Junior Kangaroo

a member of the Association Kangourou sans Frontières

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Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

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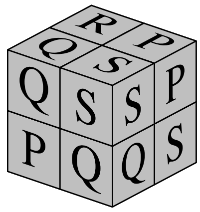
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
B	E	B	B	E	D	B	D	A	D	E	D	C	D	C	A	B	B	E	B	D	C	D	C	B

1. Claudette has eight dice, each with one of the letters P, Q, R and S written on all six faces. She builds the block shown in the diagram so that dice with faces which touch have different letters written on them.
What letter is written on the faces of the one dice which is not shown on the picture?



A P B Q C R D S
E It is impossible to say

Solution **B**

The dice above the hidden one has R marked on it, while the dice on the bottom level that touch the hidden dice have P and S marked on them. Hence the missing dice can have none of these letters on it. Therefore the missing dice has Q marked on its faces.

2. When it is 4 pm in London, it is 5 pm in Madrid and 8 am in San Francisco. Julio went to bed in San Francisco at 9 pm yesterday. What time was it in Madrid at that instant?

A 6 am yesterday
B 6 pm yesterday
C 12 noon yesterday
D 12 midnight
E 6 am today

Solution **E**

Since it is 5 pm in Madrid at the same time as it is 8 am in San Francisco, Madrid is 9 hours ahead of San Francisco. Therefore, when Julio went to bed at 9 pm yesterday in San Francisco, it was 9 hours later in Madrid and hence it was 6 am today.

3. Jacques and Gillian were given a basket containing 25 pieces of fruit by their grandmother. On the way home, Jacques ate one apple and three pears and Gillian ate three apples and two pears. When they got home they found the basket contained the same number of apples as it did pears and no other types of fruit. How many pears were they given by their grandmother?

A 12 B 13 C 16 D 20 E 21

Solution **B**

The number of pieces of fruit the children had when they arrived home was $25 - 1 - 3 - 3 - 2 = 16$. Since we are told that the basket then contained equal numbers of apples and pears, there were then $16 \div 2 = 8$ pears in the basket. Therefore, the number of pears they were given by their grandmother was $8 + 3 + 2 = 13$.

4. One standard balloon can lift a basket with contents weighing not more than 80 kg. Two standard balloons can lift the same basket with contents weighing not more than 180 kg. What is the weight, in kg, of the basket?

A 10 B 20 C 30 D 40 E 50

Solution **B**

Two balloons can lift one basket and 180 kg. One balloon can lift one basket and 80 kg. Therefore we can conclude that one balloon can lift a total of 100 kg. Hence the weight, in kg, of the basket is $(100 - 80) = 20$.

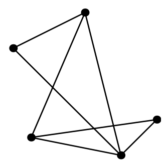
5. The positive integer 1 and every fourth integer after that are coloured red; 2 and every fourth integer after that are coloured blue; 3 and every fourth integer are coloured yellow and 4 and every fourth integer are coloured green. Peter picks a number coloured yellow and a number coloured blue and adds them together. What could the colour of his answer be?

A blue or green B only green
C only yellow D only blue
E only red

Solution **E**

When Peter picks a number coloured yellow and a number coloured blue, he picks a number that is 3 more than a multiple of 4 and a number that is 2 more than a multiple of 4. Therefore, when he adds his two numbers, he obtains a number that is $2 + 3 = 5$ more than a multiple of 4. Since 5 is itself 1 more than a multiple of 4, this means his number is 1 more than a multiple of 4 and hence is coloured red.

6. A map of Wonderland shows five cities. Each city is joined to every other city by a road. Alice's map, as shown, is incomplete. How many roads are missing?



A 6 B 5 C 4 D 3 E 2

Solution **D**

Alice's map shows seven roads. Each of the five cities is joined to four other cities. This suggests that $5 \times 4 = 20$ roads are required but this calculation counts each road twice, so 10 roads are required. Therefore there are three roads missing from Alice's map.

