Junior Kangaroo

a member of the Association Kangourou sans Frontières

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Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

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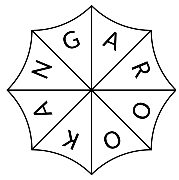
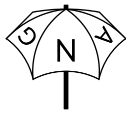
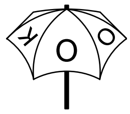
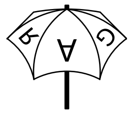
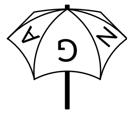
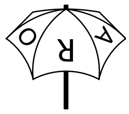
C	A	D	E	E	D	A	B	D	D	D	E	E	E	C	E	B	C	B	C	B	D	C	A	D

1. My umbrella has the word KANGAROO written on top, as shown in the diagram.
Which of the following does ***not*** represent a view of my umbrella?



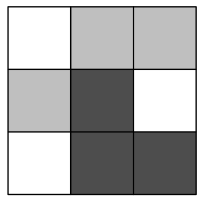
A  B  C  D 

E 

Solution **C**

In the picture in option C, the letter R should have its vertical line adjacent to the letter A but does not. Therefore the view that does not represent a view of my umbrella is the view in option C.

2. Priya painted each of the nine squares shown black, white or grey. What is the smallest number of squares that she would need to repaint so that no two squares with a common side are painted in the same colour?

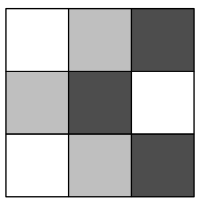


A 2 B 3 C 4 D 5 E 6

Solution **A**

There are currently two grey squares with a common side and a set of three black squares which all have a side in common with another black square. Therefore there needs to be a minimum of two changes of colour to satisfy the criterion.

If Priya repaints the square in the top right black and the square in the middle of the bottom row grey, then no squares with a common edge will be painted the same colour, as shown in the diagram.



Therefore, the smallest number of squares Priya needs to repaint is 2.

3. There are 12 ducks on Old McBride's farm. Three ducks each lay one egg every day, four ducks each lay one egg every other day and five ducks each lay one egg every three days.
How many eggs do these 12 ducks lay in a period of 12 days?

A 60 B 72 C 75 D 80 E 96

SOLUTION **D**

In twelve days, the three ducks who lay an egg every day will lay 12 eggs each, for a total of 36 eggs. In the same time, the four ducks who lay an egg every other day will lay 6 eggs each, for a total of 24 eggs. Finally, in the same time, the five ducks who lay an egg every three days will lay four eggs each, for a total of 20 eggs. Therefore the grand total of eggs that the ducks on Old McBride's farm lay is $36 + 24 + 20 = 80$.

4. Which of the following fractions is closest to 2?

A $\frac{17}{6}$ B $\frac{18}{7}$ C $\frac{19}{8}$ D $\frac{20}{9}$ E $\frac{21}{10}$

SOLUTION **E**

When you subtract 2 from each of the numbers, you obtain $\frac{5}{6}$, $\frac{4}{7}$, $\frac{3}{8}$, $\frac{2}{9}$ and $\frac{1}{10}$. Of these, the smallest is $\frac{1}{10}$. Therefore, the fraction in the list that is closest to 2 is $\frac{21}{10}$.

5. Sophie and Armaan are playing a game. The winner of each game gets 3 points and the loser gets 1 point. Armaan wins 6 games and Sophie has a total of 18 points. How many games do they play?

A 6 B 7 C 8 D 9 E 10

SOLUTION **E**

Since Armaan wins six games, Sophie loses six games and so gains six points. Since Sophie has a total of 18 points, the number of points she gets from winning games is $18 - 6 = 12$. Therefore the number of games she wins is $12 \div 3 = 4$. Hence the total number of games they play is $6 + 4 = 10$.

6. Molly and Holly each own a big dog. Molly's dog and Holly's dog weigh 80 kg in total. Molly's dog and a 20 kg bag of dog food weigh the same as Holly's dog.
What is the weight, in kg, of Holly's dog?

