OCR
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A Level Further Mathematics A

Y541/01 Pure Core 2

Practice Paper – Set 3

Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

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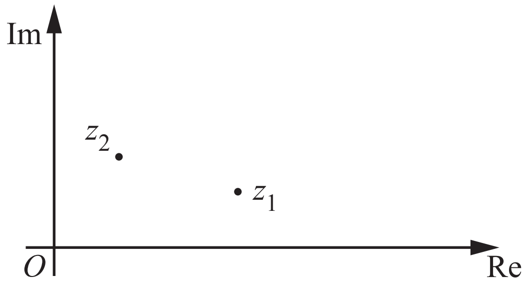
Answer **all** the questions.

1 **(a)** The Argand diagram below shows the two points which represent two complex numbers, z_1 and z_2.



On the copy of the diagram in the Printed Answer Booklet

- draw an appropriate shape to illustrate the geometrical effect of adding z_1 and z_2,
- indicate with a cross ($\times$) the location of the point representing the complex number $z_1 + z_2$. **[2]**

(b) You are given that $\arg z_3 = \frac{1}{4}\pi$ and $\arg z_4 = \frac{3}{8}\pi$.

In each part, sketch and label the points representing the numbers z_3, z_4 and $z_3 z_4$ on the diagram in the Printed Answer Booklet. You should join each point to the origin with a straight line.

(i) $|z_3| = 1.5$ and $|z_4| = 1.2$ **[2]**

(ii) $|z_3| = 0.7$ and $|z_4| = 0.5$ **[2]**

2 **In this question you must show detailed reasoning.**

S is the 2-D transformation which is a stretch of scale factor 3 parallel to the *x*-axis. **A** is the matrix which represents S.

(a) Write down **A**. **[1]**

(b) By considering the transformation represented by $\mathbf{A}^{-1}$, determine the matrix $\mathbf{A}^{-1}$. **[2]**

Matrix **B** is given by $\mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$. T is the transformation represented by **B**.

(c) Describe T. **[1]**

(d) Determine the matrix which represents the transformation S followed by T. **[2]**

(e) Demonstrate, by direct calculation, that $(\mathbf{BA})^{-1} = \mathbf{A}^{-1}\mathbf{B}^{-1}$. **[2]**

3 **In this question you must show detailed reasoning.**

Solve the equation $2\cosh^2 x + 5\sinh x - 5 = 0$ giving each answer in the form $\ln\left(p + q\sqrt{r}\right)$ where p and q are rational numbers, and r is an integer, whose values are to be determined. **[6]**

4 You are given that the matrix $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{2a-a^2}{3} & 0 \\ 0 & 0 & 1 \end{pmatrix}$, where a is a positive constant, represents the transformation R which is a reflection in 3-D.

(a) State the plane of reflection of R. **[1]**

(b) Determine the value of a. **[3]**

(c) With reference to R explain why $\mathbf{A}^2 = \mathbf{I}$, the 3×3 identity matrix. **[2]**

5 **(a)** Find the shortest distance between the point (−6, 4) and the line $y = -0.75x + 7$. **[2]**

Two lines, l_1 and l_2, are given by

$$l_1 : \mathbf{r} = \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} \text{ and } l_2 : \mathbf{r} = \begin{pmatrix} 11 \\ -1 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}.$$

(b) Find the shortest distance between l_1 and l_2. **[3]**

(c) Hence determine the geometrical arrangement of l_1 and l_2. **[2]**

6 Three matrices, **A**, **B** and **C**, are given by $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ a & -1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & -1 \\ 4 & 1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 5 & 0 \\ -2 & 2 \end{pmatrix}$ where a is a constant.

(a) Using **A**, **B** and **C** in that order demonstrate explicitly the associativity property of matrix multiplication. **[4]**

(b) Use **A** and **C** to disprove by counterexample the proposition ‘Matrix multiplication is commutative’. **[2]**

For a certain value of a, $\mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix} = 3\begin{pmatrix} x \\ y \end{pmatrix}$.

