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# Level 2 Certificate **FURTHER MATHEMATICS** **8365/1**

Paper 1 Non-Calculator

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**Mark scheme**

June 2024

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Version: 1.0 Final



Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

No student should be disadvantaged on the basis of their gender identity and/or how they refer to the gender identity of others in their exam responses.

A consistent use of 'they/them' as a singular and pronouns beyond 'she/her' or 'he/him' will be credited in exam responses in line with existing mark scheme criteria.

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## Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

|                |  |
|----------------|--|
| <b>M</b>       | Method marks are awarded for a correct method which could lead to a correct answer.  |
| <b>M dep</b>   | A method mark dependent on a previous method mark being awarded.   |
| <b>A</b>       | Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied. |
| <b>B</b>       | Marks awarded independent of method.   |
| <b>B dep</b>   | A mark that can only be awarded if a previous independent mark has been awarded.   |
| <b>ft</b>      | Follow through marks. Marks awarded following a mistake in an earlier step.  |
| <b>SC</b>      | Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.                        |
| <b>oe</b>      | Or equivalent. Accept answers that are equivalent.<br><br>eg accept 0.5 as well as $\frac{1}{2}$                                       |
| <b>[a, b]</b>  | Accept values between $a$ and $b$ inclusive.   |
| <b>3.14...</b> | Accept answers which begin 3.14 eg 3.14, 3.142, 3.1416   |

Examiners should consistently apply the following principles.

**Diagrams**

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

**Responses which appear to come from incorrect methods**

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

**Questions which ask candidates to show working**

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

**Questions which do not ask candidates to show working**

As a general principle, a correct response is awarded full marks.

**Misread or miscopy**

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

**Further work**

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

**Choice**

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

**Work not replaced**

Erased or crossed out work that is still legible should be marked.

**Work replaced**

Erased or crossed out work that has been replaced is not awarded marks.

**Premature approximation**

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

**Continental notation**

Accept a comma used instead of a decimal point (for example, in measurements or currency), provided that it is clear to the examiner that the candidate intended it to be a decimal point.

| Q | Answer  | Mark | Comments  |
|---|---|------|---|
| 1 | 2420 or 121 or $1.21 \times 10^2$   | M1   | doesn't need to be seen in a correct expression |
|   | 11  | A1   | accept $\pm 11$                                 |
|   | <b>Additional Guidance</b>  |      |   |
|   | Condone $t=11$  |      | M1A1  |
|   | Condone $1.1 \times 10^1$ or $1.1 \times 10$                                |      | M1A1  |
|   | $\frac{1210}{10}$ or $\frac{242}{2}$  |      | M1  |
|   | $\frac{\sqrt{121}}{10}$ whilst incorrect would gain the method mark for 121 |      | M1  |

| Q | Answer                                  | Mark | Comments     |
|---|---|------|--------------|
| 2 | $(x + y)(x - y)$                        | B1   | either order |
|   | <b>Additional Guidance</b>              |      |              |
|   | Do not condone missing brackets         |      |              |
|   | Penalise further working on answer line |      |              |
|   | $xy(xy^{-1} - x^{-1}y)$                 |      | B0           |
|   | $(-x + y)(-x - y)$                      |      | B1           |

| Q | Answer                     | Mark | Comments |
|---|----------------------------|------|----------|
| 3 | 3                          | B1   |          |
|   | <b>Additional Guidance</b> |      |          |
|   |                            |      |          |

