatrix} 24 & -12 & 0 \\ -48 & 15 & 6 \\ 16 & -6 & -4 \end{pmatrix}\begin{pmatrix} 6 \\ 8 \\ k \end{pmatrix}$	M1	1.1a		
			$x = -2$, $y = 7 - \frac{1}{4}k$, $z = -2 + \frac{1}{6}k$	A1 [3]	1.1		
	(ii)		$a = 2$	B1	2.2a	soi	
			$b = 5$	B1	3.1a		
			$4x - 6z = 18$	M1	3.1a	Use of $2x + 2y + 3z = 5$ and $-2x + 2y + 9z = -13$ to eliminate y or z	
			$z = \frac{2}{3}x - 3$	A1	1.1		
			$y = 7 - 2x$	A1 [5]	1.1		
	(iii)		The solution represents a straight line	B1	2.2a		
			E.g. Two of the planes are identical and the third intersects it/them (in a straight line)	B1 [2]	3.2a		

Question			Answer	Marks	AO	Guidance	
5	(i)		$V=\pi\int_{\sqrt{p}}^{\sqrt{3p}}\left(\frac{1}{\sqrt{p+x^2}}\right)^2 \mathrm{d}x$ soi	**B1**	1.2	Correct limits and function substituted in. Condone missing π.	
			$\int\left(\frac{1}{\sqrt{p+x^2}}\right)^2 \mathrm{d}x=\int\frac{1}{\left(\sqrt{p}\right)^2+x^2}\mathrm{d}x$	**M1**	2.2a	Writing in integrable form. If not seen condone incorrect constant outside or use of p for $\sqrt{p}$ inside for **M1**	
			$=\frac{1}{\sqrt{p}}\tan^{-1}\left(\frac{x}{\sqrt{p}}\right)$	**A1**	1.1		
			$\left[\tan^{-1}\left(\frac{x}{\sqrt{p}}\right)\right]_{\sqrt{p}}^{\sqrt{3p}}=\tan^{-1}\left(\frac{\sqrt{3p}}{\sqrt{p}}\right)-\tan^{-1}\left(\frac{\sqrt{p}}{\sqrt{p}}\right)$	**M1**	1.1	Correct use of limits in an integrated expression of the form $a\tan^{-1}(bx)$	
			$V=\pi\times\frac{1}{\sqrt{p}}\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\frac{\pi^2}{12\sqrt{p}}$ **oe**	**A1**	1.1		
				[5]			
	(ii)		$p=1,\ V\propto\frac{1}{\sqrt{p}}\ \Rightarrow V_{max}=\frac{\pi^2}{12}$	**B1**	2.2a		At least one of V_{max} and V_{min} must be clearly identified as such
			$\sqrt{3p}=\sqrt{48}\Rightarrow p=16$	**M1**	2.2a	Either seen	
			$p=16\Rightarrow V_{min}=\frac{\pi^2}{48}$	**A1**	1.1		If neither identified then B0M1A1 or B1M1A0 are available.
				[3]			

