OCR
Oxford Cambridge and RSA

FSMQ

Additional Mathematics

Unit **6993:** Additional Mathematics

Free Standing Mathematics Qualification

Mark Scheme for June 2017

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	**Meaning**
BP	Blank Page – this annotation **must** be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
	Method mark dependent on a previous mark, indicated by "Dep on 1st M"
cao	Correct answer only
oe	Or equivalent
soi	Seen or implied
www	Without wrong working

1. **Marking Instructions**

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Section A

Question			Answer	Marks	Guidance	
1			$-2 < 3x+1 < 7$ $\Rightarrow -3 < 3x < 6$ or $-\frac{2}{3} < x+\frac{1}{3} < \frac{7}{3}$ $\Rightarrow -1 < x < 2$ Ignore lists of integers	**M1** **M1** **A1**	Subtract 1 or divide by 3 correctly on at least one side Correct second step on at least one side Both inequalities together (allow two inequalities with "and" but not "," or "or")	In either order: candidates may work the two sides separately for the M marks (but must deal with both sides). SC1 work with **only** one side to get $x < 2$ or $x > -1$
				3		

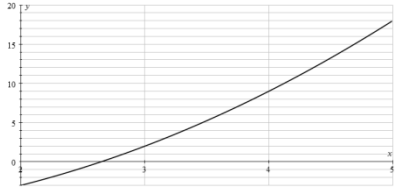
Question			Answer	Marks	Guidance	
2			Line is $\pm 3x \pm 2y = k$ $3x - 2y = k$ Satisfies $(3, -1)$ $\Rightarrow k = 9 + 2 = 11$ giving $3x - 2y = 11$ **oe**	**M1** **A1** **M1** **A1**	Swapping coefficients Correct form Substituting into *their* equation Final equation three terms only must be seen	Alt: gradient of line $= -\frac{2}{3}$ **B1** soi accept $-\frac{2}{3}x$ $\Rightarrow$ grad of perp $= \frac{3}{2}$ **M1** for finding numerical perp $\Rightarrow y = their\ \frac{3}{2}x + c$ **M1** substituting $(3, -1)$ that is not parallel to the original line $\Rightarrow y = \frac{3}{2}x - 5.5$ **oe** **A1** i.e. writing "c = – 5.5" only loses last A mark
				4		

Question			Answer	Marks	Guidance	
3			$y = x^2 - 3x \Rightarrow \frac{dy}{dx} = 2x - 3$	M1	Diffn means both powers reduced by 1	But beware division by *x*
			At (4, 4), $\frac{dy}{dx} = 5$	A1	For 5	
			$\Rightarrow y - 4 = \textit{their}\, 5(x - 4)$ **oe**	DM1	Using *their* gradient and (4, 4) to find line	*Their* result of substituting 4 into $\frac{dy}{dx}$
			$\Rightarrow y = 5x - 16$	A1	Final equation 3 terms only must be seen	
				4		

Question			Answer	Marks	Guidance	
4	(i)		$AB = \sqrt{(1--3)^2 + (5-7)^2} \left(= \sqrt{16+4}\right)$	M1	Applying Pythagoras correctly	
			$\Rightarrow AB = \sqrt{20} \quad \left(= 2\sqrt{5}\right)$	A1		
			(isw for any decimal given)			
				2		
	(ii)		$(-1, 6)$	B1		
				1		

Question			Answer	Marks	Guidance	
5	(i)		$x^2 + y^2 = r^2$ $r^2 = 50$	B1 B1	Eqn of circle centre (0,0)	Accept $(x-0)^2+(y-0)^2$ Accept $\left(5\sqrt{2}\right)^2$ (Brackets must be included) or $\sqrt{50}^2$
				2		
	(ii)		Substitute $y = 15-2x$ $\Rightarrow x^2+(15-2x)^2 = 50$ $\Rightarrow 5x^2-60x+175=0$ **oe** $\Rightarrow x^2-12x+35=0$ $\Rightarrow (x-7)(x-5)=0$ $\Rightarrow x = 5, 7$ $y = 5, 1$ $\Rightarrow (5,5)$ and $(7,1)$	M1 A1 M1 A1	Substitute into *their* quadratic from (i) 3 term quadratic Solve 3 term quadratic by factorisation that will give 2 out of 3 terms correct when expanded	Or use correct formula with at most one error Or complete square to give $(x-6)^2 = k$ SC2 for correct answers www (must be 2 and only 2 points) and no algebra (i.e. no working or trial and error)
				4		