| Q | Answer  | Mark | Comments  |
|---|---|------|---|
| 4 | <b>Alternative method 1</b>   |      |   |
|   | $y = 3x + 4$ from first line<br>or<br>$y = 3x - \frac{5}{6}$ from second line   | M1   | oe where one equation is rearranged correctly to match the other eg $6y = 18x + 24$ from first line   |
|   | $y = 3x + 4$ and $y = 3x - \frac{5}{6}$<br><br>and a statement that they have the same gradient so are parallel                               | A1   | allow $y - 3x = 4$ and $y - 3x = -\frac{5}{6}$<br><br>eg same gradient so parallel<br>same gradient but different y intercept so parallel<br><br>$m_1 = m_2$ so parallel                                    |
|   | <b>Alternative method 2</b>   |      |   |
|   | 2 correct points on at least one line found<br><br>and<br>at least one correct calculation using $\frac{y_2 - y_1}{x_2 - x_1}$ to get $m = 3$ | M1   | eg (1, 7) & (2, 10)<br>or $(0, -\frac{5}{6})$ & $(1, \frac{13}{6})$<br><br>and<br>$\frac{10 - 7}{2 - 1}$ or $\frac{\frac{13}{6} - (-\frac{5}{6})}{1 - 0}$ or $\frac{\text{rise}}{\text{run}} = \frac{3}{1}$ |
|   | $m = 3$ for both lines calculated using $\frac{y_2 - y_1}{x_2 - x_1}$<br><br>and a statement that they have the same gradient so are parallel | A1   | eg. same gradient so parallel<br>same gradient but different y intercept so parallel<br><br>$m_1 = m_2$ so parallel<br>must see two separate calculations   |
|   | <b>Additional Guidance</b>  |      |   |
|   | In Alt 1 both equations must be correctly rearranged for A mark   |      |   |
|   | Condone $3x$ as gradient rather than 3  |      |   |
|   | Gradient = 18 (must rearrange to get gradient of 3 to gain A mark in Alt 1)   |      | A0  |
|   | Condone $m$ or $\frac{dy}{dx}$ used for gradient  |      |   |
|   | A statement that only mentions gradient and doesn't say they are parallel   |      | A0  |

| Q | Answer   | Mark | Comments   |
|---|--|------|--|
| 5 | $4x^{-1}$ or $\frac{4}{x}$ or $x^3$  | M1   | either term correct (ignore additional terms)  |
|   | $-4x^{-2}$ or $3x^2$   | A1   | a correct term differentiated correctly (must be just two terms before differentiation)<br><br>accept $-\frac{4}{x^2}$ |
|   | $\left(\frac{dy}{dx} =\right) -4x^{-2} + 3x^2$   | A1   | both terms correct<br><br>accept $-\frac{4}{x^2}$  |
|   | <b>Additional Guidance</b>   |      |  |
|   | Quotient rule or product rule used to achieve correct simplified answer                    | M1A2 |  |
|   | Quotient rule or product rule used with incorrect or unsimplified answer                   | M0A0 |  |
|   | $\frac{-4 + 3x^4}{x^2}$  | M1A2 |  |
|   | Penalise additional incorrect further working eg $\frac{-1}{4x^2}$ for the final<br>A mark |      |  |

| Q | Answer            | Mark | Comments   |
|---|-------------------|------|--|
| 6 | $x^2 + y^2 = 145$ | B3   | B2 centre (0, 0) and (radius =) $\sqrt{145}$<br>or centre (0, 0) and (radius <sup>2</sup> =) 145<br>or $(x - 0)^2 + (y - 0)^2 = 145$<br>or $x^2 + y^2 = \sqrt{145}$<br>B1 $x^2 + y^2 = k$<br>or $(x - 0)^2 + (y - 0)^2 = k$<br>or $(x + \dots)^2 + (y + \dots)^2 = 145$<br>or centre (0, 0)<br>or (radius =) $\sqrt{145}$<br>or (radius =) $\sqrt{12^2 + 1^2}$<br>or (radius <sup>2</sup> =) 145 or $12^2 + 1^2$<br>or (radius =) $\frac{\sqrt{580}}{2}$<br>or (radius <sup>2</sup> =) $\frac{580}{4}$ |
|   |                   |      | <b>Additional Guidance</b>   |
|   |                   |      | <div><math>k</math> is a positive number (not 0)</div>   |
|   |                   |      | <div>Penalise further incorrect working</div>  |
|   |                   |      | <div>(0, 0) needs to be stated as centre and not just shown on a diagram nor stated as a midpoint</div>  |
|   |                   |      | <div>No marks for finding diameter on its own</div>  |