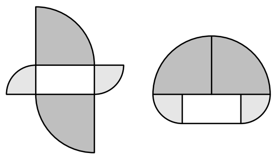
7. What is the value of $\frac{7}{6}+\frac{5}{4}-\frac{3}{2}$?

A $\frac{23}{24}$ B $\frac{11}{12}$ C 1 D $\frac{13}{12}$ E $\frac{25}{24}$

Solution **B**

The value of $\frac{7}{6}+\frac{5}{4}-\frac{3}{2}$ is $\frac{14}{12}+\frac{15}{12}-\frac{18}{12}=\frac{11}{12}$.

8. Both of the shapes shown in the diagram are formed from the same five pieces, a 5 cm by 10 cm rectangle, two large quarter circles and two small quarter circles. What is the difference in cm between their perimeters?



A 2.5 B 5 C 10 D 20 E 30

Solution **D**

Both shapes have two large quarter circles and two small quarter circles in their perimeters. Additionally, the first shape has two edges of length 10 cm and two edges of length 5 cm in its perimeter, whereas the second shape has only one edge of length 10 cm in its perimeter. Therefore the difference, in cm, between their perimeters is $10+5+5=20$.

9. Fay, Guy, Huw, Ian and Jen are sitting in a circle. Guy sits next to both Fay and Ian. Jen is not sitting next to Ian. Who is sitting next to Jen?

A Fay and Huw B Fay and Ian C Huw and Guy
D Huw and Ian E Guy and Fay

Solution **A**

Since Jen is not sitting next to Ian or to Guy, who is sitting between Ian and Fay, Jen is sitting next to the other two people. Hence Jen is sitting next to Fay and Huw.

10. The product of three different positive integers is 24. What is the largest possible sum of these integers?

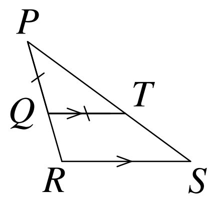
A 9 B 11 C 12 D 15 E 16

Solution **D**

The possible sets of three different positive integers with a product of 24 are 1, 2, 12; 1, 3, 8; 1, 4, 6 and 2, 3, 4 with corresponding sums 15, 12, 11 and 9. Therefore the largest possible sum of these integers is 15.

11. In the diagram, lines QT and RS are parallel and PQ and QT are equal. Angle STQ is 154°. What is the size of angle SRQ?

A 120° B 122° C 124° D 126° E 128°



Solution **E**

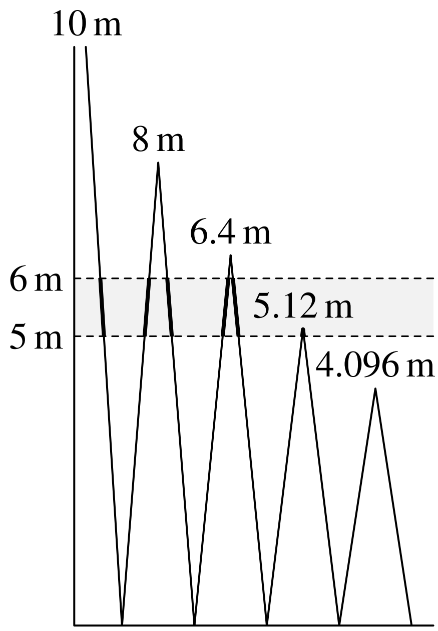
Since $\angle STQ = 154°$ and angles on a straight line add to 180°, $\angle PTQ = 26°$. Hence, since PQ and QT are equal, triangle PTQ is isosceles and so $\angle TPQ = 26°$. Therefore, since angles in a triangle add to 180°, $\angle TQP = 128°$. Finally, since QT and RS are parallel and corresponding angles on parallel lines are equal, $\angle SRQ = 128°$.

12. A rubber ball falls from the roof of a house of height 10 m. Each time it hits the ground, it rebounds to four-fifths of the height it fell from previously. How many times will the ball appear in front of a 1 m high window whose bottom edge is 5 m above the ground?

