Level 2 Certificate
FURTHER MATHEMATICS
8365/1

Paper 1 Non-Calculator

Mark scheme

June 2025

Version: 1.0 Final

2 5 6 G 8 3 6 5 / 1 / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

No student should be disadvantaged on the basis of their gender identity and/or how they refer to the gender identity of others in their exam responses.

A consistent use of 'they/them' as a singular and pronouns beyond 'she/her' or 'he/him' will be credited in exam responses in line with existing mark scheme criteria.

Further copies of this mark scheme are available from aqa.org.uk

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

M	Method marks are awarded for a correct method which could lead to a correct answer.
M dep	A method mark dependent on a previous method mark being awarded.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between *a* and *b* inclusive.
3.14…	Accept answers which begin 3.14 eg 3.14, 3.142, 3.1416

Examiners should consistently apply the following principles.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

Questions which ask candidates to show working

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

Questions which do not ask candidates to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Continental notation

Accept a comma used instead of a decimal point (for example, in measurements or currency), provided that it is clear to the examiner that the candidate intended it to be a decimal point.

Q	Answer	Mark	Comments
1(a)	$x < 4$ or $4 > x$	B1	

Additional Guidance	
Incorrect inequality symbol eg $x \leqslant 4$	B0
$\mathrm{f}(x) < 4$ or just < 4 or just 4 (even if correct answer seen in working)	B0
$=$ used in working but correct answer	B1
$a < x < 4$	B0
$x \leqslant 3$	B0
$\{x: x < 4\}$	B1
Nothing on answer line then mark final line of working	
B1 response with a list of integers on answer line	B0
Only a list of integers	B0

Q	Answer	Mark	Comments
1(b)	$-31 < \mathrm{g}(x) < 4$	B2	oe eg $4 > \mathrm{g}(x) > -31$ word 'and' or '$\cap$' must be included if writing two inequalities for B2 or B1 eg $-31 < \mathrm{g}(x)$ and $\mathrm{g}(x) < 4$ B1 $k < \mathrm{g}(x) < 4$ where k is less than 4 or $k \leqslant \mathrm{g}(x) < 4$ where k is less than 4 or $-31 < \mathrm{g}(x) < m$ where m is greater than -31 or $-31 < \mathrm{g}(x) \leqslant m$ where m is greater than -31 or -31 and 4 seen from correct working (but within parameters of the Additional Guidance notes about listing integers)

Additional Guidance

Condone $\mathrm{g}(x)$ replaced by eg y or g or gx or f or fx or G or Gx or $x^3 - 4$ eg1 $-31 < \mathrm{f}(x) < 4$ eg2 $-31 < y < 4$	B2
Condone $\mathrm{g}(x)$ replaced by eg y or g or gx or f or fx or G or Gx or $x^3 - 4$ for any of the B1 answers. eg1 $k < \mathrm{f}(x) < 4$ where k is less than 4 eg2 $-31 < y < m$ where m is greater than -31	B1
Do not condone $\mathrm{g}(x)$ replaced by x but could still score B1 for -31 and 4 seen from correct working	
$(-31, 4)$ or $\{\mathrm{g}(x)\text{: } -31 < \mathrm{g}(x) < 4\}$	B2
$[-31, 4]$ or $(-31, 4]$ or $[-31, 4)$	B1
Condone eg $\mathrm{g}(x) = -31 < \mathrm{g}(x) < 4$	B2
Just one inequality on answer line without seeing the other value in the working eg $-31 < \mathrm{g}(x)$	B0
B2 response with a list of integers on answer line	B1
B1 response with a list of integers on answer line or just a list of integers	B0

Q	Answer	Mark	Comments
1(c)	**Alternative method 1: reversing the terms then rearranging**		
	$x = \sqrt{\left(2\text{h}^{-1}(x)+1\right)}$ or $x = \sqrt{(2y+1)}$	M1	oe
	$\frac{x^2-1}{2}$	A1	oe eg $\frac{x^2}{2}-\frac{1}{2}$
	Alternative method 2: rearranging then reversing the terms		
	$(x =)\ \frac{\text{h}(x)^2-1}{2}$ or $(x =)\ \frac{y^2-1}{2}$	M1	oe
	$\frac{x^2-1}{2}$	A1	oe eg $\frac{x^2}{2}-\frac{1}{2}$
	Additional Guidance		
	Answer left as $y = \frac{x^2-1}{2}$		M1A0
	Using a flow chart with incorrect answer		M0A0
	Using a flow chart with correct answer		M1A1

