

| ALGEBRA SECTION | Topic | Learning Outcomes | Precedences | Teaching points | Integral resources | Free resources |
|---|---|--|-------------|---|--|--|
| Chapter 1. Algebraic manipulation (2 hours) | Simplifying algebraic fractions | AL2: Simplify expressions involving algebraic fractions. | GCSE Maths | <p>First remind students of the principles involved when adding, subtracting, multiplying, dividing and simplifying fractions.</p> <p>Discuss why $\frac{6+8}{10}$ is the same as $\frac{3+4}{5}$ but $\frac{6 \times 8}{10}$ is not the same as $\frac{3 \times 4}{5}$</p> | Algebraic fractions1 | Algebraic Fractions 1 - Simplifying, Adding & Subtracting |
| | Simplifying expressions containing square roots | AL2: Simplify expressions involving square roots. | GCSE Maths | <p>Use $\sqrt{3^2 + 4^2} \neq 3 + 4$ to demonstrate that $\sqrt{a^2 + b^2}$ does not simplify to $a + b$.</p> <p>Use $\sqrt{3} \times \sqrt{3} = 3$ to explain that $\sqrt{x} \times \sqrt{x} = x$</p> <p>Explain that the square root sign gives the positive root only, e.g. $\sqrt{4} = 2$, but not -2. However, the solution to $x^2 = 4$ is $x = \pm 2$</p> | Expressions involving square roots | Lesson Element Using and manipulating Surds Relevance of Surds The Root of the Problem Surds inquiry Surd activities |

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| Chapter 2. Polynomials, functions and equations (3 hours) | Addition and subtraction of polynomials | AL3: Add and subtract polynomials. | GCSE Maths | Dealing with negative terms can often lead to errors. Students should feel comfortable rearranging polynomials, e.g. $2x^3 - 5x^2 + 4x - 7$ is the same as $4x - 5x^2 - 7 + 2x^3$ | Polynomial activities | Polynomials |
| | Multiplication of polynomials | AL3: Multiply polynomials. | GCSE Maths | Encourage students to adopt a systematic approach, otherwise terms may be omitted or duplicated. | | |
| | Division of polynomials | AL3: Divide polynomials. | GCSE Maths | When using long division, students should be encouraged to set their work out in columns. The most able students should be encouraged to divide by inspection. | Operations with polynomials | Polynomial Division & Equating Coefficients How to divide polynomials using long division |
| | The factor theorem | AL4: Find linear factors of a polynomial. | AL3 | If a function has not been defined in a question, then students should be discouraged from using function notation without first defining their function, e.g. Let $f(x) = \dots$ | The factor theorem | The Factor Theorem |
| | Completing the square | AL5: Complete the square of a quadratic polynomial. | GCSE Maths | Students should be able to deal with expressions with a coefficient of x^2 other than 1. The most able students will appreciate that c is the greatest (or least) value of an expression of the form $a(x + b)^2 + c$ | Geogebra activity Completing the square activity | Completing the Square |

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|---|---------------------------|--|-------------|---|---|--|
| Chapter 3. Applications of equations and inequalities in one variable (4 hours) | Applications of equations | AL6: Set up and solve problems leading to linear, quadratic and cubic equations in one unknown, and to simultaneous equations in two unknowns. | GCSE Maths | <p>Students should appreciate that division by zero is not possible.</p> <p>For example, when solving quadratic equations such as</p> $x^2 + 3x = 0$ <p>a common error is to divide both sides of the equation by x, and consequently lose a root.</p> | <p>Simultaneous equations</p> <p>Quadratic activities</p> | <p>Modelling a Test Drive</p> <p>Runaway Train</p> |
| | Inequalities | <p>AL7: Manipulate inequalities.</p> <p>AL8: Set up and solve linear and quadratic inequalities algebraically and graphically.</p> | GCSE Maths | <p>Students should be discouraged from replacing an inequality sign with an equals sign.</p> <p>A common error when solving inequalities is to fail to reverse the sign when multiplying or dividing by a negative value.</p> <p>Students should understand why the solution to $x(x - 3) > 0$ can be written as $x < 0$ or $x > 3$ but not $x < 0$ and $x > 3$</p> | <p>Linear & quadratic inequalities</p> <p>Inequality activities</p> | <p>Inequalities in Real Life</p> <p>How do you Solve A Quadratic Inequality Algebraically?</p> <p>Solving Quadratic Inequalities</p> |