Question			Answer	Marks	AO	Guidance	
6	(i)		$\sum_{r=1}^{n}\left((r+1)^5 - r^5\right) = (n+1)^5 - 1$	B1	1.1a		
			$(r+1)^5 - r^5 = 5r^4 + 10r^3 + 10r^2 + 5r + 1$	M1	3.1a	Binomial expansion of $(r+1)^5$ with r^5 cancelling	
			Use of $\sum_{r=1}^{n}1 = n$ and $\sum_{r=1}^{n}r = \frac{n(n+1)}{2}$	B1	1.1		
			$(n+1)^5 - 1 = 5\sum_{r=1}^{n}r^4 + \frac{5n^2(n+1)^2}{2} + \frac{5n(n+1)(2n+1)}{3} + \frac{5n(n+1)}{2} + n$	M1	3.1a	Correct cancellation of sum down to two terms and substitution	
			$30\sum_{r=1}^{n}r^4 = 6n(n^4 + 5n^3 + 10n^2 + 10n + 4) -$ $n(n+1)(15n(n+1) + 15 + 10(2n+1))$	M1	1.1	Expansion and rearrangement leading to (eg) n as being a common factor	or attempt at comparable expansions of $\sum_{r=1}^{n}r^4$ expression and AG.
			Further factorisation leading to $\sum_{r=1}^{n}r^4 = \frac{1}{30}n(n+1)(2n+1)(3n^2 + 3n - 1)$	A1 **[6]**	2.1	No gaps in argument for **A1 AG**	or both expansions correctly and completely demonstrated as equal WWW.
	(ii)		$\sum_{r=1}^{80}r^4 - \sum_{r=1}^{49}r^4$	M1	2.2a	soi	
			(676 010 664 – 59 416 665 =) 616 593 999	A1 **[2]**	1.1		

Question			Answer	Marks	AO	Guidance	
7	(i)		$\alpha+\beta=-\frac{b}{a},\ \alpha\beta=\frac{c}{a}$ used	B1	1.1a		
			$\left(x--\frac{b}{a}\right)\left(x-\frac{c}{a}\right)=0$	M1	3.1a	Or $\alpha'+\beta'=-\frac{b}{a}+\frac{c}{a},\alpha'\beta'=-\frac{b}{a}\times\frac{c}{a}$	
			$x^2+\frac{b-c}{a}x-\frac{bc}{a^2}=0$	A1	1.1	Correctly expanding brackets and collecting terms	
			$a^2x^2+a(b-c)x-bc=0$	A1	2.2a	Or any non-zero integer multiple (ie correctly multiplying by a^2 (or ka^2 or a^3 etc) (could be done earlier))	
				[4]			
	(ii)		Both having repeated roots => $\alpha=\beta$ and $\alpha\beta=\alpha+\beta$ => $\alpha^2=2\alpha$	M1	3.1a	Assume repeated roots and set up equations.	
			so either $\alpha=\beta=0$ => $b=c=0$ or $\alpha=\beta=2$ => $b<0$ (or $a<0$ & $c<0$). But $a, b, c>0$	E1	2.1	For establishing contradiction	
				[2]			
	(iii)		$\Delta=\left(a(b-c)\right)^2-4a^2(-bc)$	M1	3.1a	Correct substitution into "b^2-4ac"	
			$\Delta=a^2(b^2-2bc+c^2+4bc)=a^2(b+c)^2$	A1	1.1		
			which is positive since a, b and c are real and both $a\neq 0$ and $b+c\neq 0$	E1	2.4	$a^2>0$ and $(b+c)^2>0$ alone is insufficient	
				[3]			

Question			Answer	Marks	AO	Guidance	
8	**(i)**		(6 + 5i)(7 + 5i) = 6×7 + 6×5i + 5i×7 + 5×5×-1 17 + 65i	**M1** **A1** **[2]**	1.1a 1.1	Expanding brackets with $i^2 = -1$	If no working **M0A0**
	(ii)		(6 – 5i)(7 – 5i) = 17 – 65i (17 + 65i)(17 – 65i) = $17^2 + 65^2$ = 4514 (6 + 5i)(6 – 5i) & (7 + 5i)(7 – 5i) 4514 = 61×74 so expressed as a product of prime factors 4514 = 2×37×61	**B1ft** **M1** **M1** **A1** **[4]**	1.1 3.1a 3.1a 3.2a	Conjugate of **(i)** Products considered separately Answer with no working **M0M0A0**.	 Method using complex numbers must be shown.