Q	Answer	Mark	Comments
2	$(a =)$ 4 and $(b =)$ –2	B2	B1 $(a =)$ 4 or $(b =)$ –2
	Additional Guidance		
	Condone $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$ or 15$\begin{pmatrix} 4 \\ -2 \end{pmatrix}$ in working if nothing on answer line		B2
	Condone $\begin{pmatrix} 4 \\ k \end{pmatrix}$ or 15$\begin{pmatrix} 4 \\ k \end{pmatrix}$ or $\begin{pmatrix} m \\ -2 \end{pmatrix}$ or 15$\begin{pmatrix} m \\ -2 \end{pmatrix}$ in working if nothing on answer line where k and m can be any number		B1

Q	Answer	Mark	Comments
3	$3n$ seen	M1	can be any letter eg $3n + 2$
	$3n - 1 > 500$	M1dep	oe eg $3n > 501$
	168	A1	SC1 167 on answer line if no other marks scored
	Additional Guidance		
	503 on answer line and 168 seen in working		M2A0
	Condone $3n - 1 = 500$ or $3n - 1 \geqslant 501$ or $3n - 1 \geqslant 500$		M2
	Condone $3n > 500$ oe if recovered for correct answer		M2A1
	$500 \div 3$ or $501 \div 3$ would not imply M1		
	Listing terms with correct answer		M2A1
	Listing terms with incorrect answer		M0A0

<table>
<tr><th>Q</th><th colspan="2">Answer</th><th>Mark</th><th>Comments</th></tr>
<tr><td rowspan="5">4(a)</td><td colspan="2">(x coordinate =) 27
or (y coordinate =) 18
or 12 + (12 – –3) or 10 + (10 – 2)
or –3 + 2(12 – –3) or 2 + 2(10 – 2)
or $\begin{pmatrix}15\\8\end{pmatrix}$ or $\begin{pmatrix}30\\16\end{pmatrix}$
or $\frac{x-3}{2}=12$ or $\frac{y+2}{2}=10$
or $\frac{x+3}{2}=15$ or $\frac{y-2}{2}=8$</td><td>M1</td><td>oe could be seen on a diagram</td></tr>
<tr><td colspan="2">(27, 18)</td><td>A1</td><td></td></tr>
<tr><td colspan="4">Additional Guidance</td></tr>
<tr><td colspan="3">15 and 8 both seen or 30 and 16 both seen (could then be used incorrectly) in working or on diagram</td><td>M1</td></tr>
<tr><td colspan="3">Both coordinates correct but no working</td><td>M1A1</td></tr>
</table>

Q	Answer	Mark	Comments
4(b)	**Alternative method 1: using AB**		
	$\sqrt{\left((12 - -3)^2 + (10-2)^2\right)}$	M1	oe eg $\sqrt{289}$ or may spot it's an 8, 15, 17 triangle
	17	A1	
	Alternative method 2: using AC		
	$\sqrt{\left((27 - -3)^2 + (18-2)^2\right)}$	M1	oe may spot it's an 8, 15, 17 triangle do not allow ft from incorrect answer to part (a)
	(= 34 ÷ 2 =) 17	A1	
	Additional Guidance		
	Look at part (a) if no marks awarded in part (b) eg 12 – – 3 = 9 seen in part (a) followed by $\sqrt{9^2+8^2}$ in part (b)		M1A0
	B to C would be $\sqrt{\left((27-12)^2+(18-10)^2\right)}$ oe Do not allow ft from incorrect part (a)		M1

Q	Answer	Mark	Comments
5	**Alternative method 1: by using the roots**		
	$(x - -1)(x - 5)$ or $(x + 1)(x - 5)$	M1	
	$x^2 - 5x + x - 5$ or $x^2 - 4x - 5$ or $b = -4$ or $c = -5$ with correct factors	M1dep	oe eg could be in a grid
	$b = -4$, $c = -5$	A1	
	Alternative method 2: by substitution		
	$0 = 1 - b + c$ and $0 = 25 + 5b + c$ or $1 - b + c = 25 + 5b + c$	M1	oe do not allow $0 = (-1)^2 + (-1)\,b + c$ or $0 = (5)^2 + (5)b + c$ until processed correctly
	$0 = 24 + 6b$ or $0 = 30 + 6c$	M1dep	oe eliminating either b or c
	$b = -4$, $c = -5$	A1	
	Alternative method 3: by using the minimum from completing the square		
	$(x - 2)^2 + k$ or $\left(x + \frac{b}{2}\right)^2 + k$	M1	oe k could be any letter or number or combination or 0
	$0 = (3)^2 + k$ or $k = -9$ or $\frac{-b}{2} = 2$	M1dep	oe by substituting in one pair of coordinates 3 could be -3 or $+3$ or ± 3
	$b = -4$, $c = -5$	A1	
	Alternative method 4: by using the minimum from differentiation		
	$\frac{dy}{dx} = 2x + b$	M1	
	$2(2) + b = 0$	M1dep	oe
	$b = -4$, $c = -5$	A1	
	Additional Guidance		
	Up to M1 may be awarded for correct work with no answer or incorrect answer, even if this is seen amongst multiple attempts		
	Both coefficients correct but no working		M2A1
	One coefficient correct and the other incorrect or missing and no working		M0A0