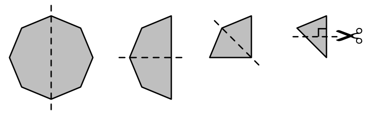
A 1 B 2 C 4 D 6 E 8

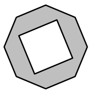
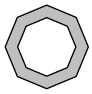
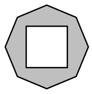
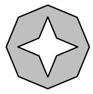
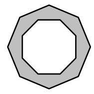
Solution **D**

As the ball falls from a height of 10 m, it will appear in front of the window on its way down. As shown in the diagram, after hitting the ground, it will rebound to a height of 8 m, which is above the upper edge of the window, and so will appear in front of the window on its way up and on its way down. After hitting the ground a second time, it will rebound to a height of 6.4 m, which is also above the upper edge of the window, and so will again appear in front of the window on its way up and on the way down. After hitting the ground a third time, it will rise to a height of 5.12 m and so will appear in front of the window before disappearing from the direction it came. After the fourth bounce, the ball only reaches a height of 4.096 m and so does not appear in front of the window. When you count up the number of times the ball appeared in front of the window, you find it has appeared six times.

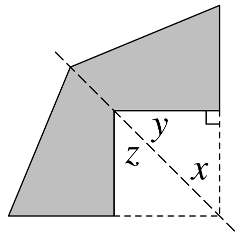


13. A regular octagon is folded exactly in half three times until a triangle is obtained. The bottom corner of the triangle is then cut off with a cut perpendicular to one side of the triangle as shown.
Which of the following will be seen when the triangle is unfolded?



A  B  C  D  E 

Solution **C**



Let the angles x, y and z be as shown in the diagram. Since the triangle containing angle x at its vertex is formed by folding the octagon in half three times, $x = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 360° = 45°$. Therefore, since angles in a triangle add to 180° and the cut is made at 90° to the side of that triangle, y is also 45°. Therefore, since the dotted line is a line along which the quadrilateral shown was folded to create the triangle, it is a line of symmetry for the quadrilateral and hence $z = y = 45°$. Hence the angle at the corner of the hole when the octagon is unfolded will be $2 \times 45° = 90°$ and therefore the hole will be in the shape of a square with its upper side horizontal, as is shown in diagram C.

14. Ayeesha had 12 guests aged 6, 7, 8, 9 and 10 at her birthday party. Four of the guests were 6 years old. The most common age was 8 years old. What was the mean age of the guests?

A 6 B 6.5 C 7 D 7.5 E 8

Solution **D**

Since there were four children aged 6 and the most common age was 8, there were at least five children aged 8 at the party. Hence, as there were 12 children in total, there were at most three other children at the party. We are told that there were children aged 7, 9 and 10 at the event so we can deduce that there were exactly five children aged 8, one child aged 7, one child aged 9 and one child aged 10 at Ayeesha's party as well as the four children aged 6. Therefore the total age of the children was $4 \times 6 + 7 + 5 \times 8 + 9 + 10 = 90$. Hence the mean age of the children was $90 \div 12 = 7.5$.

15. The volume of a cube is $V\,\text{cm}^3$. The surface area of the cube is $2V\,\text{cm}^2$. What is the value of V?

A 8 B 16 C 27 D 64 E 128

Solution **C**

Let the side-length of the cube be x cm. Since the volume is $V\,\text{cm}^3$, we have $V = x^3$. Also, since the area of one face of the cube is $x^2\,\text{cm}^2$ and the total surface area of the cube is $2V\,\text{cm}^2$, we have $2V = 6x^2$. Therefore $2x^3 = 6x^2$ and hence $x = 0$ or $2x = 6$. Therefore $x = 3$ and hence $V = 3^3 = 27$.