Question			Answer	Marks	AO	Guidance	
9	(i)	(a)	$\frac{d^2x}{dt^2} = 3\frac{dx}{dt} - 2\frac{dy}{dt} = 3\frac{dx}{dt} - 2(y+5x)$	M1	2.1	Differentiating $\frac{dx}{dt} = 3x - 2y$ and substituting in for $\frac{dy}{dt}$ from $\frac{dy}{dt} = y + 5x$	
			$\frac{d^2x}{dt^2} = 3\frac{dx}{dt} - \left(3x - \frac{dx}{dt}\right) - 10x = 4\frac{dx}{dt} - 13x$	A1 [2]	2.2a	Substituting for a second time to leave an equation in *x*, expanding and collecting. No need to see $a = 4$, $b = -13$ explicitly	
		(b)	$m^2 - 4m + 13 = 0$	M1	3.3	Finding the auxiliary equation and attempting to solve	Attempt to solve can be implied either by correct solution or by working (eg $(m-2)^2 + 9 = 0$)
			$m = 2 \pm 3i$	A1	1.1	Can be implied by correct form	
			$x = e^{2t}\left(A\sin 3t + B\cos 3t\right)$	A1ft [3]	3.3		FT on complex solutions to the AE only.

Question			Answer	Marks	AO	Guidance	
	(ii)		$\frac{d^2y}{dt^2}=\frac{dy}{dt}+5\frac{dx}{dt}=\frac{dy}{dt}+5(3x-2y)$	**M1**	2.1	Differentiating $\frac{dy}{dt}=y+5x$ and substituting in for $\frac{dx}{dt}$ from $\frac{dx}{dt}=3x-2y$	
			$\frac{d^2y}{dt^2}=\frac{dy}{dt}+3\left(\frac{dy}{dt}-y\right)-10y=4\frac{dy}{dt}-13y$	**A1**	2.2a	Substituting for a second time to leave an equation in *y*, expanding and collecting.	No need to see a = 4, b = –13 or explicitly
			Same form as for *x*, so $y=e^{2t}\left(C\sin 3t+D\cos 3t\right)$	**B1ft** **[3]**	2.2a	Arbitrary constants must not be "*A*" or "*B*" from **(i)(b)**	FT on their answer to **(i)(b)** with two arbitrary constants
	(iii)		$t=0 \Rightarrow B=4$ & $D=5$	**B1**	3.3		
			$\frac{dy}{dt}=2e^{2t}\left(C\sin 3t+D\cos 3t\right)+e^{2t}\left(3C\cos 3t-3D\sin 3t\right)$	**M1**	3.4	Attempt to differentiate expression for *x* or *y* using the product rule (could have *B* or *D* substituted out)	$\frac{dx}{dt}=2e^{2t}\left(A\sin 3t+B\cos 3t\right)+e^{2t}\left(3A\cos 3t-3B\sin 3t\right)$
			$=y+5x=e^{2t}\left(C\sin 3t+D\cos 3t\right)+5e^{2t}\left(A\sin 3t+B\cos 3t\right)$	**M1**	3.3	Using the correct equation to set up equations in *C* and *D* (could have *B* or *D* substituted out)	$=3x-2y=3e^{2t}\left(A\sin 3t+B\cos 3t\right)-2e^{2t}\left(C\sin 3t+D\cos 3t\right)$ => $2A-3B=3A-2C$ & $2B+3A=3B-2D$
			Comparing coefficients gives $2C-3D=C+5A$, $2D+3C=D+5B$. So $A=-2$, $C=5$	**A1**	1.1		
			Particular solutions are $x=2e^{2t}\left(-\sin 3t+2\cos 3t\right)$ and $y=5e^{2t}\left(\sin 3t+\cos 3t\right)$	**A1** **[5]**	1.1		

Question			Answer	Marks	AO	Guidance	
	(iv)		$x = 0 \Rightarrow \tan 3T = 2$ $\Rightarrow T = 0.369$	**B1** **[1]**	3.4	Setting $x = 0$ to and rearrange to an equation in $\tan 3T$	0.369049…
	(iv)		E.g. The model has continuous quantities, but the number of rabbits can only take integer values.	**E1** **[1]**	3.5a		