A 20 B 30 C 40 D 50 E 60

Solution **D**

Since Molly's dog and a 20 kg bag of food weigh the same as Holly's dog, we can deduce that Molly's dog, Holly's dog and a 20 kg bag of food would weigh twice as much as Holly's dog. However, we are also told that Molly's dog and Holly's dog weigh 80 kg in total. Therefore, twice the weight of Holly's dog is 80 kg + 20 kg = 100 kg. Hence the weight of Holly's dog is 100 kg $\div 2$ = 50 kg.

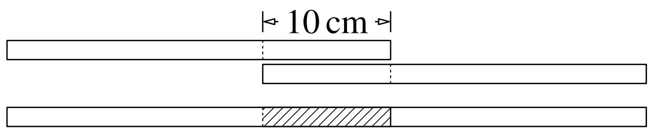
7. Every plant in my mum's window box has either two leaves and one flower or five leaves and no flowers. In total, the plants have six flowers and 32 leaves.
How many plants are in my mum's window box?

A 10 B 11 C 12 D 13 E 14

Solution **A**

Only one type of plant has a flower. Therefore, since there are six flowers in total, there must be six of that type of plant. These six plants account for 12 leaves and hence the remaining plants have $32 - 12$, that is 20, leaves in total. The second type of plant has five leaves and hence the number of this type of plant is $20 \div 5 = 4$. Therefore the total number of plants in my mum's window box is $6 + 4 = 10$.

8. Alisha has four paper strips of the same length. She glues two of them together with a 10 cm overlap to make a strip 50 cm long.



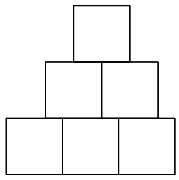
With the other two strips, she wants to make a strip 56 cm long.
How long, in cm, should the overlap be?

A 2 B 4 C 6 D 8 E 10

Solution **B**

Let the length of a strip be L cm. We can see from the diagram that $L + L - 10 = 50$ and so $2L = 60$. Therefore the length of each strip is 30 cm. Alisha wants to make a strip of length 56 cm with her other two strips. Therefore the length of the overlap, in cm, will be $2 \times 30 - 56 = 4$.

9. Elliot drew six squares, each with side-length 1 cm, to make the shape shown.
What is the perimeter, in cm, of the shape?



A 9 B 10 C 11 D 12 E 13

SOLUTION **D**

The perimeter of the shape can be thought of as being in six parts. The vertical part to the left and the vertical part to the right both have length 3 cm, as does the horizontal part on the bottom. The final parts of the perimeter are the upper horizontal parts of the squares that have nothing above them. The three squares at the bottom are partly covered by two other squares and so the upper horizontal part of the bottom two squares that forms part of the perimeter is of length 3 cm − 2 cm = 1 cm.

Similarly, the upper horizontal part of the middle two squares that forms part of the perimeter is of length 2 cm − 1 cm = 1 cm. Finally the top of the top square contributes 1 cm to the perimeter. Therefore, the total perimeter, in cm, is $3 + 3 + 3 + 1 + 1 + 1 = 12$.

10. Every day Freya writes down the date in her diary and calculates the sum of the digits she writes. For example, on 11th June, she writes 11/06 and calculates $1 + 1 + 0 + 6 = 8$.
What is the largest daily sum she calculates over the course of a year?

A 8 B 13 C 16 D 20 E 23

SOLUTION **D**

The largest digit sum from a day is $2 + 9 = 11$, when the date is the 29th of a particular month. The largest digit sum from a month is $0 + 9 = 9$, when the month is September. Therefore, the largest sum Freya will calculate is $2 + 9 + 9 = 20$ on 29th September.

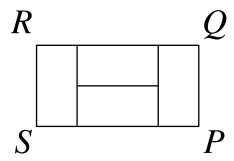
11. On Abdication Street, there are nine houses in a row. At least one person lives in each house. Any two neighbouring houses have at most six people living in them. What is the largest number of people that could be living in Abdication Street?

A 23 B 25 C 27 D 29 E 31

SOLUTION **D**

Since there is at least one person living in each house and there are at most six people living in any two neighbouring houses, the largest number of people that can live in any house is five. Therefore, the largest number of people who could be living in Abdication Street occurs when five people live in a house at one end of the street and six people live in every subsequent pair of houses along the street. Therefore the largest number of people who could live in Abdication Street is $5 + 6 + 6 + 6 + 6 = 29$.
Note: This will occur when the numbers of people in each house are 5, 1, 5, 1, 5, 1, 5, 1, 5.