| Q | Answer   | Mark  | Comments                              |
|---|--|-------|---------------------------------------|
| 7 | $\begin{pmatrix} 4 & 0 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -8 \\ 7 \end{pmatrix}$   | M1    | may be inferred from correct second M |
|   | $4x = -8$<br>and<br>$-2x + 3y = 7$   | M1dep | oe<br>may be seen in vector notation  |
|   | $(x =) -2$ and $(y =) 1$   | A1    |                                       |
|   | <b>Additional Guidance</b>   |       |                                       |
|   | $x = -2$ if no other marks scored  |       | SC1                                   |
|   | $\begin{pmatrix} 4 & 0 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} -8 \\ 7 \end{pmatrix} = \begin{pmatrix} -32 \\ 37 \end{pmatrix}$ or $(x =) -32$ and $(y =) 37$              |       | SC1                                   |
|   | Condone $\begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} 4 & 0 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} -8 \\ 7 \end{pmatrix}$ if recovered to get correct equations |       |                                       |
|   | Condone answer left as column vector $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$<br>or as coordinates $(-2, 1)$  |       |                                       |

| Q | Answer   | Mark  | Comments   |
|---|--|-------|--|
| 8 | $x(2x^2 - 9x - 5)$<br>or<br>$(2x^2 + x)(x - 5)$ or $(2x + 1)(x^2 - 5x)$<br>or states $x = 0$ and $(2x^2 - 9x - 5)$   | M1    |  |
|   | $x(2x + 1)(x - 5)$<br>or states $x = 0$ and $(2x + 1)(x - 5)$  | M1dep | do not accept $x(2x + 1)(2x - 10)$ unless recovered by correct solutions |
|   | $(x =) 0, 5, -0.5$   | A1    | oe but must come from factorising in M1dep                               |
|   | <b>Additional Guidance</b>   |       |  |
|   | $(2x + 1)(x - 5)$ if no other marks scored   |       | SC1  |
|   | $(x =) 0, 5, -0.5$ without working or by using factor theorem to find solutions without writing it as a product of factors or by using quadratic formula (must be all 3 correct answers) |       | SC1  |

| Q | Answer  | Mark  | Comments   |
|---|---|-------|--|
| 9 | $3^2 + 4^2$<br>or $3^2 + 12^2$<br>or $4^2 + 12^2$<br>or (EG $=$ ) 5                         | M1    | oe could be seen as $\sqrt{\quad}$<br>may be seen evaluated eg. 25, 153 or 160<br>may be seen on diagram |
|   | $3^2 + 4^2 + 12^2$<br>or $5^2 + 12^2$<br>or 169   | M1dep | oe could be seen as $\sqrt{\quad}$<br>must be expressions that could be evaluated to give 169            |
|   | 13  | A1    |  |
|   | <b>Additional Guidance</b>  |       |  |
|   | $4\sqrt{11}$ or $2\sqrt{44}$ or $\sqrt{176}$ or $\sqrt{162}$ or $3\sqrt{18}$ or $9\sqrt{2}$ |       | SC1  |

| Q  | Answer                                       | Mark | Comments  |
|----|--|------|---|
| 10 | $P$ is (2, 20) or $y$ value of $P$ is 20     | M1   | implied by $4a + 2b = 20$<br>could be seen on diagram   |
|    | $4a + 2b = k$                                | M1   | oe<br>$4a + 2b = 20$<br>$k$ could be any number (including 0)<br>but must be 20 if first M mark awarded |
|    | $36a + 6b = 12$                              | M1   | oe  |
|    | $a = -2$ or $b = 14$                         | A1   |   |
|    | $a = -2$ and $b = 14$                        | A1   |   |
|    | <b>Additional Guidance</b>                   |      |   |
|    | $a = -2$ <b>and</b> $b = 14$ with no working |      | M3A2  |
|    | $a = -2$ <b>or</b> $b = 14$ with no working  |      | M0A0  |