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| Chapter 4. Sequences and recurrence relationships (1 hour) | Sequences and recurrence relationships | AL10: Understand and use notation of recurrence relationships to describe and determine sequences. AL11: Use recurrence relationships in modelling. | GCSE Maths | Students often fail to realise that a sequence such as $u_{n+1} = 3u_n - 2$ could also be written as $u_m = 3u_{m-1} - 2$ | Sequences and recurrence relationships Sequence activities | Fibonacci Surprises |

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|--|---|--|-------------|---|---|-----------------------------|
| Chapter 5. Points, lines and circles (2 hours) | The line joining two points | CG1: Calculate the distance between two points. CG2: Find the mid-point of a line segment. | GCSE Maths | Problems can sometimes occur when dealing with negative coordinates. Some students may benefit from first practising the addition and subtraction of directed numbers. | Geogebra: distance Geogebra: midpoint Geogebra: Distance1 Geogebra: Midpoint | Distance2 |
| | The coordinate geometry of circles | CG3: Know and use the equation of a circle $(x - a)^2 + (y - b)^2 = r^2$ where (a, b) is the centre and r is the radius of the circle. | AL5 | Students should practise converting between the two formats: $(x - a)^2 + (y - b)^2 = r^2$ and $x^2 + y^2 + fx + gy + h = 0$ Remembering the circle theorems learned at GCSE may help when solving circle problems. | Equation of a circle | Circles2 |
| Chapter 6. Graphs (2 hours) | Linear and polynomial functions | CG4: Sketch and plot linear and polynomial functions. | GCSE Maths | Students are often uncomfortable when first asked to sketch a graph rather than plot, as it requires a deeper understanding. | Graphing polynomial functions | Cubic |
| | Trigonometric and exponential functions | CG4: Sketch and plot trigonometric and exponential functions. | GCSE Maths | Students must learn the shapes of standard graphs and remember to include the axis intercepts. A polynomial of order n has up to $n - 1$ turning points. | Geogebra: Exponential graphs Walkthrough: Trig graphs | Exponential |

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| Chapter 7. Linear inequalities in two variables (3 hours) | Illustrating linear inequalities in two variables | AL9: Illustrate linear inequalities in two variables. | AL7 & AL8 | Students should be encouraged to use a ruler when sketching straight line graphs and when drawing axes. | Inequalities in 2D | Desmos graphing |
| | Using inequalities for problem solving | CG6: Express real situations in terms of linear inequalities. | AL9 | When shading, students should be careful not to hide crucial numbers and lines. | Linear programming1 | Region |
| | Linear programming | CG7: Use graphs of linear inequalities to solve 2-dimensional maximisation and minimisation problems. CG8: Know the definition of objective function and be able to find it in 2-dimensional cases. | CG6 | When setting up an inequality related to a real situation it can be useful to substitute numbers to check that the inequality sign is correctly oriented. It can be useful to label each line with its equation. | | |

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| Chapter 8. Trigonometric functions (4 hours) | Trigonometric functions for angles of any size | PT1: Use the definitions of $\sin \theta$, $\cos \theta$ and $\tan \theta$ for any angle and their graphs. | GCSE Maths | Students should first check that their calculators are in 'degrees' mode. | Trig functions1 Trig functions2 | Trigonometry |
| | The sine and cosine rules and proof | PT2: Know the sine and cosine rules and be able to apply them, including the ambiguous case for sine. | PT1 | An accurate construction can often help students who struggle to understand why two possible triangles are sometimes possible when finding an angle from the sine rule. | Geogebra: Sin & cos rules1 | |
| | Identities involving $\sin \theta$, $\cos \theta$ and $\tan \theta$ | PT3: Know and use the identity $\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$ PT4: Know and use the identity $\sin^2 \theta + \cos^2 \theta \equiv 1$ | PT1 | Students should understand the difference between $\sin^2 x$, $\sin x^2$, $\sin^{-1} x$, $(\sin x)^{-1}$ Use Pythagoras' theorem to demonstrate the identity $\sin^2 \theta + \cos^2 \theta = 1$ | Trig identities1 | |
| | Using trigonometrical identities to solve equations | PT5: Solve simple trigonometric equations in given intervals. | PT3 & PT4 | Using knowledge of the shape of the graphs is the best way to work with the sin, cos and tan of any angle. When solving equations of the form $\sin 2x = \alpha$ a common error is to divide $\sin^{-1} \alpha$ by 2 before finding the other angles. | Geogebra: Trig equations1 | |
| Chapter 9. Applications of trigonometry (2 hours) | Applications in modelling | PT6: Apply Pythagoras' Theorem and trigonometry to 2- and 3-dimensional problems. | PT2 | A sketch is useful when solving 2-D problems. | | |
| | Working in three dimensions | | | Students should be encouraged to draw each triangle as they solve it. If necessary, students could sketch each triangle in order to decide which one(s) to use. | | |

