FSMQ

Additional Mathematics

Unit **6993:** Additional Mathematics

Free Standing Mathematics Qualification

Mark Scheme for June 2015

Annotations and abbreviations

Annotation in scoris	Meaning
BP	Blank Page – this annotation **must** be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
	Method mark dependent on a previous mark, indicated by "Dep on 1st M"
cao	Correct answer only
oe	Or equivalent
soi	Seen or implied
www	Without wrong working

1. **Marking Instructions**

   a   Annotations should be used whenever appropriate during your marking.

   The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

   For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

   b   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

   Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

   c   The following types of marks are available.

   M
   A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

   A
   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

   B
   Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d   When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e   The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f   Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g   Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h   For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

        Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Section A

Question	Answer	Marks	Guidance
1	Line is $3x - 2y = k$ Satisfied by (3, 4) ($\Rightarrow k = 1$) $\Rightarrow 3x - 2y = 1$ 	**B1** **M1** **A1** **3**	oe Substitution soi oe isw Only 3 terms 	
	Alternatively: $g = -\dfrac{2}{3} \Rightarrow$ new $g = \dfrac{3}{2}$ $\Rightarrow (y - 4) = \dfrac{3}{2}(x - 3)$ $\Rightarrow 2y = 3x - 1$	**B1** **M1** **A1**	Soi from equation *Their* normal gradient and (3,4) used oe isw Only 3 terms	Condone $g = \dfrac{3}{2}x$ Only allow if correct or if their g and the negative reciprocal is seen. i.e. only one constant term
	Alternatively: $g = -\dfrac{2}{3} \Rightarrow$ new $g = \dfrac{3}{2}$ $\Rightarrow y = \dfrac{3}{2}x + c$ $\Rightarrow y = \dfrac{3}{2}x - \dfrac{1}{2}$	**B1** **M1** **A1**	Soi from equation *Their* normal gradient and (3,4) substituted oe isw Only 3 terms	Condone $g = \dfrac{3}{2}x$ Only allow if correct or if their g and the negative reciprocal is seen. e.g. $y = \dfrac{3x - 1}{2}$

Question			Answer	Marks	Guidance	
2	**(i)**		$\tan\alpha = -1.5 \Rightarrow \alpha = 123.69(.......)°$ or $123.7°$ or $124°$	**B2**	**B1** sight of $\pm 56.3°$ or $304°$ or $303.7°$ or $303.69°....$ or $123.6°$ -1 from full marks for extra values in range. Ignore values outside range.	Ignore lack of degree sign. No marks for answers in radians.
				2		
	(ii)		$11.5°$ or anything that rounds to $11.5°$ and $168°$ or anything that rounds to $168.5°$	**B1** **B1**	-1 from full marks for extra values in range. Ignore values outside range.	Ignore lack of degree sign. No marks for answers in radians.
				2		

Question			Answer	Marks	Guidance	
3			$\dfrac{dy}{dx} = 3x^2 + 3$ $\Rightarrow g = 15$ $\Rightarrow (y-9) = 15(x-2)$ $\Rightarrow y = 15x - 21$	**M1** **A1** **A1** **M1** **A1**	Diffn Both terms isw Gradient cao Correct form for line with *their* gradient and (2,9) used. Dep on 1st M mark Three terms only	Powers reduced by 1 in at least one x term. M0 for $x^2 + 3$ Ignore any $+c$ Alt: Use $y = mx + c$ with *their* g and substitute (2,9) NB No calculus, no marks
				5		

Question			Answer	Marks	Guidance	
4	**(i)**		$\int_{1}^{2}\left(x^2+2x+3\right)dx=\left[\dfrac{x^3}{3}+x^2+3x\right]_{1}^{2}$	**M1** **A1**	Int All three terms (ignore c)	Powers increased by 1 in at least one term M0 for x^3+2x^2+3x
			$=\left(\dfrac{8}{3}+4+6\right)-\left(\dfrac{1}{3}+1+3\right)$ $=12\dfrac{2}{3}-4\dfrac{1}{3}=8\dfrac{1}{3}$ **oe**	**M1** **A1**	Apply limits and subtract in correct order. soi Dep on 1st M www	Condone lack of brackets. $-8\dfrac{1}{3}$ is by implication M0.
				4		
	(ii)		Area between $y=x^2+2x+3$, $x=1$, $x=2$ and the x axis. (Or "under" or "below" with no mention of x axis)	**B1**	Allow sketch with area shaded and $x=1$ and $x=2$ clearly seen. Curve in 1st quadrant and right way up.	May write "curve" (but not "line") instead of equation. Do not allow anything that refers to points (1,0), (2,0) and not lines.
				1		