Q	Answer	Mark	Comments
6	**Alternative method 1: by simplifying**		
	Any one of $\left(\sqrt{75}=\right)$ $5\sqrt{3}$ or $\left(\sqrt{27}=\right)$ $3\sqrt{3}$ or $\left(\sqrt{48}=\right)$ $4\sqrt{3}$ seen	M1	can be seen outside of the whole fraction
	$\dfrac{2\sqrt{3}}{4\sqrt{3}}$ or $\dfrac{2}{4}$ or $\dfrac{5-3}{4}$ or $\sqrt{\dfrac{12}{48}}$	M1dep	would imply first M mark
	$\dfrac{1}{2}$ or 0.5	A1	
	Alternative method 2: by taking out a common factor of $\sqrt{3}$		
	$\dfrac{\sqrt{25}-\sqrt{9}}{\sqrt{16}}$ or $\dfrac{\sqrt{3}\left(\sqrt{25}-\sqrt{9}\right)}{\sqrt{3}\left(\sqrt{16}\right)}$	M1	
	$\dfrac{2}{4}$ or $\dfrac{5-3}{4}$	M1dep	would imply first M mark
	$\dfrac{1}{2}$ or 0.5	A1	
	Alternative method 3: by rationalising		
	$\dfrac{\sqrt{48}\left(\sqrt{75}-\sqrt{27}\right)}{48}$ or $\dfrac{\left(\sqrt{3600}-\sqrt{1296}\right)}{48}$	M1	oe must have a denominator of 48
	$\dfrac{60-36}{48}$ or $\dfrac{24}{48}$	M1dep	oe but with all surds correctly removed
	$\dfrac{1}{2}$ or 0.5	A1	
	Additional Guidance		
	Also possible to rationalise with $\sqrt{3}$ or other surds. Follow Alt method 3 and mark to an equivalent level		

Q	Answer	Mark	Comments
7	108° or 288°	M1	can be scored in the working
	108° and 288°	A1	SC1 tan 108° and tan 288°
	Additional Guidance		
	Values outside the range that are correct should not be penalised (as long as correct answers are also given on the answer line)		M1 A1
	Incorrect values outside the range written on answer line maximum mark M1		
	Condone missing degrees sign		

Q	Answer	Mark	Comments
8	x^{12}	B1	
	Additional Guidance		

Q	Answer	Mark	Comments
9	**Alternative method 1: by considering the third digit**		
	$8 \times 7 \times 5$ or 280 or $8 \times 7 \times 4$ or 224	M1	
	$8 \times 7 \times 5$ or 280 and $8 \times 7 \times 4$ or 224 **or** $8 \times 7 \times (5 - 4)$	M1dep	
	56	A1	SC1 81
	Alternative method 2: splitting the total in the correct ratio		
	$9 \times 8 \times 7$	M1	
	$\frac{504}{9}$ **or** 280 and 224	M1dep	
	56	A1	SC1 81
	Additional Guidance		
	Up to M1 may be awarded for correct work with no answer or incorrect answer, even if this is seen amongst multiple attempts		
	Listing outcomes with incorrect answer (could still gain M1 from above if part of multiple attempts)		M0A0
	56 from clearly incorrect working		M0A0
	56 with no incorrect working but some omitted working – this could come from listing groups of outcomes but missing some groups out (that give the same answers so cancel)		M2A1
	Listing outcomes with correct answer		M2A1

Q	Answer	Mark	Comments
10	**Alternative method 1: using gradients**		
	(Gradient QR =) $\frac{12-0}{3-9}$ or –2	M1	oe condone embedded in an equation
	$\frac{-1}{\text{their} -2}$ or $\frac{1}{2}$	M1	oe condone embedded in an equation if no working for first M mark $\frac{1}{2}$ would imply M2
	$-24 + 2a = -3$ or $\frac{12-a}{3-0} = \frac{1}{2}$	M1dep	oe implies M3 and must be fully correct condone c instead of a
	$10\frac{1}{2}$ or $\frac{21}{2}$ or 10.5	A1	
	Alternative method 2: using Pythagoras		
	(QR =) $\sqrt{6^2+12^2}$ or (QR^2 =) $6^2 + 12^2$ or (PQ =) $\sqrt{3^2+(12-a)^2}$ or (PQ^2 =) $3^2 + (12 - a)^2$ or (PR =) $\sqrt{9^2+a^2}$ or (PR^2 =) $9^2 + a^2$	M1	could be seen on diagram
	(QR =) $\sqrt{6^2+12^2}$ or (QR^2 =) $6^2 + 12^2$ **and** (PQ =) $\sqrt{3^2+(12-a)^2}$ or (PQ^2 =) $3^2 + (12 - a)^2$ **and** (PR =) $\sqrt{9^2+a^2}$ or (PR^2 =) $9^2 + a^2$	M1	
	$81 + a^2 = 9 + (12-a)^2 + 180$	M1dep	oe dep on M2 must be a correct equation
	$10\frac{1}{2}$ or $\frac{21}{2}$ or 10.5	A1	
	See over for Additional Guidance		