| Q     | Answer   | Mark | Comments   |
|-------|--|------|--|
| 11(a) | <b>Alternative method 1 – by inspection or long division</b>                             |      |  |
|       | $x^2 + kx - 2$ by inspection<br>or first two terms using long division:<br>$x^2 + x + k$ | M1   | $k$ could be blank<br>can be seen in a grid method   |
|       | $x^2 + x - 2$  | A1   |  |
|       | $(x - 1)$ and $(x + 2)$<br>or $(x - 1)(x + 2)$ or $(3x - 7)(x - 1)(x + 2)$               | A1   | any order of brackets  |
|       | <b>Alternative method 2 – factor theorem</b>   |      |  |
|       | 1 or 2 or $-1$ or $-2$ substituted into function and evaluated correctly                 | M1   | $f(x) = 0$ for $x = 1$<br>$f(x) = 0$ for $x = -2$<br>$f(x) = 20$ for $x = -1$<br>$f(x) = -4$ for $x = 2$<br>ignore additional incorrect calculations |
|       | $(x + 2)$ or $(x - 1)$ identified as a factor  | A1   | must have come from a substitution from M mark   |
|       | $(x - 1)$ and $(x + 2)$<br>or $(x - 1)(x + 2)$ or $(3x - 7)(x - 1)(x + 2)$               | A1   | any order of brackets  |
|       | <b>Alternative method 3 – equating coefficients</b>                                      |      |  |
|       | $(3x - 7)(ax^2 + bx + c)$ and<br>$3ax^3 - 7ax^2 + 3bx^2 - 7bx + 3cx - 7c$                | M1   |  |
|       | $a = 1$ , $b = 1$ and $c = -2$ or $x^2 + x - 2$  | A1   |  |
|       | $(x - 1)$ and $(x + 2)$<br>or $(x - 1)(x + 2)$ or $(3x - 7)(x - 1)(x + 2)$               | A1   | any order of brackets  |
|       | <b>Additional Guidance</b>   |      |  |
|       | $x = 1$ and $-2$ on answer line or last line of working                                  |      | SC2  |
|       | Follow whichever scheme gives the best mark  |      |  |
|       | Penalise incorrect further working eg. correct answer followed by $x = 1$ and $x = -2$   |      | M1A1A0   |

| Q     | Answer   | Mark  | Comments  |
|-------|--|-------|---|
| 11(b) | <b>Alternative method 1 – by substitution</b>                                |       |   |
|       | $a \times 2^4 - 3a \times 2^3 + 5 \times 2 - 22$<br>or $16a - 24a + 10 - 22$ | M1    | oe<br>4 terms with at least 3 correct                 |
|       | $16a - 24a + 10 - 22 = 0$<br>or $-8a = 12$                                   | M1dep | oe<br>could imply first M mark                        |
|       | $a = -1\frac{1}{2}$ or $\frac{-3}{2}$ or $-1.5$                              | A1    |   |
|       | <b>Alternative method 2 – long division</b>                                  |       |   |
|       | First two terms correct  | M1    | $ax^3 - ax^2$   |
|       | Correct equation set up to find $a$  | M1dep | eg $-8a = 12$ or $5 - 4a = 11$                        |
|       | $a = -1\frac{1}{2}$ or $\frac{-3}{2}$ or $-1.5$                              | A1    |   |
|       | <b>Alternative method 3 – equating coefficients or inspection</b>            |       |   |
|       | $d = 11$ and $ax^3$<br>and<br>$c = 3$ or $b = 1.5$ or $c = -2a$ or $b = -a$  | M1    | could be seen in a grid<br>for $ax^3 + bx^2 + cx + d$ |
|       | Correct equation set up to find $a$  | M1dep | eg $4a + 11 = 5$ or $2a + 3 = 0$                      |
|       | $a = -1\frac{1}{2}$ or $\frac{-3}{2}$ or $-1.5$                              | A1    |   |
|       | <b>Additional Guidance</b>   |       |   |
|       | Follow whichever scheme gives the best mark                                  |       |   |

| Q  | Answer  | Mark  | Comments  |
|----|---|-------|---|
| 12 | $12^2 + 10^2 - 2 \times 12 \times 10 \times \frac{3}{4}$  | M1    | oe in a fully correct substitution<br>could already be under $\sqrt{\quad}$ |
|    | $144 + 100 - 180$ or 64   | M1dep |   |
|    | 8   | A1    | $\pm 8$ is A0   |
|    | <b>Additional Guidance</b>  |       |   |
|    | $\cos \frac{3}{4}$ used in the working will not score unless recovered into<br>a fully correct calculation that scores to at least the first M mark |       |   |

