OCR

Friday 6 June 2014 – Afternoon

FSMQ ADVANCED LEVEL

6993/01 Additional Mathematics

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 6993/01

Other materials required:
- Scientific or graphical calculator

Duration: 2 hours

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given correct to three significant figures where appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **100**.
- The Printed Answer Book consists of **20** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

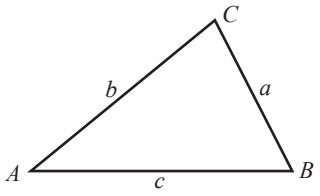
INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Formulae Sheet: 6993 Additional Mathematics

In any triangle *ABC*

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$



Binomial expansion

When n is a positive integer

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n$$

where

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Section A

1 Solve the following.

$$-6 < 2x - 1 < 7$$ **[3]**

2 The gradient function of a curve that passes through the point (1, 2) is given by

$$\frac{dy}{dx} = 3x^2 - 4x + 7.$$

Find the equation of the curve. **[4]**

3 **(i)** Find the area enclosed between the curve $y = 8x^3$, the x-axis and the line $x = 2$. **[3]**

(ii) Hence, or otherwise, deduce the area between the x-axis, the y-axis, the line $x = 2$ and the curve $y = 8x^3 + 5$. **[1]**

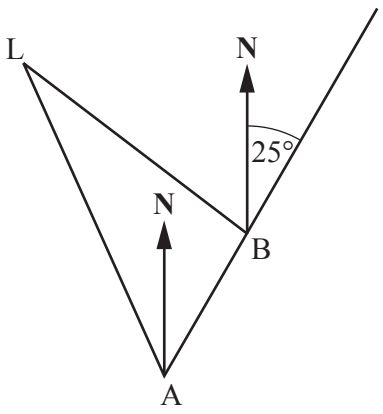
4 A train travels from station A to station B. It starts from rest at A and comes to rest again at B. The displacement of the train from A at time t seconds after starting from A is s metres where

$$s = 0.09t^2 - 0.0001t^3.$$

(i) Find the velocity at time t seconds after leaving A and hence find the time taken to reach B. Give the units of your answer. **[4]**

(ii) Find the distance between A and B. Give the units of your answer. **[2]**

5 A ship is moving on a bearing of 025º at 14 knots (1 knot = 1 nautical mile per hour). As it passes point A, a lighthouse L is seen on a bearing of 340º. After 30 minutes, the ship passes point B from where the lighthouse is seen on a bearing of 320º.



Not to scale

(i) Find the angle BAL and the angle ALB. **[3]**

(ii) Hence, or otherwise, calculate the distance BL in nautical miles. **[3]**

6 The function $f(x) = x^3 - 4x^2 + ax + b$ is such that

- $x = 3$ is a root of the equation $f(x) = 0$,
- when $f(x)$ is divided by $(x - 1)$ there is a remainder of 4.

