OCR
Oxford Cambridge and RSA

FSMQ

Additional Mathematics

Unit **6993:** Additional Mathematics

Free Standing Mathematics Qualification

Mark Scheme for June 2014

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations and abbreviations

Annotation in scoris	**Meaning**
BP	Blank Page – this annotation **must** be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for 6993

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation **isw**. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument. Unless otherwise stated (by for instance, **cao** usually apply **isw**

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation **ft** implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Section A

Question			Answer	Marks	Guidance	
1			$-6 < 2x - 1 < 7$ $\Rightarrow -5 < 2x < 8 \Rightarrow -\frac{5}{2} < x < 4$ **isw** **S.C**. If solution only given as number line with two clear open circles at ends then B2	**B1** **B1** **B1** **[3]**	One end point The other Both together - accept 2 separate inequalities linked by "and" **cao** **B3** Final answer seen **www**	Condone incorrect signs (including equals) for first 2 marks. Ignore listings. but not "or" or comma or nothing

Question			Answer	Marks	Guidance	
2			$\frac{dy}{dx} = 3x^2 - 4x + 7$ $\Rightarrow \ y = \ x^3 - 2x^2 + 7x \ + c$ **oe** Substitute (1, 2) $\Rightarrow 2 = 1 - 2 + 7 + c$ $\Rightarrow c = -4$ $\Rightarrow y = x^3 - 2x^2 + 7x - 4$	**M1** **A1** **M1** **A1** **[4]**	Integrate (at least 2 powers increased by 1) Ignore lack of $y =$ and c Dependent on 1st M Final equation must be seen	Beware just multiplying by x Must include $y =$ and -4 Condone f(x) =

Question			Answer	Marks	Guidance	
3	(i)		Area $= \int_0^2 8x^3\,dx = \left[2x^4\right]_0^2$ $= 32 \ (-0) = 32$	**M1** **M1** **A1** **[3]**	Integrate correct function (power of 4 **soi**). Ignore wrong or no limits Substitute $x = 2$. Dependent on first M	Beware just multiplying by x i.e. $8x^4$
	(ii)		Add a rectangle of area 10 $\Rightarrow$ Total area = 42	 **B1** **[1]**	**ft** Addition of 10 must be correct!	Can be by integration

Question			Answer	Marks	Guidance	
4	(i)		$s = 0.09t^2 - 0.0001t^3$ $\Rightarrow \ v = \ 0.18t - 0.0003t^2$ **isw** When $v = 0$, $\quad t = 0$ or $\quad t = \dfrac{0.18}{0.0003} = 600$ Time = 600 seconds or 10 mins **isw**	 **M1** **A1** **M1** **A1** **[4]**	 Diffn (both powers reduced by 1). Set $v = 0$ and any attempt to solve. Dependent on first M. Units required.	Beware division by t. Condone division by t or a constant when expression not set to 0
	(ii)		Substitute *their t* into s $\Rightarrow s$ = 10800 m or 10.8 km **isw**	**M1** **A1** **[2]**	Units required, but only withhold this mark for units wrong or missing if not already withheld in part **(i)**.	

Question			Answer	Marks	Guidance	
5	(i)		Angle BAL = 20 + 25 = 45	**B1**		BAL and ALB must be correctly identified (not from use in (ii))
			Angle ABL = 180 – 65 = 115 **soi** OR exterior angle = 65 **soi** OR angle LBN = 40 **soi**	**B1**		
			Angle ALB = 20	**B1** **[3]**	Or **B2** for ALB **www**	
	(ii)		AB = 7 **soi**	**B1**		
			$\frac{LB}{\sin 45} = \frac{AB}{\sin 20}\left(= \frac{7}{\sin 20}\right)$	**M1**	For *their* AB and *their* angles.	
			$\Rightarrow LB = \frac{7\sin 45}{\sin 20} = 14.5$	**A1** **[3]**	Anything that rounds to 14.5	

Question			Answer	Marks	Guidance	
6	(i)		f(3) = 0 $\Rightarrow$ 27 – 36 + 3a + b = 0 or better	**B1**	e.g. 3a + b = 9.	Powers need to be evaluated
			f(1) = 4 $\Rightarrow$ 1 – 4 + a + b = 4 or better	**B1**	e.g. a+ b = 7	Powers need to be evaluated
			Solve *their* simultaneous eqns from above $\Rightarrow a = 1, b = 6$	**M1** **A1** **[4]**	Attempt to find a and b from *their* eqns	Their working need not be correct
	(ii)		Sight of $(x – 3)(x^2 + px + q)$ for any p, q Or algebraic division	**M1**	Or attempt to find another root of *their cubic* by remainder theorem **soi**	Algebraic division seen by x^2 + ... in quotient and $x^3 – 3x^2$ in division
			$\Rightarrow (x – 3)(x + 1)(x – 2) = 0$	**A1**	For correct complete factorisation **soi** by final answer	
			$\Rightarrow (x =) 3, 2, –1$	**A1** **[3]**	Correct solution	