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| Chapter 10. Permutations and combinations (3 hours) | Probability diagrams | EN2: Construct and use tree diagrams, two-way tables, Venn Diagrams to enumerate outcomes. EN6: Solve problems about outcomes, including problems in the context of probability. | GCSE Maths | Students often forget that $P(A \cap B) = P(A) \times P(B)$ can only be used if A and B are independent, and that $P(A \cup B) = P(A) + P(B)$ can only be used if A and B are mutually exclusive. | Products, tree & Venn diagrams | Sets and Venn diagrams |
| | Factorials and the product rule | EN3: Use the product rule for counting numbers of outcomes of combined events. EN6: Solve problems about outcomes, including problems in the context of probability. | GCSE Maths | When solving such problems, it is sometimes easier to first solve a similar problem with smaller numbers. Then, if necessary, all outcomes can be listed. | | Product rule |
| | Permutations and combinations | EN4: Enumerate the number of ways of obtaining an ordered linear subset (permutation) of r elements from a set of n distinct objects. EN5: Enumerate the number of ways of obtaining an unordered subset (combination) of r elements from a set of n distinct objects. EN6: Solve problems about outcomes, including problems in the context of probability. | EN3 | ${}_nC_r$ and ${}_nP_r$ are sometimes confused. The number of arrangements when the order is considered (${}_nP_r$) will be greater than when the order is not considered (${}_nC_r$). | Perms & Combs1 | Ways to choose |

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| SELECTIONS SECTION Chapter 11. The binomial distribution (2 hours) | Binomial expansion | EN1: Understand and be able to apply the binomial expansion of $(a + b)^n$ where n is a positive integer. | GCSE Maths | Mistakes often occur when students simplify their expansion prematurely. | The binomial expansion | Binomial expansion2 |
| | The binomial distribution | EN2: Use the binomial distribution to enumerate outcomes. EN6: Solve problems about outcomes, including problems in the context of probability. | EN1 & EN5 | Students find a binomial distribution easier to use when they define their variable clearly before attempting to answer the question. | The binomial distribution | Binomial distribution2 |

| POWERS and ITERATION SECTION | Topic | Learning Outcomes | Precedences | Teaching points | Integral resources | Free resources |
|---|--|---|-------------|--|---|---|
| Chapter 12. Exponentials and logarithms (4 hours) | Properties of the exponential function | EL1: Know and use the function ka^x and its graph, where a is positive. | GCSE Maths | When sketching the exponential graph, students will find it easier to start near the asymptote and draw away from it. | Geogebra: Exponential graphs | Exponentials and logarithms |
| | Logarithms | EL2: Know and use the definition of $\log_a x$ as the inverse of a^x . EL3: Understand and use the laws of logarithms. | EL1 | Old logarithm books could be used to demonstrate how base ten was used to carry out difficult calculations before the advent of the calculator. The anti-log pages at the back could be used to illustrate the link with the exponential. | Exponentials & logs Logs intro Log laws | |
| | Reduction to linear form | EL4: Convert equations of the form $y = ka^x$ and $y = kx^n$ to a linear form using logarithms. EL5: Estimate values of k and a (or k and n) from graphs. | EL1-3 | Students could be encouraged to discuss ways to improve the reliability of their estimates. This could then lead to a discussion on the accuracy of the estimates produced. | | |
| | Equations involving exponentials | EL6: Solve equations of the form $a^x = b$ for $a > 0$. EL7: Use exponentials and logarithms in problems involving exponential growth and decay. | EL1-3 | Students could explore the use of different bases to solve such equations. | | |

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| Chapter 13. Numerical methods (7 hours) | Locating a root of an equation | NM1: Solve equations approximately by considering the change of sign. | EL2 | The use of graph plotting software will provide students with opportunities to experiment and to predict when a change of sign may fail. | Solution of equations1 | Interval bisection |
| | Improving a root | NM3: Recognise when these numerical methods may fail. | | | | |
| | Iterative sequences | NM2: Use a simple iterative method to solve equations approximately. NM3: Recognise when these numerical methods may fail. | EL2 | Scientific calculators allow students to experiment with a variety of sequences for one equation, varying the start point for each sequence. A spreadsheet will help when dealing with a sequence which converges too slowly. | | Numerical methods |
| | Gradients of tangents | NM4: Use a chord to estimate gradient of a tangent to a curve at a point. NM5: Recognise how to improve an estimate for the gradient of a curve at a point. | EL2 | The use of graph plotting software will help to illustrate how a gradient estimate can be improved, and why moving closer to a point can sometimes produce a worse estimate. | Approximating gradients | Introducing calculus |
| Area from rectangles | NM6: Use rectangular strips to estimate the area between a curve and the x -axis. NM8: Recognise whether an estimate would be an over- or underestimate, and understand how to calculate an improved estimate. | EL2 | Students should be encouraged to write their coordinates in a table to avoid confusion. The use of graph plotting software will help to illustrate how an estimate could be an underestimate or an overestimate. It could also be used to demonstrate how an estimate could be improved. | | | |