Question			Answer	Marks	Guidance	
5	**(i)**		$s=\dfrac{3}{2}t^2-2t+3 \Rightarrow \left(v=\dfrac{ds}{dt}=\right)3t-2$ $\Rightarrow a$ or $\dfrac{dv}{dt}$ or $\dfrac{d^2s}{dt^2}=3$	**M1** **B1** **A1**	Diffn twice For sight of $3t-2$ www correctly defined or in words	Powers decreased by 1 in at least one term. M0 for $\dfrac{3}{2}t-2$
				3		
	(ii)		$v=3\times5-2=13$ Velocity = 13 (m s^{-1})	**M1** **A1**	Substitute into *their* v Ignore units	Note that use of SUVAT formulae will give different answers. Use of $v=u+at$ to give 15 M1 A1 You may also see 12.2, 15, 13.6, $\sqrt{183}\approx13.5(....)$
				2		

Question			Answer	Marks	Guidance	
6	**(i)**		$(n-1)+n+(n+1)=99$ or $3n=99$ $\Rightarrow n=33$ 	**B1** **B1** **2**	Must be seen isw Isw 32,33,34 unidentified is B0 $(n)=33$ without the eqn is B1	
	(ii)		$(n-1)(n+1)+5n=203$ $\Rightarrow n^2+5n-204\,(=0)$ $\Rightarrow (n+17)(n-12)=0$ $\Rightarrow n=12$ 	**B1** **B1** **M1** **A1** **4**	 Correct 3 term quadratic either as expression or equation implies first B1 To give $(n\pm12)(n\pm17)$ soi $n=-17$ must be rejected (by ignoring it is acceptable) isw	 e.g. $n^2+5n=204$ Or correct sub from correct quadratic in correct formula Or sight of 6.25 (soi) in completing the square to give $(n\pm2.5)^2=204+6.25$ If no equation seen then $n=12$ is M1 A1, $n=12$ and -17 is M1 A0 For last two marks: If trial on correct equation gives $n=12$ then M1 A1

	Question		Answer	Marks	Guidance	
7	(i)		$\Rightarrow x+7=3+5x-x^2$	M1	Substitute, eliminating x or y.	Or $y^2-18y+81=0$
			$\Rightarrow x^2-4x+4=0$ oe	A1	3 term quadratic.	
			$\Rightarrow x=2,$	A1	$x($ or $y)$	
			$y=9$	A1	Substitute and find y (or x).	
				4		
	(ii)		Line is tangent to curve (at (2, 9))	B1	Allow "touches".	Or a sketch with any parabola touched by any line
				1		

	Question		Answer	Marks	Guidance	
8	(i)		$f(-3)=(-3)^3-3a+6=0$	M1	Sub -3 and equating to 0. Allow $27-3a+6=0$	Or equivalent by long division and equating remainder to 0. Or Use $(x+3)(x^2+px+q)=f(x)$ and equate coefficients
			$\Rightarrow 3a=-21$			
			$\Rightarrow a=-7$	A1		
				2		
	(ii)		x^2-3x+2 or x^2+x-6 or x^2+2x-3	B1	Quadratic	Alternative method by factor theorem
			$(x+3)(x-2)(x-1)$	B1	$(x\pm2)$	Any trial except f(-3) B1
				B1	$(x\pm1)$	Obtain f(1) = 0 B1
						Obtain f(2) = 0 B1
			$x=1, 2, -3$	B1	Ans only B4	Ans B1
					NB. Quadratic can be recovered even if cubic is wrong.	
				4		

Question			Answer	Marks	Guidance	
9	**(i)**		$(x-4)^2$	**B1**	Sight of $(x\pm4)^2$	
			$(y+1)^2$	**B1**	Sight of $(y\pm1)^2$	
			36	**B1**	Sight of -36 on lhs or 36 on rhs	
			$\Rightarrow (x-4)^2+(y+1)^2=6^2$	**B1**	Allow 36. isw	
				4		
	(ii)		Distance from *their* centre to (8, 3)	**M1**	By pythagoras	Or sub into either form of the circle.
			$=\sqrt{(8-4)^2+(3+1)^2}=\sqrt{16+16}=\sqrt{32}$			
			and $\sqrt{32}<6$ or $32<36$ so inside	**A1**	Reason must be given	<0 (or <36) therefore inside
			NB. Their centre is either stated or assumed to be their $(4, -1)$ in (i)			
				2		