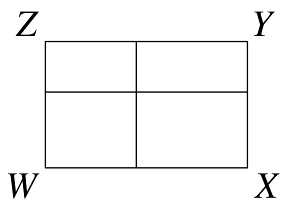
16. There are more than 20 and fewer than 30 children in Miss Tree's class. They are all standing in a circle. Anna notices that there are six times as many children between her and Zara going round the circle clockwise, as there are going round anti-clockwise. How many children are there in the class?

A 23 B 24 C 25 D 26 E 27

Solution **A**

Let the number of children between Anna and Zara going round the circle anti-clockwise be n. Therefore, there are $6n$ children between Anna and Zara going round the circle clockwise. Hence, including Anna and Zara, there are $7n + 2$ children in the circle. The only number more than 20 and less than 30 that is of the form $7n + 2$ is 23. Therefore there are 23 children in Miss Tree's class.

17. Rectangle $WXYZ$ is cut into four smaller rectangles as shown. The lengths of the perimeters of three of the smaller rectangles are 11, 16 and 19. The length of the perimeter of the fourth smaller rectangle lies between 11 and 19. What is the length of the perimeter of $WXYZ$?



A 28 B 30 C 32 D 38 E 40

Solution **B**

From the diagram, we see that the sum of the perimeters of two diagonally opposite smaller rectangles is equal to the perimeter of the large rectangle. We are told that the rectangle whose perimeter we do not know has neither the largest nor the smallest of the perimeters of the smaller rectangles. Hence the rectangles with perimeters 11 and 19 are the ones with the largest and smallest perimeters respectively and so are two diagonally opposite rectangles. Therefore the perimeter of the large rectangle is $19 + 11 = 30$.

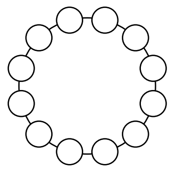
18. The sum $3 + 5 \times 7 - 9 = 36$ is incorrect. However, if one of the numbers is increased by 1, it becomes a correct calculation. Which number should be increased?

A 3 B 5 C 7 D 9 E 36

Solution **B**

The value of $3 + 5 \times 7 - 9$ is 29. If this value is increased by 7, it would equal the value on the right-hand side of the original equation. Hence, if we increase the number of 7s on the left-hand side by 1, the two sides will be equal. Therefore, the number which should be increased by 1 is 5.// It is left to the reader to check that increasing any other number by 1 will not produce a correct equation.

19. Joseph writes the numbers 1 to 12 in the circles so that the numbers in adjacent circles differ by either 1 or 2. Which pair of numbers does he write in adjacent circles?

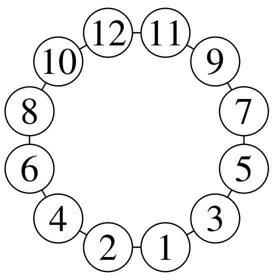


A 3 and 4 B 5 and 6 C 6 and 7 D 8 and 9 E 8 and 10

Solution **E**

Consider first the placement of the number 12. Since the only numbers that differ from 12 by 1 or 2 are 10 and 11, the two numbers on either side of 12 are 10 and 11. Now consider the number 11. The numbers that differ by 1 or 2 from 11 are 12, 10 and 9 and hence, since 12 and 10 are already placed, the number on the other side of 11 from 12 is 9. Similarly, consider the number 10. The numbers that differ by 1 or 2 from 10 are 12, 11, 9 and 8 and hence, since all of these except 8 have already been placed, the number on the other side of 10 from 12 is 8. Therefore Joseph writes 8 and 10 in adjacent circles.

The diagram can then be completed in the way shown below, which is unique apart from rotations and reflection about a line of symmetry. From this it can be seen that the only instances where an odd number is adjacent to an even number are when 11 is next to 12 and when 1 is next to 2. Therefore options A to D are all incorrect.

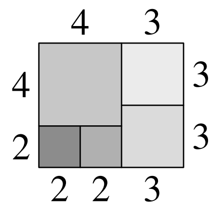


20. Sacha wants to cut a 6×7 rectangle into squares that all have integer length sides. What is the smallest number of squares he could obtain?