12. The rectangle $PQRS$ shown is divided into four smaller congruent rectangles. The length of PQ is 1 cm.
What is the length, in cm, of RQ?



A 4 B 3.5 C 3 D 2.5 E 2

SOLUTION **E**

From the diagram, you can see that twice the length of a shorter side of the smaller rectangles is the same as the length of PQ. The length of RQ is equal to the sum of length of a longer side of the smaller rectangles and the twice the length of a shorter side of the smaller rectangles. Therefore the length, in cm, of RQ is $1 + 1 = 2$.

13. Diego noticed that 20% of 30% of a number was 3 less than 30% of 40% of the same number.
What was that number?

A 30 B 35 C 40 D 45 E 50

SOLUTION **E**

Let the number be N. Since 20%, 30% and 40% have equivalent decimals 0.2, 0.3 and 0.4 respectively, the information in the question tells us that $0.2 \times 0.3 \times N = 0.3 \times 0.4 \times N - 3$. Therefore $0.06 \times N = 0.12 \times N - 3$ and hence $3 = 0.06 \times N$. Therefore the value of N is $3 \div 0.06 = 300 \div 6 = 50$.

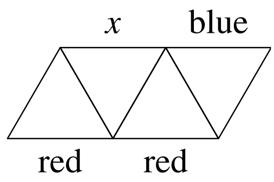
14. The area of a rectangle is $12\,\text{cm}^2$. The lengths of its sides, measured in cm, are integers.
Which of the following could be the perimeter of the rectangle?

A 18 cm B 20 cm C 22 cm D 24 cm E 26 cm

SOLUTION **E**

Since the lengths of the sides of the rectangle, in cm, are integers, the only possibilities for the dimensions of a rectangle with area $12\ \text{cm}^2$ are $1\,\text{cm} \times 12\,\text{cm}$, $2\,\text{cm} \times 6\,\text{cm}$ and $3\,\text{cm} \times 4\,\text{cm}$. The perimeters, in cm, of these rectangles are 26, 16 and 14. Therefore the only one of the values given that could be a perimeter of such a rectangle is 26 cm.

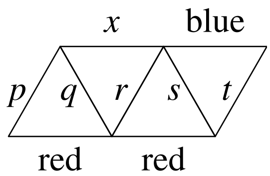
15. Mabel wants to colour each of the nine line segments shown in the diagram red, blue or green. The sides of every triangle should be coloured with three different colours. She has already coloured three of the segments, as shown.
What colour can the line segment marked x be coloured?



A only blue
B only green
C only red
D any of red, blue or green
E such a colouring is not possible

SOLUTION **C**

Label the six line segments whose colours are not yet known p, q, x, r, s and t, as shown in the diagram.



Since the segment labelled s is in a triangle with a red side and in a triangle with a blue side, it must be coloured green. Therefore the segment labelled r must be coloured blue.
Now consider the segment labelled q. This is in a triangle with a side coloured red and in a triangle with a side coloured blue, so must be coloured green. Therefore the segment labelled x is in a triangle with a side coloured green and a side coloured blue so can only be coloured red.

(For completeness, note that the segment labelled p can only be coloured blue and the segment labelled t can only be coloured red.)

16. A bag contains 3 green apples, 5 yellow apples, 7 green pears and 2 yellow pears. George takes out pieces of fruit at random, one piece at a time. How many pieces of fruit must he take to be certain he has at least one apple and one pear of the same colour?

A 9 B 10 C 11 D 12 E 13

Solution **E**

If George removed just the 7 green pears and 5 yellow apples, he would not have an apple and a pear of the same colour. Hence removing 12 pieces of fruit does not guarantee that he has removed an apple and a pear of the same colour.
Now suppose George removes 13 pieces of fruit. These must include at least three green pears, because the total number of the other fruit is 3 + 5 + 2 = 10, and at least 6 pieces of fruit that are not green pears, since there are only 7 green pears. Now consider what these 6 pieces of could be. There could be 5 yellow apples but the sixth piece would then either be a green apple or a yellow pear. In either case George would then have an apple and a pear of the same colour.
So George needs to remove 13 pieces of fruit.