Question			Answer	Marks	Guidance	
6	(i)		$f(-3)=0$ $\Rightarrow (x=)-3$ (is the root)	M1 A1	Use of remainder theorem soi Root must be stated Answer only is M1 A1	If A0, trials on negative numbers must be seen at least twice for M1. NB. (x + 3) is a factor gets M1 A0
				2		
	(ii)		$x^2-4x+2=0 \Rightarrow x=\frac{4\pm\sqrt{16-8}}{2}$ $f(x)=(x+3)(x^2-4x+2)$ $\Rightarrow x=2\pm\sqrt{2}\,(=3.41 \text{ and } 0.586)$ and $x=-3$	M1 A1 DM1 A1	Find quadratic using (x + 3) Use correct formula with at most one error to solve *their* quadratic All 3; can be left in surd form or 3sf or 2dp or better.	Observation or long division by (x + 3) obtaining at least 2 terms correct Or complete square to give $(x-2)^2=k$ SC: All 3 roots correct with no working and no quadratic seen B2 If only irrational roots B1
				4		

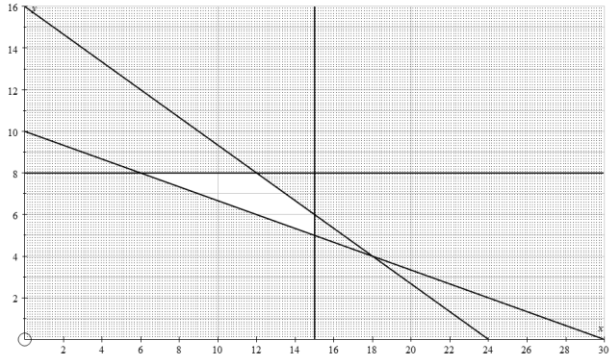
Question			Answer	Marks	Guidance	
7	(i)		$\int_3^5 \left(x^2 - 7\right) \mathrm{d}x$ $= \left[\frac{x^3}{3} - 7x\right]_3^5$ $= \left(\frac{125}{3} - 35\right) - \left(9 - 21\right)$ $= 18\frac{2}{3}$	**M1** **A1** **DM1** **A1**	Integrate – powers increased by 1 in both terms. Beware multiplication by x. Ignore limits. Both terms Use correct limits in correct order Accept $\frac{56}{3}$ or anything that rounds to 18.7 www	MR – lower limit of 2 gives 18. This would get 3/4. SC2 for answer only www Two values obtained must be subtracted
				4		
	(ii)		 Because the curve cuts the *x*-axis and so part of the area will be negative	**B1** **B1**	Curve having positive gradient and cutting the axis between *x* = 2 and 5 and nearer to *x* = 2 Comment must mention negative area or difference or "cancel out"	NB The curve does not need to go up to *x* = 5 because of the size of *y*
				2		

Question			Answer	Marks	Guidance	
8			$P(0)=\left(\frac{5}{6}\right)^4 \quad (=0.4823)$ $P(1)=4\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) \quad (=0.3858)$ P(at least two) = 1 –(0.4823 + 0.3858) = 0.1319	**B1** **M1** **A1** **A1** **M1** **A1**	P(0) soi. P(1) with correct powers Coefficient soi 1 – sum of *their* two terms Accept answer www that rounds to 0.132	If evaluated, accept 3dp or better. If evaluated, accept 3dp or better. $=\frac{19}{144}$
				6		
			Alternative: Obtain last three terms $P(2)+P(3)+P(4)$ $=6\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^2+4\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)^3+\left(\frac{1}{6}\right)^4$ $=0.1157+0.01543+0.00077$ $=0.1319$		B1 P(4) M1 Remaining two terms with correct powers A1 At least one coefficient (4 or 6) soi A1 One term soi M1 adding *their* three terms A1 answer Accept answer www that rounds to 0.132	SC P(exactly 2) M1 correct powers A1 coefficient soi A1 0.1157 - final answer accept 3sf or better. Mark to candidate's advantage if there is a mixture of methods