<table>
<tr><td rowspan="3">10</td><td colspan="2">Additional Guidance</td></tr>
<tr><td>Condone further incorrect simplification of fraction on answer line</td><td></td></tr>
<tr><td>Common incorrect gradients of QR in Alt 1 will give answers of:
13.5 from gradient of $QR = 2$
18 from gradient of $QR = \frac{1}{2}$
6 from gradient of $QR = -\frac{1}{2}$
These could be M2M0A0 or M0M1M0A0 depending on whether the first mark can be awarded</td><td></td></tr>
</table>

<table>
<tr><th>Q</th><th>Answer</th><th>Mark</th><th>Comments</th></tr>
<tr><td rowspan="14">11</td><td colspan="3">Alternative method 1: quadratic formula</td></tr>
<tr><td>$x(x^2 + 10x + 8)$ $(= 0)$
or $x = 0$</td><td>M1</td><td></td></tr>
<tr><td>$(x =)$ $\dfrac{-10 \pm \sqrt{(10^2 - 4(\times 1) \times 8)}}{2(\times 1)}$</td><td>M1</td><td>oe correct substitution into formula from correct quadratic</td></tr>
<tr><td>$(x =)$ $\dfrac{-10 \pm \sqrt{100 - 32}}{2}$ or $\dfrac{-10 \pm \sqrt{68}}{2}$</td><td>A1</td><td>oe simplification eg $-5 \pm \dfrac{\sqrt{68}}{2}$
dep on second M mark only
implied by final A1</td></tr>
<tr><td>$(x =)$ 0, $-5 \pm \sqrt{17}$</td><td>A1</td><td>could be written as two separate surds.</td></tr>
<tr><td colspan="3">Alternative method 2: completes the square</td></tr>
<tr><td>$x(x^2 + 10x + 8)$ $(= 0)$
or $x = 0$</td><td>M1</td><td></td></tr>
<tr><td>$x\ [(x + 5)^2 \ldots\ldots.]$</td><td>M1</td><td>oe eg may not include the common factor of x at this stage
must be from correct quadratic</td></tr>
<tr><td>$x\ [(x + 5)^2 - 17]$</td><td>A1</td><td>oe eg $(x + 5)^2 = 17$
dep on second M mark only
condone common factor of x not being there at this stage</td></tr>
<tr><td>$(x =)$ 0, $-5 \pm \sqrt{17}$</td><td>A1</td><td>could be written as two separate surds.</td></tr>
<tr><td colspan="3">Additional Guidance</td></tr>
<tr><td colspan="2">If $x = 0$ stated as final answer then it doesn't need to appear in the working</td><td></td></tr>
<tr><td colspan="2">If $x = 0$ not stated on answer line then second A mark cannot be awarded: eg
$x = 0$ is seen early in the working with everything else correct
$x = 0$ or $x(x^2 + 10x + 8)$ not seen in the working but with everything else correct</td><td>M2A1A0
M0M1A1A0</td></tr>
<tr><td colspan="2">Factor theorem used correctly to get complete answer</td><td>M2A2</td></tr>
<tr><td></td><td colspan="2">Factor theorem used without complete answer</td><td>M0A0</td></tr>
</table>

Q	Answer	Mark	Comments
12	**Alternative method 1: reflex angle *AOC* from angle at circumference**		
	$2(y + 3x)$ or $2y + 6x$	M1	using 2 × *ABC*
	$4x + 2(y + 3x) = 360$	M1dep	oe implies first M mark
	$2y + 10x = 360$ or $2y = 360 - 10x$ Leading to $y = 180 - 5x$	A1	M2 must be scored either of the first two statements is sufficient or a correct equivalent statement that improves on M2
	Angle at centre = 2 × angle at circumference **and** angles at a point = 360 stated and used correctly	B1	
	Alternative method 2: $2x$ on circumference		
	Lines drawn to make angle at circumference and stated or seen on the diagram as $2x$ or $180 - (y + 3x)$	M1	If stated in working then lines not necessary on diagram
	$y + 3x + 2x = 180$ or $4x = 2(180 - (y + 3x))$	M1dep	oe implies first M mark
	$y = 180 - 5x$	A1	M2 must be scored
	Angle at centre = 2 × angle at circumference **and** opposite angles of a cyclic quad total 180 stated and used correctly	B1	
	Alternative method 3: reflex angle *AOC* from angles at the centre		
	$360 - 4x$	M1	using angles at the centre
	$360 - 4x = 2(y + 3x)$	M1dep	oe implies first M mark
	$2y + 10x = 360$ or $2y = 360 - 10x$ Leading to $y = 180 - 5x$	A1	M2 must be scored either of the first two statements is sufficient or a correct equivalent statement that improves on M2
	Angle at centre = 2 × angle at circumference **and** angles at a point = 360 stated and used correctly	B1	
	See over for Alternative method 4 and Additional Guidance		

