

Practice Paper – Set 1

A Level Further Mathematics A Y542/01 Statistics

MARK SCHEME

Duration: 1 hour 30 minutes

MAXIMUM MARK 75

DRAFT

This document consists of 13 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
√and x	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
۸	Omission sign
MR	Misread
Highlighting	
Other abbreviations in	Meaning
mark scheme	
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
rot soi	Rounded or truncated Seen or implied
	Rounded or truncated
soi	Rounded or truncated Seen or implied
soi www AG awrt	Rounded or truncated Seen or implied Without wrong working Answer given Anything which rounds to
soi www AG	Rounded or truncated Seen or implied Without wrong working Answer given

2. Subject-specific Marking Instructions for AS Level Further Mathematics A

- Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
- An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

 If you are in any doubt whatsoever you should contact your Team Leader.
- c The following types of marks are available.

М

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

Α

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

В

Mark for a correct result or statement independent of Method marks.

Ε

Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

 Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.
- g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.
- For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
- If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

	Questic	on	Answer	Marks	AO	Guidano	ce
1	(i)	(a)	$X + Y \sim Po(31)$	M1	3.3	Stated or implied	
			$P(X + Y) > 40 = 1 - P(X + Y \le 40)$	M1	1.1a	Allow M1A0 for 0.0678	
			= 0.048728	A1	1.1	Awrt 0.0487	BC
				[3]			
	(i)	(b)	4×17 + 14	M1	1.1a	Allow M1 for 2×17 + 14	
			= 82	A1	1.1		
				[2]			
	(ii)		X and Y are independent	B1	1.2	Doesn't need to be contextualised	
				[1]			
2			$f(x) = \frac{3}{x^4} \qquad (x \ge 1)$	M1	1.1a	Attempt to differentiate $F(x)$	
			x^4 $(x=1)$	A1	1.1		
			$E(\sqrt{X}) = \int_{1}^{\infty} \sqrt{x} f(x) dx$	M1FT	1.1a	Limits wrong or omitted: M1A0	
			$= \int_{1}^{\infty} 3x^{-3.5} \mathrm{d}x$	A1FT	1.1	FT on their 3 and power of x	
			$=\frac{6}{5}$ or 1.2	A1	2.2a	Awrt 1.2 BC	
				[5]			

3	(i)	(a)	$^{10}C_{2} \times ^{40}C_{2}$	M1	3.1b	At least one correct ${}^{n}C_{r}$ seen	Or probability method
			$\frac{3}{50}C_5$	A1	1.1	Correct expression	$=\frac{120\times780}{2118760}$
			$=\frac{2340}{52969}$ or 0.044176	A1	1.1	Exact or awrt 0.0442	
				[3]			
	(i)	(b)	$1/5! = \frac{1}{120}$	B1	1.1	Any equivalent form, allow awrt 0.00833	
				[1]		0.00833	
	(ii)		$\frac{{}^{7}C_{2} \times {}^{33}C_{8}}{{}^{40}C_{10}} \times \frac{{}^{5}C_{3} \times {}^{25}C_{7}}{{}^{30}C_{10}}$	M1	3.1b	Correct structure for B	$= \frac{21 \times 13884156}{847660528} \times \frac{10 \times 480700}{30045015}$
			$^{40}C_{10}$ $^{30}C_{10}$	A1	1.1a	Correct numbers for B	647000326 30043013
				M1	3.1b	Two terms of this sort multiplied	
				A1	1.1	Fully correct expression	
			$=\frac{450}{8177}$ or 0.055032	A1	1.1	Exact or awrt 0.0550	
			01//	[5]			

4	(i)	Distribution of times needs to be symmetric	B1	2.4		
			[1]			
	(ii)	Uses more information	B1	3.5b	e.g. "uses magnitudes of differences"	
			[1]		differences	
	(iii)	$n = 32, \ \mu = 264$	B1	3.3		
		$\sigma^2 = 2860$	B1	1.1a		
		$264 - 1.645\sqrt{2860} - 0.5$	M1	2.1	$\mu - z\sigma$, allow σ^2 , ignore cc	
			A1	1.1	Correct, with correct or no cc	
		= 175.52	A1	2.2a	Allow 176.02 <i>only</i> if later rounded down to 175	
		$W_{-} \leq 175$	A1	3.4	Needs any letter, 175 and ≤	Allow e.g. $W_{-} < 176$
		where W_{-} is the sum of the ranks	B1	2.4	Allow W_{-} and "smaller of sum of	Allow any letter if
		corresponding to the negative differences	[7]		positive rankings and sum of negative rankings"	properly explained

5	(i)	(a)	20	B1	1.2	
				[1]		
	(i)	(b)	16	B1	1.2	
				[1]		
	(ii)		N(20, 16) and $P(X < x) = 0.98$	M1	3.3	Correct calc ⁿ seen or implied
			[= 28.7]	A1	1.1	28.7 seen or implied
			n=29	A1	3.2a	Allow $n \ge 29$
				[3]		
	(iii)		B(100, 0.2): $P(\ge 29) = 0.2002$,	M1	1.1a	One probability from B(100, 0.2)
			$P(\ge 30) = 0.01125$	A1	1.1	Both correct
			So correct answer is $n = 30$	A1	2.2a	Correct conclusion, cwo
				[3]		