A 4 B 5 C 7 D 9 E 42

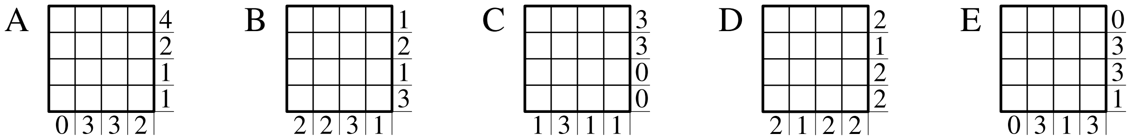
Solution **B**

The largest single square with integer length sides that can be cut from a 6×7 rectangle is a 6×6 square. This would leave a 6×1 strip to be cut into squares, which would require six 1×1 squares, meaning the rectangle is cut into seven squares. Next consider the case when the largest square cut out is a 5×5 square. The remaining region to be cut up would consist of a 5×1 strip, made up of five 1×1 squares and either two 6×1 strips made up of twelve 1×1 squares or one 6×2 strip which could be cut into three 2×2 squares. In each case, more than seven squares would be created.



Now consider the case where the largest square is a 4×4 square. It is possible to cut out this square to leave a 4×2 strip, which could be further cut into two 2×2 squares and a 3×6 strip, which could be further cut into two 3×3 squares, making five squares in total, as shown in the diagram. In the case where the largest square is a 3×3 square, since the total area of the original rectangle is 42 square units and $42 = 4 \times (3 \times 3) + 6$, the rectangle could not be cut into fewer than 5 squares. Similarly, if the largest squares are 2×2, then since $42 = 10 \times (2 \times 2) + 2$, more than 10 squares are required and if the largest square is a 1×1 square, at least 42 squares would be needed.. Hence, the smallest number of squares that Sacha could obtain is 5.

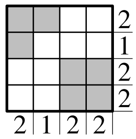
21. Patricia painted some of the cells of a 4×4 grid. Carl counted how many red cells there were in each row and in each column and created a table to show his answers. Which of the following tables could Carl have created?



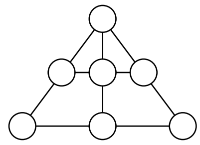
Solution **D**

The top row of the table in option A indicates that all four cells in that row need to be painted but this is not possible if the left hand column is to have no cells painted. In each table, the sum of the row totals and the sum of column totals represents the total number of cells painted. The table in option B has a row total sum of 7 and a column total sum of 8, so this is not a possible table. The table in option C indicates no coloured cells in the bottom two rows which contradicts the three coloured cells in the second column. Hence table C is not possible. The table in option E indicates no coloured cells in the top row but three coloured cells in each of the second and fourth columns which contradicts the one coloured cell in the fourth row. Hence table E is not possible.

Therefore, the only table that Carl could have created is the table in option D. One possible arrangement that would give this table is shown in the diagram.



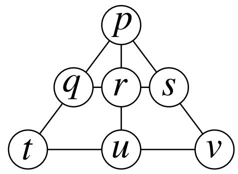
22. Andrew wants to write the numbers 1, 2, 3, 4, 5, 6 and 7 in the circles in the diagram so that the sum of the three numbers joined by each straight line is the same. Which number should he write in the top circle?



A 2 B 3 C 4 D 5 E 6

SOLUTION **C**

Let the numbers in the circles be p, q, r, s, t, u and v, as shown. Since these numbers are 1, 2, 3, 4, 5, 6 and 7 in some order,



$$p + q + r + s + t + u + v = 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$$

.