(i) Find the value of a and the value of b. **[4]**

(ii) Solve the equation $f(x) = 0$. **[3]**

7 The points A and B have coordinates (3, 7) and (5, 11) respectively.

(i) Find the exact length of AB. **[2]**

(ii) Find the equation of the circle with diameter AB. **[3]**

8 Four points have coordinates A(−5, −1), B(0, 4), C(7, 3) and D(2, −2).

(i) Using gradients of lines, prove that ABCD is a parallelogram. **[2]**

(ii) Using lengths of lines, prove further that ABCD is a rhombus. **[2]**

(iii) Prove that ABCD is not a square. **[2]**

9 **(i)** Show that $\frac{1-\cos^2 x}{1-\sin^2 x} = \tan^2 x$. **[1]**

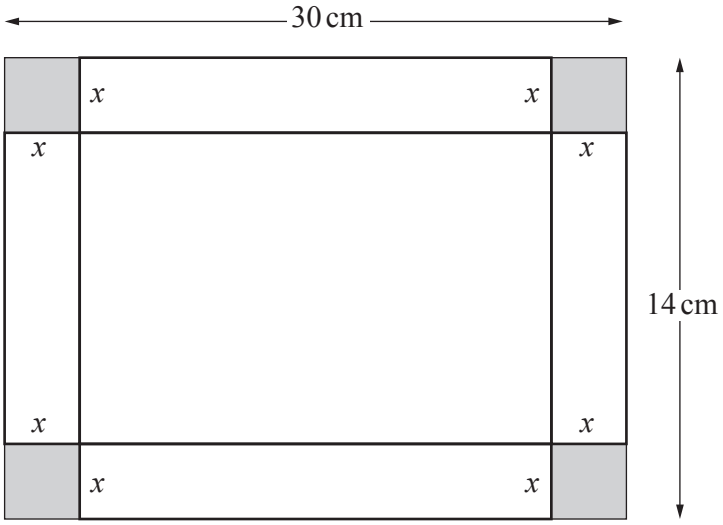
(ii) Hence solve the equation $\frac{1-\cos^2 x}{1-\sin^2 x} = 3 - 2\tan x$ for values of x in the range $0° \leqslant x \leqslant 180°$. **[4]**

10 **(i)** Find the coordinates of the point P on the curve $y = 2x^2 + x - 5$ where the gradient of the curve is 5. **[3]**

(ii) Find the equation of the normal to the curve at the point P. **[3]**

Section B

11 Kala is making an open box out of a rectangular piece of card measuring 30 cm by 14 cm. She cuts squares of side x cm out of each corner and turns up the sides to form the box.



(i) Find an expression in terms of x for the volume, $V\text{cm}^3$, of the box and show that this reduces to

$$V = 4x^3 - 88x^2 + 420x.$$ **[4]**

(ii) Find the two values of x that give $\frac{\mathrm{d}V}{\mathrm{d}x} = 0$. **[5]**

(iii) Explain why one of these values should be rejected and find the maximum volume of the box using the other value. **[3]**

12 Paul walked from Anytown to Nexttown, a distance of 15 km. When he got there he then walked back. His average speed on the return journey was 2 km per hour less than on the outward journey.

Let Paul's average speed on the outward journey be x km hr^{-1}.

(i) Write down an expression for the time, in hours, taken for the whole journey. **[2]**

The time taken by Paul for the whole journey was 6 hours.

(ii) Use your expression in **(i)** to form an equation in x and show that it simplifies to

$$x^2 - 7x + 5 = 0.$$ **[4]**

(iii) Solve this equation to find Paul's average speed on the outward journey. **[3]**

(iv) Find the difference in time between the outward and return journeys. Give your answer to the nearest minute. **[3]**

13 A company needs to buy some storage units. There are two types of unit available, type X and type Y. The cost of each type of unit, the floor space required and the volume for storage are given in the following table.

	Cost per unit (£)	**Floor space required (m^2)**	**Volume for storage (m^3)**
X	100	2	3.5
Y	120	1.5	3

The maximum cost allowed for the purchase of the units is £1200 and the maximum floor space available is $18\,m^2$.
The company wants to maximise the volume for storage.

Let x and y be the number of each type of unit, X and Y, respectively.

(i) Write down an inequality for the total cost and an inequality for the total floor space required. **[3]**

(ii) Draw the inequalities you gave in **(i)** on the grid provided in the answer book. Given that $x \geqslant 0$ and $y \geqslant 0$, indicate the region for which the inequalities hold by shading the area that is **not** required. **[4]**

(iii) Write down the objective function for the volume for storage and find the combination of units that should be bought to maximise the volume for storage.
Write down this maximum volume. **[5]**

14 Mugs are packed in boxes of 10. On average, 5% of the mugs are imperfect. A box of mugs is classified as "unsatisfactory" if it contains two or more imperfect mugs.

(i) State two conditions that must be satisfied for the number of imperfect mugs in a box to have a binomial distribution. **[2]**

(ii) Assuming that these two conditions are satisfied, calculate the probability that a box chosen at random is "unsatisfactory". **[6]**

A shop receives a delivery of a large number of boxes of mugs. The delivery is checked as follows.

A box is chosen at random.
- If there are no imperfect mugs in the box then the whole delivery is accepted.
- If the box is "unsatisfactory" then the whole delivery is rejected.
- If there is exactly one imperfect mug in the box then a second box is chosen at random. The delivery is accepted only if this box contains no imperfect mugs.

(iii) Calculate the probability that the delivery is accepted. **[4]**

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