Question			Answer	Marks	Guidance	
9	(i)		$v = \frac{7}{128}\left(12t^2 - t^3\right)$ $\Rightarrow a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{7}{128}\left(24t - 3t^2\right)$ When $t = 0,\ 24t - 3t^2 = 0$ When $t = 8,\ 24t - 3t^2 = 0$	**M1** **A1** **A1**	Differentiate once only to obtain acceleration Both terms including 7/128 Condone lack of 7/128 in these calculations	$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{21t}{16} - \frac{21t^2}{128}$ Alternative: $24t - 3t^2 = 0$ gives $t = 0$ and 8
				3		
	(ii)		$v = \frac{7}{128}\left(12t^2 - t^3\right) \Rightarrow s = \int v\mathrm{d}t = \frac{7}{128}\left(4t^3 - \frac{t^4}{4}\right)$ When $t = 8,\ s = \frac{7}{128}\left(4\times 8^3 - 2\times 8^3\right) = \frac{14\times 8^3}{128} = 56$	**M1** **A1** **DM1** **A1**	Integrate 2 terms Both terms Use limits or substitute t = 8	Constant dropped M1 A0 Constant changed (e.g. 7^2) M0 $\frac{7t}{128}\left(4t^3 - \frac{t^4}{4}\right)$ gets M0 This may be indicated by an answer of 448. NB. 56 is gained from using suvat formulae because of symmetry. This gets 0.
				4		

Question			Answer	Marks	Guidance	
10	(i)		$\cos \text{ADC} = \dfrac{\left(\frac{a}{2}\right)^2 + d^2 - b^2}{2\left(\frac{a}{2}\right)d}$ **oe isw**	**B1**		
				1		
	(ii)		$\cos \text{ADB} = \dfrac{\left(\frac{a}{2}\right)^2 + d^2 - c^2}{2\left(\frac{a}{2}\right)d}$ **oe isw**	**B1**		
				1		
	(iii)		$\text{ADC} = 180 - \text{ADB} \Rightarrow \cos \text{ADC} = -\cos \text{ADB}$ $\Rightarrow \dfrac{a^2 + 4d^2 - 4b^2}{4ad} = -\dfrac{a^2 + 4d^2 - 4c^2}{4ad}$ **oe** $\Rightarrow 8d^2 = 4b^2 + 4c^2 - 2a^2$ $\Rightarrow d^2 = \dfrac{2b^2 + 2c^2 - a^2}{4}$	**B1** **M1** **A1**	Soi AG	Can't get last mark if (i) or (ii) wrong
				3		
	(iv)		$b = 7$, $c = 9$, $a = 10$ $\Rightarrow d^2 = \dfrac{2b^2 + 2c^2 - a^2}{4} = \dfrac{2\times 7^2 + 2\times 9^2 - 10^2}{4}$ $= 40$ $\Rightarrow d = \sqrt{40} \quad \left(= 2\sqrt{10}\right)$ Isw any decimal	**M1** **A1**	Substituting 3 values into correct formula	Allow any method that ignores (iii) but uses correctly the cosine formula twice
				2		

Question			Answer	Marks	Guidance	
11	(i)		Substitute $x = 0$, $y = 24$ gives $c = 24$	B1		
				1		
	(ii)		Substitute (2, 34) and (4, 32) $\Rightarrow 34 = -8 + 4a + 2b + 24 \Rightarrow 2a + b = 9$ **oe** (3 terms) and $\Rightarrow 32 = -64 + 16a + 4b + 24 \Rightarrow 4a + b = 18$ **oe** (3 terms) $\Rightarrow 2a = 9 \Rightarrow a = \frac{9}{2}$, $b = 0$	M1 A1 A1 M1 A1	Substituting both Solve simultaneously both	SC M1 Substitute $x = 2$ and $x = 4$ into given formula A1 obtain $y = 34$ A1 obtain $y = 32$
				5		
	(iii)		$y = -x^3 + \frac{9}{2}x^2 + 24 \Rightarrow \frac{dy}{dx} = -3x^2 + 9x$ $= 0$ when $3x^2 = 9x$ $\Rightarrow x = 3$ $\Rightarrow y = 37.5$ $x = 0$ gives minimum so discard	M1 A1 DM1 A1 A1 A1	Diffn Set = 0 Discard $x = 0$ (must be stated)	 Alt: find $x = 3$ by trial Alternative: Determine that $x = 3$ gives maximum (i.e. 2nd derivative, values of gradient, values of y)
				6		