Question			Answer	Marks	Guidance	
7	(i)		Distance^2 = $5-3\ ^2+\ 11-7\ ^2$ $(=20)$ $\Rightarrow$ distance $=\sqrt{20}\ \ =2\sqrt{5}$	**M1** **A1** **[2]**	**Soi** e.g. by 4.47….. Must be exact **isw**	d = 20 is M0
	(ii)		Centre = midpoint = (4, 9) Radius = $\sqrt{5}$ or decimal equivalent $\Rightarrow\ x-4\ ^2+\ y-9\ ^2=5$ OR $x^2+y^2-8x-18y+92=0$	**B1** **B1** **B1**	Centre Radius **soi** by for e.g. r^2 = 5 ft *their* identified centre (but don't accept A or B) **isw**	Rhs must be 5, not $\sqrt{5}\ ^2$
			Alternative: $\frac{y-11}{x-5}.\frac{y-7}{x-3}=-1$ $\Rightarrow\ y-11\ \ y-7\ +\ x-5\ \ x-3\ =0$ $\Rightarrow x^2+y^2-8x-18y+92=0$	**B1** **B1** **B1** **[3]**	Use of $m_1m_2 = -1$	

Question			Answer	Marks	Guidance	
8	(i)		Grad AB = Grad CD $=1$ $\left(=\frac{4--1}{0--5}\right)$ and $\left(=\frac{-2-3}{2-7}\right)$ **oe** Grad BC = Grad AD $=-\frac{1}{7}$ $\left(=\frac{3-4}{7-0}\right)$ and $\left(=\frac{-2--1}{2--5}\right)$ Two pairs of parallel sides (means ABCD parallelogram)	**B1** **B1** **[2]**	For showing one pair of gradients equal and correct **www** For showing other pair of gradients equal and correct plus completion	**bod** no working but care about seeing $\frac{\delta x}{\delta y}$ Final statement is necessary. Condone 2 pairs of equal gradients (providing they are correct)
	(ii)		$AB^2 = 5^2 + 5^2$ (=50) **oe** for any side $BC^2 = 1^2 + 7^2$ (=50) $\Rightarrow AB^2 = BC^2$ (=50) Equal sides (means rhombus)	**B1** **B1** **[2]**	One length (or squared length) For adjacent length plus completion **www**	Final statement is necessary
	(iii)		Gradients do not fulfil $m_1 . m_2 = -1$ **oe** ie $1\times-\frac{1}{7}\neq-1$ Therefore lines not perpendicular **Alternatives:** **A:** Use of cosine rule Does not give 90^0 **B:** Use of Pythagoras Not satisfied therefore not 90^0 **C:** Use of pythagoras to find length of diagonals (i.e. $\sqrt{160}$ and $\sqrt{40}$) Diagonals not equal	**M1** **A1** **[2]** **M1** **A1** **M1** **A1** **M1** **A1**	For use of $m_1 . m_2 = -1$ Gradients must be correct. **www** **www** **www**	i.e. 2nd gradient not the negative reciprocal of the other Final statement is necessary.

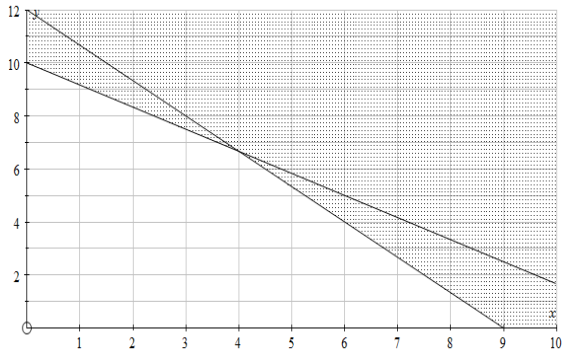
Question			Answer	Marks	Guidance	
9	(i)		$\frac{1-\cos^2 x}{1-\sin^2 x} = \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$	**B1** **[1]**	Use of $\frac{\sin x}{\cos x} = \tan x$ and use of $\sin^2 x + \cos^2 x = 1$	
	(ii)		$\frac{1-\cos^2 x}{1-\sin^2 x} = 3 - 2\tan x$ $\Rightarrow \tan^2 x = 3 - 2\tan x$ $\Rightarrow \tan^2 x + 2\tan x - 3 = 0$ $\Rightarrow \quad \tan x + 3 \quad \tan x - 1 \; = 0$ $\Rightarrow \tan x = -3$ or $\tan x = 1$ $\Rightarrow x = 108(.4...)$ or 45	**M1** **M1** **A1A1** **[4]**	Correct use of **(i)** Factorise their three term quadratic or insertion of *their* values into correct formula Dep on 1st M –1 for any other values inside range, ignore extra values outside range	(Check the two linear factors by whether they multiply out to give the first and last terms of their quadratic.)