| Q  | Answer   | Mark  | Comments   |
|----|--|-------|--|
| 13 | $7x + 8(x - 3)$  | M1    | oe<br>eg $7 \times 3 + 15(x - 3)$<br>or $(8 + 7)x - 8 \times 3$                                |
|    | A correct inequality formed for $x$  | M1dep | eg. $7x + 8x - 24 < 51$<br>or $7x + 8(x - 3) < 51$<br>or $15x < 75$<br>or $21 + 15x - 45 < 51$ |
|    | $x < 5$  | A1    |  |
|    | $3 < x (< k)$  | B1    | $k$ greater than 3 or blank on<br>answer line  |
|    | <b>Additional Guidance</b>   |       |  |
|    | $3 < x < 5$ with no or incorrect working can only score the B1   |       |  |
|    | Trial and Improvement: stop marking once this begins (may<br>have scored M2 and may go on to score B1) |       |  |
|    | Ignore any units stated  |       |  |
|    | Inequalities replaced with = or other inequalities are penalised<br>unless recovered                   |       |  |

| Q     | Answer  | Mark  | Comments  |
|-------|---|-------|---|
| 14(a) | <b>Alternative method 1</b>   |       |   |
|       | $2(x^2 - 8x + \dots)$<br>$2(x - 4)^2 + \dots$   | M1    | oe  |
|       | $2[(x - 4)^2 - 4^2] + \dots$<br>or $2[(x - 4)^2 - 16] + \dots$<br><br>or $2[(x - 4)^2 - 16 + \frac{7}{2}]$<br>or $2[(x - 4)^2 - \frac{25}{2}]$<br>or $2(x - 4)^2 - 2 \times \frac{25}{2}$<br>or $2(x - 4)^2 - 32 + 7$ | M1dep | oe the bracket is after the $4^2$ and the 16 here. If they put something else inside the bracket it is incorrect unless it is equivalent to one of the fully complete versions listed |
|       | $2(x - 4)^2 - 25$   | A1    |   |
|       | <b>Alternative method 2 – using identities</b>  |       |   |
|       | $kx^2 + 2kmx + km^2 + n$  | M1    | RHS oe  |
|       | $k = 2$ and $2km = -16$<br>and $km^2 + n = 7$   | M1dep | oe  |
|       | $2(x - 4)^2 - 25$   | A1    |   |
|       | <b>Additional Guidance</b>  |       |   |
|       | Condone answer followed by $k = 2$ $m = -4$ $n = -25$   |       | M2A1  |
|       | $k = 2$ $m = -4$ $n = -25$ without correct answer is penalised  |       | M2A0  |

| Q     | Answer  | Mark | Comments |
|-------|---|------|----------|
| 14(b) | $(x =) 1 \pm \sqrt{5}$  | B1   |          |
|       | <b>Additional Guidance</b>  |      |          |
|       | Condone roots written separately (and/or are both fine)               |      |          |
|       | Do not condone $1 + \sqrt{5}$ on its own or $1 - \sqrt{5}$ on its own |      |          |
|       | Do not accept oe answers – must be simplified                         |      |          |

| Q     | Answer   | Mark | Comments |
|-------|--|------|----------|
| 15(a) | $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ | B1   |          |
|       | Additional Guidance                              |      |          |
|       |  |      |          |

| Q     | Answer  | Mark | Comments  |
|-------|---|------|---|
| 15(b) | $(\mathbf{N}^2 =) \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$<br>or<br>describes $\mathbf{N}$ as a rotation $90^\circ$ clockwise about the origin (or (0, 0)) | M1   | can be implied from A1<br><br>condone rotation of $-90^\circ$ or $270^\circ$ about the origin (or (0, 0)) |
|       | Rotation $180^\circ$ about the origin<br>(or (0, 0))<br>or<br>Enlargement sf. $-1$ about the origin<br>(or (0, 0))  | A1   | clockwise or anticlockwise can be stated  |
|       | Additional Guidance   |      |   |
|       | ( $\mathbf{N} =$ ) Rotation $90^\circ$ (without clockwise) about the origin recovered with a correct answer   |      | M1A1  |
|       | ( $\mathbf{N} =$ ) Rotation $90^\circ$ anticlockwise about the origin with correct answer would be from incorrect working   |      | M0A0  |
|       | ( $\mathbf{N} =$ ) Rotation $90^\circ$ (without clockwise) about the origin without the correct answer  |      | M0A0  |
|       | Missing 'about the origin' in M mark can be recovered in A mark   |      |   |
|       | Condone missing brackets in matrices  |      |   |