Let the sum of the numbers in each line of three circles be X. Consider the five lines of three circles. We have

$$(p + q + t) + (p + r + u) + (p + s + v) + (q + r + s) + (t + u + v) = 5X$$

. Therefore

$$3p + 2q + 2r + 2s + 2t + 2u + 2v = 5X$$

and hence

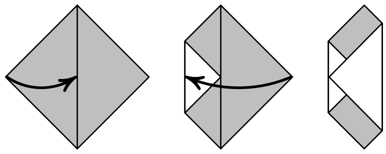
$$p + 2(p + q + r + s + t + u + v) = 5X$$

. Therefore

$$5X = p + 2 \times 28 = p + 56$$

, and hence $p + 56$ is a multiple of 5. Since p is one of the numbers 1, 2, 3, 4, 5, 6 and 7, the only number for which this is possible is $p = 4$.
Note: This shows that if a solution exists, then $p = 4$. It can then be shown that such a solution is possible, for example with $p = 4, q = 3, r = 7, s = 2, t = 5, u = 1, v = 6$.

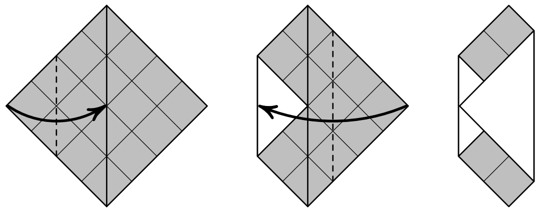
23. A square piece of paper of area $64\,\text{cm}^2$ is folded twice, as shown in the diagram. What is the sum of the areas of the two shaded rectangles?



A $10\ \text{cm}^2$ B $14\ \text{cm}^2$ C $15\ \text{cm}^2$ D $16\ \text{cm}^2$ E $24\ \text{cm}^2$

Solution **D**

Consider the square divided up into 16 congruent smaller squares, as shown. Since the original square has area $64\,\text{cm}^2$, each of the smaller squares has area $4\,\text{cm}^2$. When the square is folded twice in the manner shown, it can be seen that each of the shaded rectangles is made up of two small squares. Therefore the sum of the areas of the two shaded rectangles is $2 \times 2 \times 4\,\text{cm}^2 = 16\,\text{cm}^2$.



24. The non-zero digits p, q and r are used to make up the three-digit number 'pqr', the two-digit number 'qr' and the one-digit number 'r'. The sum of these numbers is 912. What is the value of q?

A 3 B 4 C 5 D 6 E 0

Solution **C**

The three-digit number 'pqr' is equal to $100p + 10q + r$ and the two-digit number 'qr' is equal to $10q + r$. Hence the sum of 'pqr', 'qr' and 'r' is equal to $100p + 20q + 3r$. Since this sum is 912, $3r$ ends in '2' and hence, as r is a single digit, $r = 4$. Therefore $100p + 20q = 900$ and hence $10p + 2q = 90$. Therefore $2q$ ends in '0' and, as the question tells us q is non-zero. $q = 5$.

It is then easy to work out that $p = 8$ to show that a full solution to the question does exist.

25. I gave both Ria and Sylvie a piece of paper. Each piece of paper had a positive integer written on it. I then told them that the two integers were consecutive. Ria said “I don’t know your number”. Then Sylvie said “I don’t know your number”. Then Ria said “Ah, I now know your number”. Which of these could be the integer on Ria’s piece of paper?

A 1 B 2 C 4 D 7 E 11

SOLUTION **B**

If Ria’s number were 1, she would know, since she and Sylvie have consecutive positive integers written on their pieces of paper, that the number on Sylvie’s piece of paper is 2. Therefore, when Ria says she does not know Sylvie’s number, Sylvie can deduce that Ria’s number is not 1. Similarly, when Sylvie says she does not know Ria’s number, Ria can deduce that Sylvie’s number is not 1. Also if Sylvie’s number were 2, she would be able to deduce that Ria’s number is 3 since she knows it is not 1. Therefore, since Sylvie says she does not know Ria’s number, Ria can deduce that Sylvie’s number is not 2.

Therefore, since Ria can now say that she knows Sylvie’s number, it follows that either Ria’s number is 2 and she can deduce that Slvie’s number is 3, or that Ria’s number is 3 and she can deduce that Sylvie’s number is 4. If Ria’s number is greater than 3 she would not be able to deduce Sylvie’s number.

Therefore, of the options given, the only possibility for Ria’s number is 2.