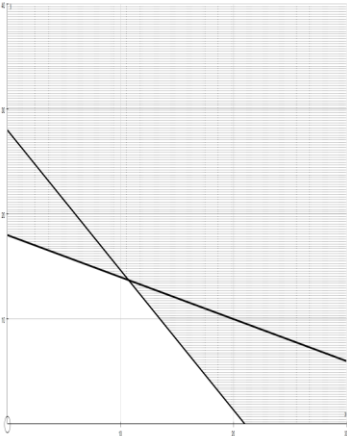| Question			Answer	Marks	Guidance	
10	**(i)**		$\cos\theta = \dfrac{10^2 + 8^2 - 7^2}{2 \times 8 \times 10}$ $= 0.71875$ $\Rightarrow \theta = 44^\circ$ or $44.0...^\circ$ or 44.1°	**M1** **A1** **A1** **3**	Correct substitution into correct formula to give correct angle. Accept 3sf Soi by answer	Could be done in 2 stages
	(ii)		$7^2 = x^2 + 8^2 - 2 \times x \times 8 \times \cos 30$ $= x^2 + 64 - 13.86x$ $\Rightarrow x^2 - 13.86x + 15 = 0$ $x = 12.7$ cm	**M1** **A1** **M1** **A1** **4**	Correct substitution into correct formula Oe. Accept 13.9 or better Solve *their* quadratic with correct substitutions into correct formula. Dep on 1st M1	
			Alternatively: Sin rule to find one angle D = 34.8(5)°, C = 115.1(5)° Find other angle and use sin rule or cosine rule	**M1** **A1** **M1** **A1**	Correct substitution into correct formula Correct substitution into correct formula	

Section B

	Question		Answer	Marks	Guidance	
11	**(i)**		Substitute A(1, 4) into both equations Substitute B(4, 7) into both equations NB. Answer given so working must be seen	**B1** **B1** **2**		Alternatively by solving: Forming the correct three term quadratic by equating B1 e.g. $2x^2 - 10x = -8$ Both pairs of coordinates found B1
	(ii)		$PQ = \left(6x - x^2 - 1\right) - \left(x^2 - 4x + 7\right)$ $= 10x - 2x^2 - 8$ 	**M1** **A1** **2**	Allow QP. Allow no brackets Cao isw	
	(iii)		$\dfrac{d PQ}{dx} = 10 - 4x$ $= 0$ when $x = 2.5$ $\Rightarrow PQ = 4.5$	**M1** **A1** **M1** **A1** **4**	In *their* quadratic PQ at least one power decreased by 1. Allow $x = 2.5$ from diffn of QP and PQ/2 and QP/2 Substitute *their* x into *their* quadratic. Dep on first M mark Cao www (NB only from $10 - 4x$)	NB Non calculus methods get M0 Functions can be diffn separately and equated for marks Look out for the abandonment of the $-$ve sign from use of QP
	(iv)		$A = \displaystyle\int_{1}^{4}\left(10x - 2x^2 - 8\right)dx$ $= \left[5x^2 - 2\dfrac{x^3}{3} - 8x\right]_{1}^{4}$ $= 5\dfrac{1}{3} - \left(-3\dfrac{2}{3}\right) = 9$	**M1** **A1** **M1** **A1** **4**	Int of *their* PQ; at least one power increased by 1. Correct integrand. From correct PQ only. Ignore *c* and limits. Apply limits and subtract in correct order. soi Dep on 1st M (need not be correct integrand) www	Alternatively: Finding the area under each curve Integrate each function M1 Correct integrands A1 Apply limits and subtract in correct order and then subtract answers in correct order M1 Ans $21 - 12 = 9$ A1 www