Question			Answer	Marks	Guidance	
10	(i)		$y = 2x^2 + x - 5 \Rightarrow \left(\frac{dy}{dx} =\right) \; 4x + 1$ $= 5$ $\Rightarrow x = 1, y = -2$	**B1** **M1** **A1** **[3]**	 Equating to 5 and solving For both	
	(ii)		gradient normal = $-\frac{1}{5}$ $\Rightarrow (y+2) = -\frac{1}{5} \; x - 1$ $\Rightarrow 5y + x + 9 = 0$	**B1** **M1** **A1** **[3]**	Gradient of normal **soi** Equation using *their* (1, –2) and *their* normal gradient (which may only be $\pm\frac{1}{5}$ or -5) **oe**, but only 3 terms **isw**	

Section B

Question			Answer	Marks	Guidance	
11	(i)		Length = $30 - 2x$ Breadth = $14 - 2x$ (Height = x) $\Rightarrow V = (30-2x)(14-2x)x$ $= 4x^2 - 88x + 420\ \ x$ **oe** $= 4x^3 - 88x^2 + 420x$	**B1** **B1** **M1** **A1** **[4]**	**Soi** **Soi** Product of *their* length, breadth and x **www;** must show at least one product of any two lengths step (N.B. Answer given)	N.B. dimensionally correct e.g $= (30-2x)(14x-2x^2)$ Length and breadth must be functions of x
	(ii)		$V = 4x^3 - 88x^2 + 420x$ $\frac{dV}{dx} \Rightarrow 12x^2 - 176x + 420$ **isw** $= 0$ when $12x^2 - 176x + 420 = 0$ $\Rightarrow 3x^2 - 44x + 105 = 0$ $\Rightarrow\ 3x - 35\ \ x - 3\ = 0$ $\Rightarrow x = \frac{35}{3}$ or anything that rounds to 11.7, $x = 3$ **S.C.** Answers only B1, B1	**M1** **A1** **M1** **M1** **A1** **[5]**	Diff (at least two powers reduced by 1) Set *their function* = 0 Dep on 1st M **Soi** by solution Factorise three term quadratic or insertion of *their* values into correct formula Both	Beware division by x. Condone prem div by a constant. (Check the two linear factors by whether they multiply out to give the first and last terms of their quadratic.)
	(iii)		$x = \frac{35}{3}$ should be rejected as it is over half of 14 Substitute an acceptable x into V ($0 < x < 7$) Volume = 576	**B1** **M1** **A1** **[3]**	**ft** from *their* incorrect x Explanation necessary (e.g. one length is -ve) Alt: use of second derivative acceptable. Ignore units	"V is -ve" as the only explanation not accepted. Nor is "x is too big".

Question			Answer	Marks	Guidance	
12	(i)		Time out = $\frac{15}{x}$, Time back = $\frac{15}{x-2}$	B1	For one	
			Total time = $\frac{15}{x}+\frac{15}{x-2}$	B1 [2]	Addition of two correct terms **isw**	
	(ii)		$\frac{15}{x}+\frac{15}{x-2}=6$	B1	Equate *their* time to 6	3 might be divided throughout here
			$\Rightarrow 15\ x-2\ +15x=6x(x-2)$	M1	Multiply throughout by LCM	LCM implies 2 different algebraic denominators
			$15x-30+15x=6x^2-12x$ **oe**	A1	Brackets cleared	N.B. Algebra might have been done in (i)
			$\Rightarrow 6x^2-42x+30=0$ $\Rightarrow x^2-7x+5=0$	A1	**www** At least one interim step must be seen. N.B. Answer given	
				[4]		
	(iii)		$x^2-7x+5=0$			
			$\Rightarrow x=\frac{7\pm\sqrt{49-20}}{2}=\frac{7\pm\sqrt{29}}{2}$ **or** 6.19 and 0.807	M1 A1	Use of correct formula with given equation	Condone 0.81 or 0.8 but not 6.2
			(Paul's speed) = 6.19 km hr $^{-1}$	A1	+ units but only if *their* 0.807 is discarded	N.B. If 3 marks not awarded then look for the S.C.
				[3]		
			S.C (Paul's speed) is 6.19 km hr $^{-1}$ B2 or x = 6.19 B1			
	(iv)		$\frac{15}{x-2}-\frac{15}{x}=\frac{15}{4.19}-\frac{15}{6.19}$	M1	Sub *their x* into correct expression Or sub *their x* into the two separate correct expressions for time and then subtract	Or the simplified expression Accept other way round giving a negative value
			Sight of 3.58 or 2.42 or 1.16	B1		
			69 or 70 mins or 1hr 9 mins or 1 hr 10 mins	A1 [3]	Answer correct in minutes	