12	**Alternative method 4: isosceles triangles**		
	Radius drawn from *O* to *B* to make two isosceles triangles and shows that *OAB* is equal to *OBA* and *OCB* is equal to *OBC* in working or on diagram	M1	
	$y + 3x + 4x + OAB + OCB = 360$ and $OAB + OCB = y + 3x$	M1dep	oe implies first M mark
	$y + 3x + 4x + y + 3x = 360$ or $2y = 360 - 10x$ Leading to $y = 180 - 5x$	A1	M2 must be seen either of the first two statements is sufficient or a correct equivalent statement that improves on M2
	Angles of a quadrilateral add to 360 **and** isosceles triangles formed from radii have two equal angles stated and used correctly	B1	
	Additional Guidance		
	A correct angle on the diagram from any Alt will score the first M mark then follow that Alt		
	Stop marking as soon as $y = 180 - 5x$ is substituted in and used		
	If angles are stated in the working without being indicated on the diagram (the diagram will take precedence) they must be labelled accurately eg Reflex *O* or Concave *AOC* = Omission of labels or incorrect labels (if not seen on diagram either) will lose the A mark but M marks can still be awarded		

Q	Answer	Mark	Comments
13	C	B1	
	Additional Guidance		

Q	Answer	Mark	Comments
14	**Alternative method 1**		
	$2(x-3)(x+1)$ or $(2x-6)(x+1)$ or $(x-3)(2x+2)$	M1	
	$x(x-3)$ or $4(x+1)$ or $2(2x+2)$	M1	oe for either of the other expressions factorised
	$\frac{2(x-3)(x+1)}{x(x-3)} \times \frac{7x^2}{4(x+1)}$	M1dep	oe depends on both previous M marks and must all be correctly factorised into something that can be fully cancelled could imply previous M marks
	$\frac{7}{2}x$ or $3.5x$ or $3\frac{1}{2}x$	A1	
	Alternative method 2: cancelled factor of 2 first		
	$\frac{x^2-2x-3}{x^2-3x} \div \frac{2x+2}{7x^2}$	M1	oe where 2 has been cancelled
	$(x-3)(x+1)$ or $x(x-3)$ or $2(x+1)$	M1	oe for any expression factorised correctly
	$\frac{(x-3)(x+1)}{x(x-3)} \times \frac{7x^2}{2(x+1)}$	M1dep	oe depends on both previous M marks and must all be correctly factorised into something that can be fully cancelled could imply previous M marks
	$\frac{7}{2}x$ or $3.5x$ or $3\frac{1}{2}x$	A1	
	See over for Alternative method 3 and Additional Guidance		

<table>
<tr><td rowspan="9">14</td><td colspan="3">Alternative method 3: multiplying first</td></tr>
<tr><td>$\frac{7x^2(2x^2-4x-6)}{(x^2-3x)(4x+4\ )}$</td><td>M1</td><td>oe eg $\frac{14x^4-28x^3-42x^2}{4x^3-8x^2-12x}$
where fractions have been multiplied and combined into a single fraction</td></tr>
<tr><td>Numerator or Denominator further factorised correctly as a step towards the final answer</td><td>M1</td><td>eg $14x^2(x^2-2x-3)$
or $14x^2(x-3)(x+1)$
or $2x(2x^2-4x-6)$
or $4x(x^2-2x-3)$
or $4x(x-3)(x+1)$ etc
could be implied by cancellation of a common factor
neither $7x^2(2x^2-4x-6)$ nor $(x^2-3x)(4x+4\ )$ would score this M mark</td></tr>
<tr><td>$\frac{7x^2(2x^2-4x-6)}{2x(2x^2-4x-6)}$ or
$\frac{14x^2(x^2-2x-3)}{4x(x^2-2x-3)}$</td><td>M1dep</td><td>oe depends on both previous M marks and must all be correctly factorised into something that can be fully cancelled</td></tr>
<tr><td>$\frac{7}{2}x$ or $3.5x$ or $3\frac{1}{2}x$</td><td>A1</td><td></td></tr>
<tr><td colspan="3">Additional Guidance</td></tr>
<tr><td colspan="2">Condone factorising outside of the fractions for first two M marks for Alt 1 and Alt 2 and on the second mark for Alt 3</td><td></td></tr>
<tr><td colspan="2">Alt 2 it is possible to cancel with x or $2x$. Mark as an equivalent to Alt 2</td><td></td></tr>
<tr><td colspan="2">Follow whichever Alternative method gives the best mark</td><td></td></tr>
</table>