Question			Answer	Marks	Guidance	
12	(i)		$10x + 30y \geq 300$ **oe**	B1		Don't accept = or >
				1		
	(ii)		$x \leq 15, \quad y \leq 8$ **oe**	B1		Don't accept = or <
				1		
	(iii)		$100x + 150y \leq 2400$ **oe**	B1		Don't accept = or <
				1		
	(iv)			B1 B1 B1 B1 B1	One oblique line The other one Shading of one Shading of the other Horizontal and vertical lines plus shading	Allow intercepts on axes up to one small square out Allow these marks if wrong oblique lines have negative gradient. NB. It is possible to interchange the axes to get full marks, but 0 if axes are not labelled
				5		
	(v)		15 minibuses and 5 coaches i.e. 20 vehicles.	B1 B1	Allow (15, 5)	Do not allow $15x + 5y$
				2		
	(vi)		6 minibuses and 8 coaches £1800	B1 B1	Allow (6, 8)	Do not allow $6x + 8y$
				2		

Question			Answer	Marks	Guidance	
13	(a)	(i)	Time = $\frac{4}{5}+\frac{3}{2}=2.3$ hrs (2 hr 18 mins = 138 mins)	M1 A1	For sight of one of them correct (could see 48 and 90) isw	
				2		
		(ii)	Distance AC = 5 km $\Rightarrow$ Time = $\frac{5}{2}=2.5$ hrs (2 hr 30 mins = 150 mins)	B1 M1 A1		
				3		
	(b)	(i)	XC = $\sqrt{3^2+(4-x)^2}$ **oe** $\Rightarrow$ Time = $\frac{x}{5}+\frac{\sqrt{3^2+(4-x)^2}}{2}$ **oe**	M1 A1 M1 A1	Use of Pythagoras to find XC May be seen in expression for time *x*/5 + *their* XC/2. Can be a multiple of 60	Could be *x* or 4 – *x* Could be $\text{XC}=\sqrt{x^2-8x+25}$ Accept XC^2 SC2 Interchanging *x* and 4 – *x* throughout
				4		
		(ii)	*x* XC *t* 2.6 3.310589 2.175295 2.7 3.269557 2.174778 2.8 3.231099 2.175549 *t* decreasing from *x* = 0 to 2.7 and then seems to increase to *x* = 4	B2,1 DB1	Three times – 1 each error Justification i.e. *t* is smallest at 2.7 Dep on previous B marks	Needs more than 4sf. Accept 2.175.... etc SC B1 for correct 4sf NB. If worked in minutes (and hrs not seen) then 130.52, 130.49, 130.53 acceptable
				3		

Question			Answer	Marks	Guidance	
14	(i)		$\frac{h}{100} = \sin 28$ $\Rightarrow h = 46.95$	M1 A1	Correct trig ratio Accept 46.9, 47.0 (but not 47) or anything that rounds to 46.95	Accept correct sin rule
				2		
	(ii)		Find length BE in order to find XE: $\frac{200}{\text{BE}} = \sin 28 \Rightarrow \text{BE} = 426$ $\Rightarrow \text{XE} = 326$ $FX = \sqrt{1000^2 + their 326^2}$ $= 1051.8$	M1 A1 A1 M1 A1	Correct method to find BE BE XE Find FX by Pythagoras Accept 1052 or better or 1050	Be aware of valid alternative methods to find FX.
				5		
	(iii)		FY = 200 –their h FY= 153.(05) or XY = 1040.(6) $\text{Angle of slope} = \sin^{-1}\left(\frac{their\ \text{FY}}{their\ \text{FX}}\right)$ **oe** $= \sin^{-1}(0.1455) = 8.37$	M1 A1 M1 M1 A1	For FY or XY For correct ratio for angle YFX or YXF For identifying correct angle, possibly on diagram Accept anything that rounds to 8.4 Mark last answer	Correct ratios are: $\sin^{-1}\left(\frac{153}{1052}\right)$ or $\cos^{-1}\left(\frac{153}{1052}\right)$ $\sin^{-1}\left(\frac{1041}{1052}\right)$ or $\cos^{-1}\left(\frac{1041}{1052}\right)$ $\tan^{-1}\left(\frac{153}{1041}\right)$ or $\tan^{-1}\left(\frac{1041}{153}\right)$
				5		

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