OCR
Oxford Cambridge and RSA

A Level Further Mathematics A

Y541/01 Pure Core 2

Practice Paper – Set 2

Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

1 Line l_1 has Cartesian equation

$$l_1: \quad \frac{-x}{2} = \frac{y-5}{2} = \frac{-z-6}{7}.$$

(i) Find a vector equation for l_1. **[2]**

Line l_2 has vector equation

$$l_2: \quad \mathbf{r} = \begin{pmatrix} 2 \\ 7 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}.$$

(ii) Find the point of intersection of l_1 and l_2. **[3]**

(iii) Find the acute angle between l_1 and l_2. **[3]**

2 **In this question you must show detailed reasoning.**

(i) Find $\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} 2\tan x \mathrm{d}x$ giving your answer in the form $\ln p$. **[3]**

(ii) Show that $\int_{0}^{\frac{1}{2}\pi} 2\tan x \mathrm{d}x$ is undefined explaining your reasoning. **[2]**

3 The equation of a plane, Π, is

$$\Pi: \quad \mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}.$$

(i) Find a vector which is perpendicular to Π. **[2]**

(ii) Hence find an equation for Π in the form $\mathbf{r.n} = p$. **[2]**

(iii) Find in the form $\sqrt{q}$ the shortest distance between Π and the origin, where q is a rational number. **[2]**

4 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} a & 2 & 3 \\ 4 & 4 & 6 \\ -2 & 2 & 9 \end{pmatrix}$ where a is a constant. It is given that if $\mathbf{A}$ is not singular then

$$\mathbf{A}^{-1} = \frac{1}{24a-48}\begin{pmatrix} 24 & -12 & 0 \\ -48 & 9a+6 & 12-6a \\ 16 & -2a-4 & 4a-8 \end{pmatrix}.$$

(i) Use $\mathbf{A}^{-1}$ to solve the simultaneous equations below, giving your answer in terms of k.

$$\begin{aligned} x+2y+3z &= 6 \\ 4x+4y+6z &= 8 \\ -2x+2y+9z &= k \end{aligned}$$

[3]

(ii) Consider the equations below where a takes the value which makes $\mathbf{A}$ singular.

$$\begin{aligned} ax+2y+3z &= b \\ 4x+4y+6z &= 10 \\ -2x+2y+9z &= -13 \end{aligned}$$

b takes the value for which the equations have an infinite number of solutions.

- Determine the value of b.
- Find the solutions for y and z in terms of x.

[5]

(iii) For the equations in part **(ii)** with the values of a and b found in part **(ii)** describe fully the geometrical arrangement of the planes represented by the equations. **[2]**

5 The region R between the x-axis, the curve $y = \frac{1}{\sqrt{p+x^2}}$ and the lines $x = \sqrt{p}$ and $x = \sqrt{3p}$, where p is a positive parameter, is rotated by 2π radians about the x-axis to form a solid of revolution S.

(i) Find and simplify an algebraic expression, in terms of p, for the exact volume of S. **[5]**

(ii) Given that R must lie entirely between the lines $x = 1$ and $x = \sqrt{48}$ find in exact form

- the greatest possible value of the volume of S
- the least possible value of the volume of S.

[3]

6 **(i)** By considering $\sum_{r=1}^{n}\left((r+1)^5 - r^5\right)$ show that $\sum_{r=1}^{n} r^4 = \frac{1}{30}n(n+1)(2n+1)(3n^2+3n-1)$. **[6]**

(ii) Use the formula given in part **(i)** to find $50^4 + 51^4 + \ldots + 80^4$. **[2]**

7 The roots of the equation $ax^2 + bx + c = 0$, where a, b and c are positive integers, are α and β.

(i) Find a quadratic equation with integer coefficients whose roots are $\alpha + \beta$ and $\alpha\beta$. **[4]**

(ii) Show that it is not possible for the original equation and the equation found in part **(i)** both to have repeated roots. **[2]**

(iii) Show that the discriminant of the equation found in part **(i)** is always positive. **[3]**

8 In this question you must show detailed reasoning.

(i) Express $(6 + 5\text{i})(7 + 5\text{i})$ in the form $a + b\text{i}$. **[2]**

(ii) You are given that $17^2 + 65^2 = 4514$. Using the result in part **(i)** and by considering $(6 - 5\text{i})(7 - 5\text{i})$ express 4514 as a product of its prime factors. **[4]**

9 The quantity of grass on an island at time t years is x, in appropriate units. At time $t = 0$ some rabbits are introduced to the island. The population of rabbits on the island at time t years is y, in units of 100s of rabbits.

An ecologist who is studying the island suggests that the following pair of simultaneous first order differential equations can be used to model the population of rabbits and quantity of grass for $t \geqslant 0$.

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 3x - 2y,$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = y + 5x$$

(i) **(a)** Show that $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = a\frac{\mathrm{d}x}{\mathrm{d}t} + bx$ where a and b are constants which should be found. **[2]**

(b) Find the general solution for x in real form. **[3]**

(ii) Find the corresponding general solution for y. **[3]**

At time $t = 0$ the quantity of grass on the island was 4 units. The number of rabbits introduced at this time was 500.

(iii) Find the particular solutions for x and y. **[5]**

(iv) The ecologist finds that the model predicts that there will be no grass at time T, when there are still rabbits on the island.

Find the value of T. **[1]**

(v) State one way in which the model is not appropriate for modelling the quantity of grass and the population of rabbits for $0 \leqslant t \leqslant T$. **[1]**

END OF QUESTION PAPER

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