Q	Answer	Mark	Comments
15	$p = -2$	B1	
	$q = -1$	B1ft	ft their $p + 1$ or correct answer
	$r = -14$	B1ft	ft their $p - 12$ or correct answer
	Additional Guidance		
	Fully correct expansion (could be seen in a grid) if no other marks scored $x^3 + px^2 + x^2 + p\,x - 12\,x - 12\,p$ oe		B1

Q	Answer	Mark	Comments
16	**Alternative method 1: sine rule**		
	$\dfrac{10}{\sin 45} = \dfrac{AC}{\sin 30}$	M1	oe could be seen by implication in further working
	$\sin 30 = \dfrac{1}{2}$ and $\sin 45 = \dfrac{1}{\sqrt{2}}$ or $\dfrac{\sqrt{2}}{2}$	B1	oe could be seen by implication in working must be clearly selected if part of a list of values
	$5\sqrt{2}$	A1	
	Alternative method 2: splitting the triangle into 2 right angled triangles with a perpendicular from C to the side AB at a point D		
	$CD = 10\sin 30$ **and** $AC = \dfrac{CD}{\sin 45}$ or $\sqrt{CD^2 + CD^2}$	M1	oe
	$\sin 30 = \dfrac{1}{2}$ and $\sin 45 = \dfrac{1}{\sqrt{2}}$ or $\dfrac{\sqrt{2}}{2}$	B1	oe could be seen by implication in further working must be clearly selected if part of a list of values if Pythagoras used in M mark sin45 not needed so $\sin 30 = \dfrac{1}{2}$ is enough for B1
	$5\sqrt{2}$	A1	
	Additional Guidance		
	Condone untidy notation		

Q	Answer	Mark	Comments
17	$4 \times 4a \ (\times x^3)$ or $(-2x)^3$ or $-8(x^3)$	M1	oe 4 could be written as $\begin{pmatrix}4\\3\end{pmatrix}$ or $\begin{pmatrix}4\\1\end{pmatrix}$ x^3 doesn't need to be seen
	$4 \times 4a = -8$ or $(-2)^3$ or $16a = -8$	M1dep	oe both terms correct x^3 needs to be on both sides of the equation or neither
	-0.5	A1	oe
	Additional Guidance		
	Coefficients must be part of a correct term eg $16a + x^3$		M0
	Errors in negatives can be recovered		
	Condone correct terms seen in full expansions (even if there are other errors in the expansions)		

Q	Answer	Mark	Comments
18(a)	$2x + y = 60$ or $y = 60 - 2x$ seen	M1	oe
	$A = xy = x(60 - 2x)$ seen	A1	must be clear that the answer has come from an understanding that the area is x multiplied by y eg A = length × width
	Additional Guidance		
	Consider working in part (b) if not contradictory		
	Condone $A = x \times y = x \times (60 - 2x)$		

Q	Answer	Mark	Comments
18(b)	**Alternative method 1: differentiation**		
	$60x - 2x^2$	M1	expanding the brackets
	$\left(\frac{dA}{dx} =\right) 60 - 4x$	M1dep	oe condone $\left(\frac{dy}{dx} =\right)$
	$x = 15$	A1ft	dependent on first M mark x value from their 2-term derivative
	450	A1	
	Alternative method 2: midpoint of roots		
	$x(60 - 2x) = 0$ or $2x(30 - x) = 0$	M1	only award if it is clearly an attempt to find the midpoint of the roots
	$x = 0$ and $x = 30$	M1dep	with M1 awarded
	$x = 15$	A1ft	dependent on first M mark correct midpoint from their roots
	450	A1	
	Alternative method 3: completes the square		
	$60x - 2x^2$ or $-2(x^2 - 30x)$	M1	
	$-2(x - 15)^2$……..	M1dep	
	$x = 15$	A1ft	dependent on first M mark correct value from their bracket
	450	A1	implies A1ft
	Additional Guidance		
	Consider working in part (a) and on diagram if not contradictory		
	Answer only without any working gains full marks (students may know that the maximum Area is when the length is twice the width)	M2A2	
	450 from Trial and Improvement (must be on answer line)	M2A2	
	Incorrect answer from Trial and Improvement (but M marks could have been awarded before T&I started)	M0A0	
	On Alt 1 condone $A = 60 - 4x$ and $y = 60 - 4x$ in Alt 1		