Question			Answer	Marks	Guidance	
13	(i)		$100x + 120y \leq 1200$ **oe** $2x + 1.5y \leq 18$ **oe**	**M1** **A1** **A1** **[3]**	Attempting to use information to create an inequality Ignore extra inequalities	Seen by one LH side Condone use of <
	(ii)			**B1** **B1** **B1** **B1** **[4]**	One line (allow intercepts ±0.1) Other line (allow intercepts ±0.1) Shading one line – 1st quad only Shading other line – 1st quad only N.B. Shading below a line gets B0	N.B. Candidates may get inequalities wrong in (i) but get the shading correct in (ii) - this should be allowed. Allow ft for shading of wrong line but only if the gradient is -ve Allow ft for shading of wrong line but only if gradient is -ve and the two lines intersect in the 1st quadrant (not on axes)
	(iii)		$(P =)\ 3.5x + 3y$ e.g. (9, 0) gives 31.5, (0, 10) gives 30 (4, 6) gives 32, (5, 5) gives 32.5 (3, 7) gives 31.5 (6, 4) gives 33 (6, 4) gives 33 **S.C**. for last 4 marks. (6,4) gives 33 B2 Either 33 or (6,4) B1	**B1** **M1** **A1** **A1** **A1** **[5]**	test at least two integer points in correct feasible region in correct OF Both points correct. Ignore any others For (6, 4) chosen For 33	Ignore any equating to a number i.e. the point must be identified as the maximum.

Question			Answer	Marks	Guidance	
14	(i)		Probability remains constant Imperfection of mugs independent	**B1** **B1** **[2]**		Not "Random"
	(ii)		$p = \frac{1}{20}, q = \frac{19}{20}$ $P(0 \text{ or } 1) = \left(\frac{19}{20}\right)^{10} + 10\left(\frac{19}{20}\right)^{9}\left(\frac{1}{20}\right)$ $= 0.5987 + 0.3151$ $= 0.9139$ $P(\geq 2) = 1 - 0.9139$ $= 0.086$	**B1** **M1** **A1** **A1** **M1** **A1** **[6]**	**Soi** $q^{10} + kpq^9$ attempted, k an integer > 0 $p + q = 1$ $k = 10$ **soi** **soi** accept rounding to 3dp Dependent on previous M (Anything that rounds to 0.086)	For k Not $\binom{10}{1}$ or ${}^{10}C_1$
	(iii)		P(accepted) = P(0 imperfect) + P(1 imperfect) × P(0 imperfect) $\left(\frac{19}{20}\right)^{10} + 10\left(\frac{19}{20}\right)^{9}\left(\frac{1}{20}\right)\left(\frac{19}{20}\right)^{10}$ $= 0.5987 + 0.1887$ $= 0.787(4...)$	**M1** **A1** **A1** **A1** **[4]**	Correct plan **soi** by both correct terms Correct 1st term (as an expression) **soi** Correct 2nd term (as an expression including the 10) **soi** Accept 0.788	Words only sufficient **soi** by final answer Anything that rounds to 0.787 or 0.788
			Alternative: P(accept) = 1 – (Ans to (ii) + P(1)×P(at least 1)) = 1 – (ans to (ii) + P(1)×(1–P(0)) = 1 - (0.0861+ 0.3151 × 0.4013) = 1 - (0.0861+ 0.1264) =1 - 0.213 = 0.787(4...)	**M1** **A1** **A1** **A1**	Correct plan soi by both correct terms taken from 1 **Ft** Using *their* ans to (ii) 2nd term as an expression **soi** by final answer	Words only sufficient

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