| Q  | Answer  | Mark | Comments  |
|----|---|------|---|
| 16 | $(BOD =) 104^\circ$   | B1   |   |
|    | $(BCD =) 128^\circ$   | B1   |   |
|    | $(OBC + ODC =)$<br>$360^\circ - (\text{their})128^\circ - (\text{their})104^\circ$<br><br>or $OBC + ODC = 128^\circ$<br>or $128 = 8x$   | M1   | oe eg $360^\circ - 232^\circ$<br>one of $128^\circ$ and $104^\circ$ must be correct<br><br>this could imply B1 B1<br>oe |
|    | $(OBC =) 80^\circ$  | A1   | can only be awarded with evidence of B2M1   |
|    | <b>Additional Guidance</b>  |      |   |
|    | Angles may be seen on diagram   |      |   |
|    | Check they are not splitting $BCD$ in the ratio 5:3 (benefit of doubt may need to be applied). It is possible to draw a line from O to C which splits the quadrilateral into two isosceles triangles. This method does allow for $BCD$ to be split in the ratio 5:3 |      |   |
|    | $OBC = 80^\circ$ and $ODC = 48^\circ$ both on answer line   |      | A1  |
|    | $80^\circ$ and $48^\circ$ both on answer line would be choice   |      | A0  |
|    | Condone other forms of angle notation   |      |   |

| Q  | Answer   | Mark  | Comments  |
|----|--|-------|---|
| 17 | <b>Alternative method 1</b>  |       |   |
|    | $(3x + 4)(x - 2)$  | M1    | oe for correct factorisation  |
|    | $\frac{21x - 7(3x + 4)}{(3x + 4)(x - 2)}$                            | M1dep | oe<br>fractions can be written separately<br>denominator must be<br>$(3x + 4)(x - 2)$ or $3x^2 - 2x - 8$  |
|    | $\frac{-28}{3x^2 - 2x - 8}$  | A1    |   |
|    | <b>Alternative method 2</b>  |       |   |
|    | $\frac{21x(x - 2) - 7(3x^2 - 2x - 8)}{(3x^2 - 2x - 8)(x - 2)}$       | M1    | oe<br>fractions can be written separately   |
|    | $\frac{21x^2 - 42x - 21x^2 + 14x + 56}{(3x^2 - 2x - 8)(x - 2)}$      | M1dep | oe<br>must be one fraction and brackets expanded in the numerator (could have been simplified and factorised)<br>eg $\frac{-28(x - 2)}{(3x^2 - 2x - 8)(x - 2)}$<br>expanded denominator is<br>$3x^3 - 8x^2 - 4x + 16$ |
|    | $\frac{-28}{3x^2 - 2x - 8}$  | A1    |   |
|    | <b>Additional Guidance</b>   |       |   |
|    | Students may use a combination of methods which are covered by oe    |       |   |
|    | $k = -28$<br>Could come from using identities or substitutions       |       | SC1   |
|    | The minus can be written before the fraction rather than with the 28 |       |   |

| Q  | Answer   | Mark  | Comments   |
|----|--|-------|--|
| 18 | $\times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}}$   | M1    | oe eg $\frac{3\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} \times \frac{-\sqrt{5} - \sqrt{3}}{-\sqrt{5} - \sqrt{3}}$  |
|    | Denominator 5 – 3 or 2   | M1dep |  |
|    | Numerator 15 + 3 + $3\sqrt{15} + \sqrt{15}$  | M1dep | oe<br>three terms correct<br>could be simplified without writing 4 terms which would imply original terms if done correctly<br>eg $18 + 2\sqrt{15} + \sqrt{15}$<br>or $18 + 3\sqrt{15}$<br>whereas $17 + 3\sqrt{15} + \sqrt{15}$ could be two errors so wouldn't get M1<br>accept $\sqrt{3}\sqrt{5}$ for $\sqrt{15}$ for this mark<br>do not accept $\sqrt{3}\sqrt{3}$ or $\sqrt{9}$ for 3<br>or $\sqrt{5}\sqrt{5}$ or $\sqrt{25}$ for 5 this would count as one error<br>dep on first M mark only |
|    | $9 + 2\sqrt{15}$   | A1    |  |
|    | <b>Additional Guidance</b>   |       |  |
|    | For M dep marks numerator and denominator can be seen separately   |       |  |
|    | Follow an equivalent MS for any student using<br>$\frac{3\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} \times \frac{-\sqrt{5} - \sqrt{3}}{-\sqrt{5} - \sqrt{3}}$ |       |  |
|    | M marks may be seen in a grid method   |       |  |
|    | Untidy mathematical notation can be condoned as long as it's recovered   |       |  |
|    | Missing brackets can be recovered  |       |  |