Q	Answer	Mark	Comments
19(a)	**Alternative method 1: inspection or long division**		
	$3x^2 + kx + 2$ by inspection or first two terms using long division: $3x^2 + 5x + k$	M1	k could be blank. Can be seen in a grid method
	$3x^2 + 5x + 2$	M1dep	all terms correct written as a single expression
	$(x - 1)(3x + 2)(x + 1)$	A1	any order of brackets
	Alternative method 2: factor theorem		
	Any one of f(–1), f(2), f(–2), $\text{f}\left(-\frac{2}{3}\right)$ attempted and evaluated correctly	M1	f(–1) = 0, f(2) = 24, f(–2) = –12, $\text{f}\left(-\frac{2}{3}\right) = 0$
	$(3x + 2)$ or $(x + 1)$ identified as a factor	M1dep	oe
	$(x - 1)(3x + 2)(x + 1)$	A1	any order of brackets
	Alternative method 3: equating coefficients		
	$(x - 1)(ax^2 + bx + c)$ and $ax^3 - ax^2 + bx^2 - bx + cx - c$	M1	
	$a = 3$, $b = 5$ and $c = 2$ or $3x^2 + 5x + 2$	M1dep	
	$(x - 1)(3x + 2)(x + 1)$	A1	any order of brackets
	Alternative method 4: factorising without using $(x - 1)$		
	$x^2(3x + 2) - 1(3x + 2)$	M1	1 could be omitted
	$(3x + 2)(x^2 - 1)$	M1dep	
	$(x - 1)(3x + 2)(x + 1)$	A1	any order of brackets
	See over for Additional Guidance		

19(a)	**Additional Guidance**	
	Penalise further working to find solutions to f(x) = 0	M2A0
	Correct answer with no working	M2A1
	(x – 1) or (3x + 2) or (x + 1) on answer line on their own with no working	M0A0
	(3x + 2) (x + 1) on its own on answer line	M2A0
	Condone missing brackets recovered Do not award A mark for missing brackets not recovered	
	In Alt 1 synthetic division can be used	

Q	Answer	Mark	Comments
19(b)	x or c = 3 used	M1	eg 1 × 19 × 11 or 19 and 11 seen together in working
	209 or 19 × 11	A1	
	Additional Guidance		
	c = 209 would imply method		M1A0
	x or c = 3 could be one of a number of trials		M1
	19 × 11 seen in working but then worked out incorrectly for answer		M1A0
	(c =) 3 on answer line not from incorrect working		M1A0

Q	Answer	Mark	Comments
20	**Alternative method 1: completes the square**		
	$\left(\frac{dy}{dx}=\right) x^2 - 10x + 28$	M1	oe 3 terms with at least two terms correct condone $y = x^2 - 10x + 28$
	$(x - 5)^2$……	M1	must be ft from a quadratic derivative
	$(x - 5)^2 + 3$	M1dep	dep on both previous M marks must be from correct $\frac{dy}{dx}$ and not ft
	$(x - 5)^2 \geqslant 0$ **and** so increasing function **or** the gradient or $(x - 5)^2 + 3$ or $\frac{dy}{dx}$ is always positive	A1	oe condone a statement that squares are always positive or $(x - 5)^2 > 0$
	Alternative method 2: uses the discriminant		
	$\left(\frac{dy}{dx}=\right) x^2 - 10x + 28$	M1	oe 3 terms with at least two terms correct condone $y = x^2 - 10x + 28$
	$10^2 - 4 \times 1 \times 28$	M1	oe must be ft from a quadratic derivative
	No real roots stated or $\sqrt{-12}$ **or** the gradient is never 0	M1dep	dep on both previous M marks must be from correct $\frac{dy}{dx}$ and not ft
	Statement to explain why this means the cubic is always an increasing function **or** the gradient is always positive	A1	eg gradient can't be 0 and when $x = 1$ gradient is 19 so increasing function or graph of $\frac{dy}{dx}$ against x is always above the x axis so $\frac{dy}{dx}$ is always positive
	Additional Guidance		

Q	Answer	Mark	Comments
21	**Alternative method 1**		
	Denominator $3 + \cos^2 x - 3(1 - \cos^2 x)$	M1	oe after identity used eg $\cos^2 x + 3\cos^2 x$
	$\frac{20\sin^2 x}{4\cos^2 x} = 5\tan^2 x$	A1	
	Alternative method 2		
	Denominator $4 - 4\sin^2 x$	M1	from $\cos^2 x = 1 - \sin^2 x$
	$\frac{20\sin^2 x}{4\cos^2 x} = 5\tan^2 x$	A1	
	Alternative method 3		
	Denominator $3(\cos^2 x + \sin^2 x) + \cos^2 x - 3\sin^2 x$	M1	oe after identity used eg $\cos^2 x + 3\cos^2 x$
	$\frac{20\sin^2 x}{4\cos^2 x} = 5\tan^2 x$	A1	
	Additional Guidance		
	Only mark using one of the alts – once the candidate starts to treat the solution as an equation by moving terms around from one side of the identity to the other then stop awarding marks		
	Correct notation for $5\tan^2 x$ needed to score A mark		
	M mark can be awarded for denominator worked out separately but not the A mark (unless recovered)		