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| Chapter 13. Numerical methods (continued) | Area under a curve | <p>NM7: Use the trapezium rule to estimate the area between a curve and the x-axis.</p> <p>NM8: Recognise whether an estimate would be an over or under estimate, and understand how to calculate an improved estimate.</p> | EL2 | <p>Students should be encouraged to write their coordinates in a table to avoid confusion.</p> <p>The use of graph plotting software will help to illustrate how an estimate could be an underestimate or an overestimate. It could also be used to demonstrate how an estimate could be improved.</p> | Approximating area | Introducing calculus |
| | Applications of numerical methods | NM9: Apply numerical methods in context where appropriate. | NM4-8 | Students should be encouraged to check the consistency of units. | | |

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| Chapter 14. Differentiation (4 hours) | Differentiation | CA1: Differentiate kx^n where n is a positive integer or 0, and the sum of such functions. | NM4 | Students should understand that $\frac{d}{dx}$ is the operator, and that y is the operand, when $\frac{dy}{dx}$ is written. | Gradient of a curve Tangents, normal & stationary points | Introducing calculus |
| | The gradient of a curve | CA2: Know that the gradient function gives the gradient of the curve and measures the rate of change of y with x . CA3: Know that the gradient of the function is the gradient of the tangent at that point. CA4: Find the equation of a tangent and normal at any point on a curve. | CA1 | Students may find it useful to remember that a constant term added to a function simply translates its graph vertically and has no effect on its gradient. Hence a constant term differentiates to zero. A diagram can often be helpful when solving problems related to tangents and/or normal to curves. | | |
| | Stationary points | CA5: Use differentiation to find stationary points on a curve. | CA2 | The use of graph plotting software would help students to gain a better understanding of stationary points. | | |
| | Extension - the second derivative | CA6: Determine the nature of a stationary point. CA7: Sketch a curve with known stationary points. | CA5 | Students should understand that a maximum or minimum is 'local' and is not necessarily a greatest or least value of the function. | | |

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| Chapter 15. Integration (5 hours) | The rule for integrating x^n where n is a positive integer | CA8: Integrate kx^n where n is a positive integer or 0, and the sum of such functions. CA9: Be aware that integration is the reverse of differentiation. | CA1 | Graph plotting software could be used to illustrate the family of curves produced when a polynomial is integrated. | Reversing differentiation | Introducing calculus |
| | Definite integrals | CA10: Know what is meant by an indefinite and a definite integral. CA11: Evaluate definite integrals. | CA8 | It is useful for students to see why a constant is not required when evaluating a definite integral. | | |
| | Areas between a curve and the x -axis | CA12: Find the area between a curve, two ordinates and the x -axis. | CA11 | Students could use this to compare with estimates calculated using the trapezium rule and/or rectangles. | | |
| | Areas below the x -axis | | | | | |
| | The area between two curves | CA13: Find the area between two curves. | CA12 | Students may find it useful to see why this technique works when parts of the required area are below the x -axis. This is easily demonstrated by translating both graphs vertically upwards. | Finding areas | |

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| Chapter 16. Application to kinematics (3 hours) | Motion in a straight line | CA15: Recognise the special case where the use of constant acceleration formulae is appropriate. | GCSE Maths | <p>Before using a suvat formula, students should:</p> <ul style="list-style-type: none"> • check that the acceleration is constant; • check that the units are consistent; • decide on a start position and a finish position; • choose a positive direction. | suvat1 | suvat3 |
| | Acceleration due to gravity | CA14: Use differentiation and integration with respect to time to solve simple problems involving variable acceleration. | CA1 & CA8 | <p>It is useful for students to see</p> $s = ut + \frac{1}{2}at^2$ differentiated to produce $v = u + at$ | Variable acceleration1 | Variable acceleration3 |
| | Finding displacement from velocity and velocity from acceleration | | | | | |