6	(i)		H_0 : no association between weather and results, H_1 : association	B1	2.5		
			Expected frequencies	M1	1.1a	Find expected frequencies	
			8 6 6 12 9 9	A1	1.1		
			$X^2 = 2 + 1.5 + 0.167 + 1.333 + 1 + 0.111$	M1	3.3	Use $\Sigma (O-E)^2/E$	
			= 6.111	A1	3.4	Awrt 6.11	
			6.111 > 5.991	A1	1.1	Compare with 5.991, needs M1M1	
			Reject H ₀ . Significance evidence of association	M1ft	1.1a	ft on 6.111 only	
			between results and weather	A1ft	2.2b	Contextualised, not too definite	
				[8]			
	(ii)	(a)	Sunny/Win	B1	2.2b	FT on X^2 if M1M1 and evidence	
				[1]			
	(ii)	(b)	The team wins more than average in sunny	B1	1.1	Or equivalent	FT from part (a)
			conditions	[1]			

7	(i)	$\int_0^2 \frac{1}{4} x (4 - x^2) dx = 1$	M1	3.1a	Attempt to show this	
			A1	2.2a	Algebra needed for "show that"	"By calculator": M1A0
		$f(x) \ge 0 \text{ for } 0 \le x \le 2$	B 1	2.4	Explicitly stated – not just diagram	
			[3]		drawn	
	(ii)	$\int_0^a \frac{1}{4} x (4 - x^2) dx = \frac{15}{16}$	M1	3.1b	Integral, correct limits somewhere,	
		J ₀ 4 \			and equate to $\frac{15}{16}$	
		$\left[\frac{x^2}{2} - \frac{x^4}{16}\right]_0^a = \frac{15}{16}$	A1	1.1	Correct indefinite integral	
		$a^4 - 8a^2 + 15 = 0$	M1	2.1	Reduce to quadratic	
		$a^2 = 3 \text{ or } 5$	M1	3.1a	Solve for a^2	BC
		$a \neq -\sqrt{3}$ or $\pm \sqrt{5}$ (outside range)	A1	2.3	Explicitly rejected	No reason needed
		$a = \sqrt{3}$ and no other solutions	A1	2.2a	Needn't obtain previous A1	
			[6]			

8	(i)		H ₀ : no association between rankings	B 1	2.5		
			H ₁ : positive association				
			Rankings 1 2 3 4 5 6 7 2 4 1 3 5 6 7	M1	1.1a	Attempt to rank	Allow rankings in opposite order
			$r_s = 1 - \frac{6 \times 10}{7 \times 48} = 0.8214$	M1	3.1a		
			7×48	A1	1.1	Or $\frac{23}{28}$	
			0.8214 < 0.8929	B1	1.1a	Correct comparison	
			Do not reject H ₀ . Insufficient evidence of	M1ft	2.2b	FT on their TS or on 0.9286	
			agreement between judges' rankings.	A1ft	3.5a	Contextualized, not too definite	NOT "there is agreement"
				[7]			
	(ii)	(a)	1/7!	M1	3.1a		
			$=\frac{1}{5040}$ or 0.000198	A1	1.1a	Exact or awrt 0.000198	
				[2]			
	(ii)	(b)	$1 - \frac{6\Sigma d^2}{7 \times 48} = \frac{55}{56} \implies \qquad \Sigma d^2 = 1$	M1	2.1	Find Σd^2	
			Impossible as 1 incorrect ranking implies	A1	2.2a	Justify impossible	
			another	[2]			

9		$S_{xx} = \frac{42767}{50} - \left(\frac{1270}{50}\right)^2 = 210.18$ $S_{xy} = \frac{173161}{50} - \frac{1270}{50} \times \frac{5173}{50} = 835.336$	M1	2.1	Correct method for b
		$b = S_{xy}/S_{xx} = 3.97438$	A1	1.1	Correct value of <i>b</i> , in [3.97, 3.98]
		$a = 103.46 - 25.4 \times b = 2.510$	A1	1.1	Correct value of a, awrt 2.51
		$y = 2.510 + 20 \times 3.974$	M1	3.3	Estimate y
		= 82.0	A1	1.1	Correct value
		$\hat{y} \pm 1.96 \sqrt{\frac{1.9}{10}}$	M1	1.1a	$y \pm z \sqrt{\sigma^2/n}$
		$\sqrt{\frac{y\pm 1.90\sqrt{10}}{10}}$	A1	1.1a	z = 1.96
		= (81.14, 82.85)	A1 [8]	2.2a	Both correct, awrt 4 sf. Allow [81.1, 81.2] and [82.8, 82.9] only if all other numbers fully correct

BLANK PAGE