Q	Answer	Mark	Comments
22	**Alternative method 1**		
	$y = 2x + 1$	B1	oe equation of straight line eg $y - 7 = 2(x - 3)$
	$(x - 2)^2 + (y - 3)^2 = 5^2$	B1	oe eg = 25 or brackets expanded correctly
	Substitutes their $(2x + 1)$ into their equation of a circle in the form $(x - a)^2 + (y - b)^2 =$ their 5^2 where a and $b \neq 0$	M1	eg $(x - 2)^2 + ((2x + 1) - 3)^2 = 5^2$ or $x^2 - 4x + 4 +$ $(2x + 1)^2 - 6(2x + 1) + 9 = 25$
	$5x^2 - 12x - 17$ (= 0)	A1	condone $5x^2 - 12x = 17$ if completing the square
	$(5x - 17)(x + 1)$ (=0) or $\dfrac{12 \pm \sqrt{144 + 340}}{10}$ or $5\left[\left(x - \dfrac{6}{5}\right)^2 - \dfrac{121}{25}\right]$	M1dep	dep on B2M1A1
	(–1, –1) and (3.4, 7.8)	A1	oe
	See over for Alternative method 2 and Additional Guidance		

22	**Alternative method 2**		
	$x = \dfrac{y-1}{2}$	B1	oe equation of straight line eg $y - 7 = 2(x - 3)$
	$(x - 2)^2 + (y - 3)^2 = 5^2$	B1	oe eg = 25 or brackets expanded correctly
	Substitutes their $\dfrac{y-1}{2}$ into their equation of a circle in the form $(x - a)^2 + (y - b)^2 =$ their 5^2 where a and $b \neq 0$	M1	eg $\left(\left(\dfrac{y-1}{2}\right)-2\right)^2 + (y-3)^2 = 5^2$ or $\left(\dfrac{y-1}{2}\right)^2 - 4\left(\dfrac{y-1}{2}\right) + 4$ $+ y^2 - 6y + 9 = 25$
	$5y^2 - 34y - 39$ (= 0)	A1	condone $5y^2 - 34y = 39$ if completing the square
	$(5y - 39)(y + 1)$ (=0) or $\dfrac{34 \pm \sqrt{1156 + 780}}{10}$ or $5\left[\left(y - \dfrac{17}{5}\right)^2 - \dfrac{484}{25}\right]$	M1dep	dep on B2M1A1 would imply B2M1A1
	$(-1, -1)$ and $(3.4, 7.8)$ or $\left(\dfrac{17}{5}, \dfrac{39}{5}\right)$	A1	
	Additional Guidance		
	First M mark can imply the B marks if correct		
	Expanding the circle equation and then substituting in can gain marks as an oe but won't if the circle equation has been expanded incorrectly		
	Condone $(5x - 17)(5x + 5)$ (=0) and $(5y - 39)(5y + 5)$ (=0) for the fifth mark		
	Correct answer from Trial and Improvement		B2M2A2

Q	Answer	Mark	Comments
23	(Area =) $\frac{1}{2} \times (x + 5 + 3x - 8) \times 2x$ or $4x^2 - 3x$	M1	
	$4x^2 - 3x - 22 < 0$	M1dep	oe needs to be a quadratic < 0 condone $4x^2 - 3x < 22$ if completing the square
	$(4x - 11)(x + 2)$ or $\frac{3 \pm \sqrt{9+352}}{8}$ or $4\left[\left(x - \frac{3}{8}\right)^2 - \frac{361}{64}\right]$	M1	must be from the correct quadratic oe could imply first 2 M marks if < 0 seen
	$(k<)\ x < 2\frac{3}{4}$	A1	oe for $2\frac{3}{4}$ where $k < 2\frac{3}{4}$ or k could be blank could imply M3A1
	$2\frac{2}{3} < x \ (< k)$	B1	oe for $2\frac{2}{3}$ where $k > 2\frac{2}{3}$ or k could be blank

Additional Guidance	
Inequalities changed to = without being recovered correctly will have a maximum mark of M2	M1M0M1A0B0
Inequalities changed to = then recovered can score full marks	
Condone $(4x - 11)(4x + 8)(< 0)$ for third M mark	