	Question		Answer	Marks	Guidance	
12	(a)	(i)	$\binom{10}{4}\left(\dfrac{3}{4}\right)^4\left(\dfrac{1}{4}\right)^6 = 210\left(\dfrac{3}{4}\right)^4\left(\dfrac{1}{4}\right)^6$ $= 0.0162$	**M1** **A1** **A1** **A1** **4**	$0.75^n \times 0.25^{10-n}$ seen, $n \neq 0$ or 10 Coefficient evaluated soi by answer Powers soi by answer SC. Use of $^1/_3$, $^2/_3$ in correct order gives 0.0569 B2	Answers throughout to 3sf or better. Allow % probabilities NB 0.75 and 0.25 interchanged is max 2/4 (gives ans = 0.146)
	(a)	(ii)	$P(0 \text{ tulip bulbs}) = \left(\dfrac{3}{4}\right)^{10} (= 0.0563)$ $P(\text{at least 1 tulip bulb}) = 1 - \left(\dfrac{3}{4}\right)^{10} = 1 - 0.0563$ $= 0.944$	**B1** **M1** **A1** **3**	For $\left(\dfrac{3}{4}\right)^{10}$ seen (possibly amongst other terms) For $1 - p^{10}$ used. p can also be $^1/_4$, $^2/_3$	Alternatively: Add 10 terms M1 Correct powers and coefficients soi A1 Ans A1
	(b)		$P(3 \text{ tulips in } 20) = \binom{20}{3}\left(\dfrac{3}{4}\right)^{17}\left(\dfrac{1}{4}\right)^3 = 1140\left(\dfrac{3}{4}\right)^{17}\left(\dfrac{1}{4}\right)^3$ $= 0.134$	**B1** **M1** **A1** **A1** **A1** **5**	Sight of 20 anywhere.soi $0.75^n \times 0.25^{20-n}$ seen, $n \neq 0$ or 20 Coefficient evaluated soi by answer Powers soi by answer	
			Alternatively: 4 terms or 2 terms 0 in one and 3 in the other + 1 in one and 2 in the other 0.0141 ($\times 2 = 0.0282$) 0.0529 ($\times 2 = 0.1057$) 0.134	**B1** **M1** **A1** **A1** **A1**	Sight of 0,3 and1,2 One of these pairs multiplied, each term of the form $0.75^n \times 0.25^{10-n}$ rounds to 0.014 soi rounds to 0.053 soi	So e.g. 0.067 from 2 terms is 4/5

Question			Answer	Marks	Guidance	
13	(i)		OA = 0.5 Pythagoras $\left(OV^2 \right) = 2.4^2 - their\, OA^2$ $\Rightarrow OV = 2.35\,m$ 	B1 M1 A1 3	Or any line from centre to corner of base Do not accept $\sqrt{5.51}$ for final answer. Accept better than 3sf	
	(ii)		$\cos^{-1} \dfrac{their\, OA}{2.4}$ $= 78.(0)°$	M1 A1 2	OR $\sin^{-1} \dfrac{their\, OV}{2.4} = 78.0°$ OR $\tan^{-1} \dfrac{their\, OV}{their\, OA} = 78.0°$ Accept 77.9° – 78.3°	
	(iii)		Angle is VMO where M is the midpoint of AB. $OM^2 = their\, OA^2 - 0.25^2$ $\Rightarrow OM = 0.433$ or $VM = 2.39$ oe Angle VMO $= \tan^{-1} \dfrac{their\, OV}{their\, OM} = 79.5°$	 M1 A1 M1 A1 4	 OM or VM Using triangle VMO Accept 79.5° – 79.6°	
	(iv)		$\dfrac{1}{their\ OV}$ up $\dfrac{their\ OV - 1}{their\ OV} = \dfrac{1.35}{2.35}\left(= \dfrac{27}{47} = 0.575 \right)$ $\times 3 = 1.72$	M1 A1 A1 3	Attempt by correct ratio Ratio using slant edges $\dfrac{2.04 - 1.02}{2.4}$	NB. 1.4/2.4 is incorrect and gives 1.75 so M0 Alternatively: Use of trigonometry in truncated hexagon (ht 1.35) M1 gives length of base 0.287 A1 $\times 6 = 1.72$ A1 Beware lots of alternatives!

Question			Answer	Marks	Guidance	
14	(i)		x is the number of bottles of X (produced), y is the number of bottles of Y (produced). (The constraint) models the quantity of A	**B1** **B1** **2**	Both	
	(ii)		$2x + 5y \leq 90$	**B1** **1**	Do not accept $<$	
	(iii)			**B1** **B1** **B1** **3**	One line (0, 28) to (21,0) Other line (0,18) to (30,6) Shading ft for two lines with negative gradients and which intersect in 1st quadrant.	Allow up to one small square out at each edge of grid.
	(iv)		(12, 12) or 12 (of each)	**B2** **2**	Give B1 for attempt to find by drawing line $y = x$ or testing (n,n) or an answer of 24.	Line $y = x$ can be seen on previous graph
	(v)		Maximum is 24 (10, 14), (11, 13), (12, 12)	**B1** **B3** **4**	B1 for each. If all 3 given then − 1 for each extra one.(Ignore same point given twice)	Condone for e.g. 10X + 14Y

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