| Q  | Answer  | Mark  | Comments  |
|----|---|-------|---|
| 19 | $\left(\frac{dy}{dx}\right) 3x^2 - 6x$  | M1    | either term correct<br>could have additional incorrect terms  |
|    | Their $\frac{dy}{dx} = 0$   | M1dep | follow through an incorrect differentiation as long as M1 scored  |
|    | $x = 2$ and $x = 0$   | A1    | these values could imply M1dep if first M mark awarded  |
|    | $\left(\frac{d^2y}{dx^2}\right) 6x - 6$ and<br>$\left(\frac{d^2y}{dx^2}\right) 6$ and/or positive (for $x = 2$ )<br>or<br>$\left(\frac{d^2y}{dx^2}\right) -6$ and/or negative (for $x = 0$ )<br>or<br>$\left(\frac{dy}{dx}\right) 3x^2 - 6x$<br>and<br>any correct check for both sides of one correct solution to give one side with a negative gradient and one side with a positive gradient | M1dep | a correct $x$ value assessed correctly<br><br><br><br><br><br><br><br><br><br><br>eg<br>$x = -1 \frac{dy}{dx} > 0$ and $x = 1 \frac{dy}{dx} < 0$<br>or<br>$x = 1 \frac{dy}{dx} < 0$ and $x = 3 \frac{dy}{dx} > 0$ |
|    | $(0, 5)$ Maximum<br>$(2, 1)$ Minimum  | A1    | both points must have been assessed correctly   |
|    | <b>Additional Guidance</b>  |       |   |
|    | Final A mark must have everything correct including both stationary points assessed correctly   |       |   |
|    | Just a sketch is not enough to determine the nature of the stationary points  |       |   |
|    | Condone incorrect writing of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ even if it's just $y =$ as long as it's recovered to get the correct nature of the turning points  |       |   |

| Q  | Answer   | Mark | Comments   |
|----|--|------|--|
| 20 | $(y =) \frac{-x}{2} + 3$   | B2   | oe eg $2y = -x + 6$<br>can be implied from correct line B1<br>for $(y =) \frac{x}{2} - 3$<br>can be implied from drawing this line<br>or<br>$m = -\frac{1}{2}$ or $c = +3$ not from<br>incorrect working for either $m$ or $c$ |
|    | $y = \frac{-x}{2} + 3$ accurately drawn on diagram                     | M1   | ft their stated linear function in $x$<br>accuracy is half a square at $x = 0$<br>and $x = 4$<br>needs to be a straight line (mark intention)  |
|    | $[-0.6, -0.8]$ and $[4.1, 4.3]$  | A1   | dependent on B2M1  |
|    | <b>Additional Guidance</b>   |      |  |
|    | Factorising by use of the quadratic formula alone                      |      | B0M0A0   |
|    | Ignore any $y$ values for A1   |      |  |
|    | Do not allow M mark for $\frac{dy}{dx} = \text{linear function drawn}$ |      |  |
|    | Graph of $x^2 - \frac{7}{2}x - 3 = 0$                                  |      | B0M0A0   |