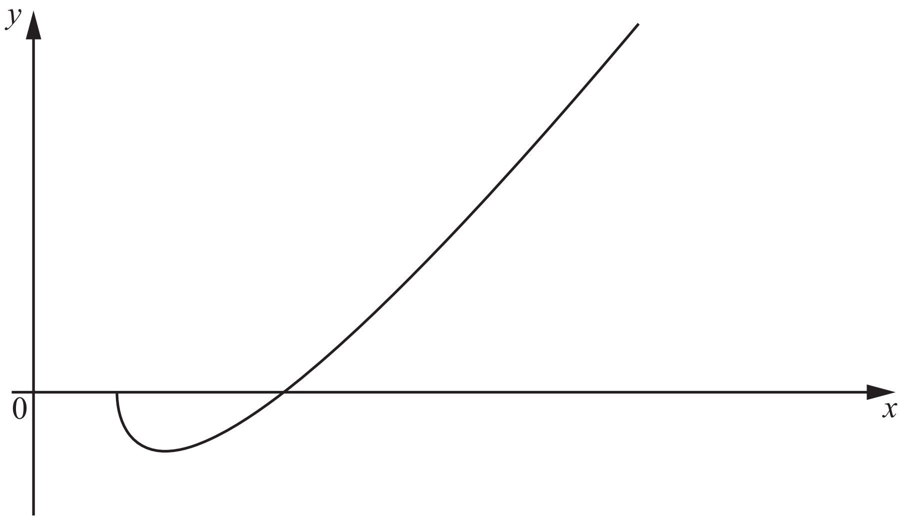
(c) Find

- y in terms of x,
- the value of a. **[3]**

7 C is the locus of numbers, z, for which $\operatorname{Im}\left(\frac{z+7\mathrm{i}}{z-24}\right)=\frac{1}{4}$.

By writing $z=x+\mathrm{i}y$ give a complete description of the shape of C on an Argand diagram. **[7]**

8



The figure shows part of the graph of $y=(x-3)\sqrt{\ln x}$. The portion of the graph below the x-axis is rotated by 2π radians around the x-axis to form a solid of revolution, S.

Determine the exact volume of S. **[7]**

9 **(a)** By using Euler's formula show that $\cosh(\mathrm{i}z)=\cos z$. **[3]**

(b) Hence, find, in logarithmic form, a root of the equation $\cos z=2$.
[You may assume that $\cos z=2$ has complex roots.] **[2]**

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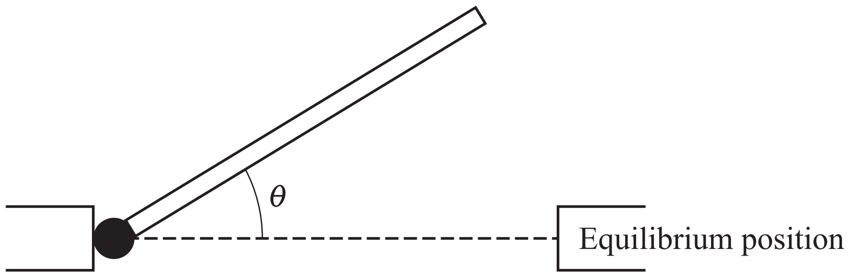
10 A swing door is a door to a room which is closed when in equilibrium but which can be pushed open from either side and which can swing both ways, into or out of the room, and through the equilibrium position. The door is sprung so that when displaced from the equilibrium position it will swing back towards it.

The extent to which the door is open at any time, t seconds, is measured by the angle at the hinge, θ, which the plane of the door makes with the plane of the equilibrium position. See the diagram below.



In an initial model of the motion of a certain swing door it is suggested that θ satisfies the following differential equation.

$$4\frac{d^2\theta}{dt^2} + 25\theta = 0 \quad (*)$$

(a) **(i)** Write down the general solution to (*). **[2]**

(ii) With reference to the behaviour of your solution in part **(a)(i)** explain briefly why the model using (*) is unlikely to be realistic. **[1]**

In an improved model of the motion of the door an extra term is introduced to the differential equation so that it becomes

$$4\frac{d^2\theta}{dt^2} + \lambda\frac{d\theta}{dt} + 25\theta = 0 \quad (\dagger)$$

where λ is a positive constant.

(b) In the case where $\lambda = 16$ the door is held open at an angle of 0.9 radians and then released from rest at time $t = 0$.

(i) Find, in a real form, the general solution of (†). **[3]**

(ii) Find the particular solution of (†). **[4]**

(iii) With reference to the behaviour of your solution found in part **(b)(ii)** explain briefly how the extra term in (†) improves the model. **[2]**

(c) Find the value of λ for which the door is critically damped. **[2]**

END OF QUESTION PAPER

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