| Q  | Answer  | Mark  | Comments  |
|----|---|-------|---|
| 21 | $2^{(4x+1)(x+3)}$ or $2^{4x^2+13x+3}$   | M1    | lhs   |
|    | $(2^3)^{x-1}$ or $2^{3(x-1)}$ or $2^{3x-3}$   | M1    | rhs   |
|    | $4x^2 + 12x + x + 3 = 3x - 3$<br>or<br>$4x^2 + 10x + 6 (= 0)$<br>or<br>$2x^2 + 5x + 3 (= 0)$            | M1dep | oe with expanded brackets<br>dependent on both M marks<br>$4x^2 + 12x + x + 3$ implies first M mark<br>$3x - 3$ implies second M mark |
|    | $(2x + 3)(x + 1) (= 0)$<br>or if quadratic formula used<br>$\frac{-5 \pm \sqrt{5^2 - 24}}{4}$ or better | M1dep | oe factorised   |
|    | $-1$<br>and<br>$-1\frac{1}{2}$ or $-\frac{3}{2}$ or $-1.5$  | A1    |   |
|    | <b>Additional Guidance</b>  |       |   |
|    | Both answers correct without working will get all marks   |       | M4A1  |
|    | Only one solution correct without working scores nothing  |       | M0A0  |
|    | No oe for A mark  |       |   |
|    | Possible to make both sides powers of 8. Mark to the equivalent of this MS                              |       |   |

| Q  | Answer   | Mark  | Comments                     |
|----|--|-------|------------------------------|
| 22 | <b>Alternative method 1 – common factor</b>  |       |                              |
|    | $\frac{\sin x(3 \cos x + \sin x)}{4 \cos x(3 \cos x + \sin x)}$  | M1    |                              |
|    | $\frac{\tan x}{4}$   | M1dep |                              |
|    | $\tan x = -\sqrt{3}$   | M1dep | could imply first M1dep mark |
|    | $120^\circ$  | A1    |                              |
|    | <b>Alternative method 2 - divide by <math>\cos^2 x</math></b>  |       |                              |
|    | $\frac{3 \tan x + \tan^2 x}{12 + 4 \tan x}$  | M1    |                              |
|    | $\frac{\tan x(3 + \tan x)}{4(3 + \tan x)}$ or $\frac{\tan x}{4}$   | M1dep |                              |
|    | $\tan x = -\sqrt{3}$   | M1dep | could imply first M1dep mark |
|    | $120^\circ$  | A1    |                              |
|    | <b>Alternative method 3 – setting up an equation</b>   |       |                              |
|    | $4 \sin x(3 \cos x + \sin x)$<br>$= -4\sqrt{3} \cos x(3 \cos x + \sin x)$<br>or $3 \tan x + \tan^2 x = -3\sqrt{3} - \sqrt{3} \tan x$ | M1    |                              |
|    | $\tan x(3 \cos x + \sin x)$<br>$= -\sqrt{3}(3 \cos x + \sin x)$<br>or $\tan x(3 + \tan x) = -\sqrt{3}(3 + \tan x)$                   | M1dep |                              |
|    | $\tan x = -\sqrt{3}$   | M1dep | could imply first M1dep mark |
|    | $120^\circ$  | A1    |                              |
|    | <b>Additional Guidance</b>   |       |                              |
|    | Additional solutions   |       | A0                           |
|    | Untidy mathematical notation can be condoned   |       |                              |
|    | Correct answer with no working   |       | MOA0                         |

| Q  | Answer               | Mark | Comments  |
|----|----------------------|------|---|
| 23 | $x = 6$ and $y = 32$ | B5   | B4<br>$x = 6$ or $y = 32$<br>and<br>$a = 3$ or $c = 4$<br>(could be embedded in $3n^2 - 5n + 4$ )<br>B3<br>$a = 3$ or $c = 4$<br>(could be embedded in $3n^2 - 5n + 4$ )<br>B2<br>any two different correct equations<br>set up for a combination of $a$ , $c$ , $x$<br>and $y$<br>(may include $b$ . Do not include<br>$b = -5$ )<br>B1<br>any correct equation set up for a<br>combination of $a$ , $c$ , $x$ and $y$<br>(may include $b$ . Do not include<br>$b = -5$ )<br>or (second difference)<br>$= 18 - 2x$ or $x + y - 32$<br>(must be correctly simplified) |
|    |                      |      | <b>Additional Guidance</b>  |
|    |                      |      | Do not accept any equations with $n$  |
|    |                      |      | $x = 6$ on answer line with no working and $y \neq 32$<br>B0  |
|    |                      |      | $x = 6$ and $y = 32$ on answer line with no working<br>B5   |