17. In the sum shown, equal letters represent equal digits and different letters represent different digits.
What is the value of $X + Y + Z$?

$$\begin{array}{r} X \\ +\quad X \\ +\; YY \\ \hline ZZZ \\ \hline \end{array}$$

A 15 B 16 C 17 D 18 E 19

Solution **B**

Since $X \leq 9$ and $YY \leq 99$, the answer $ZZZ = X + X + YY \leq 9 + 9 + 99 = 117$. Therefore the only possible value of Z is 1 and $ZZZ = 111$.
Now note that largest answer that can be obtained if $Y < 9$ is $88 + 9 + 9 = 106$ which is less than 111. Hence the two-digit number is 99. Therefore $X + X = 111 - 99 = 12$ and hence $X = 6$. Therefore the value of $X + Y + Z$ is $6 + 9 + 1 = 16$.

18. The number 100 is multiplied by either 2 or 3. The result then has 1 or 2 added to it. Finally the new result is divided by either 3 or 4. The final result is an integer.
What is this integer?

A 50 B 51 C 67 D 68
E More than one final answer is possible

Solution **C**

Consider the possible outcomes from the process. Starting with 100, the result of the first step is 200 or 300. Then the result of the second step is either 201, 202, 301 or 302. We are told that the third step involves dividing by 3 or 4 and that the final result is an integer. However, none of the possible values from the second step are divisible by 4 and only 201 is divisible by 3. Therefore, the final result is $201 \div 3 = 67$.

19. In the four-digit number '$PQRS$', the digits P, Q, R and S are all non-zero and are in increasing order from left to right. What is the largest possible difference 'QS' − 'PR' between the two two-digit numbers 'QS' and 'PR'?

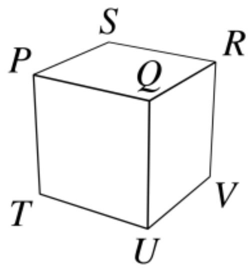
A 86 B 61 C 56 D 50 E 16

SOLUTION **B**

Since we want the difference 'QS' − 'PR' to be as large as possible, we want 'PR' to be as small as possible and 'QS' to be as large as possible. The largest possible value of 'QS' occurs when $Q = 7$ and hence $R = 8$ and $S = 9$. Then the smallest value of 'PR' is obtained when $P = 1$. This gives 'QS' − 'PR' = 79 − 18 = 61. If $Q < 7$, then 'QS' < 70 and 'PR' ≥ 13 and hence 'QS' − 'PR' ≤ 70 − 13 = 57. Therefore the largest value of 'QS' − 'PR' is 61.

20. Harvey writes a number on each face of a cube. Then, for each vertex, he adds the numbers on the three faces that meet at that vertex. The answers he gets for vertices R, S and T are 14, 16 and 24, respectively.
What answer does he get for vertex U?

A 15 B 19 C 22 D 24 E 26



SOLUTION **C**

Since R and T are diagonally opposite vertices, the three faces that meet at R and the three faces that meet at T together make up every face of the cube. We can also see that this is the case for vertices S and U. Therefore, the total for vertex U can be obtained by subtracting the total for vertex S from the sum of the totals for vertices R and T. Hence the total for vertex U is $14 + 24 - 16 = 22$.

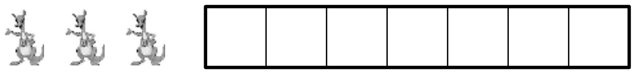
21. A train has 12 carriages. Each carriage has the same number of compartments. Iliana is travelling in the 7th carriage and in the 50th compartment from the front.
How many compartments are there in each carriage?

A 7 B 8 C 9 D 10 E 12

SOLUTION **B**

Let the number of compartments in each carriage be N. Since Iliana is in the 7th carriage and the 50th compartment from the front, we can conclude that the front six carriages contain fewer than 50 compartments in total and that the front seven carriages contain 50 or more compartments. Therefore $6N < 50$ and $7N \geq 50$. Hence $N < 8\frac{1}{3}$ and $N \geq 7\frac{1}{7}$. The only integer value of N which satisfies both of these inequalities is $N = 8$. Therefore the number of compartments in each carriage is 8.

22. In how many ways can three kangaroos be placed in three different cells of the grid shown so that no two adjacent cells both contain kangaroos?



A 7 B 8 C 9 D 10 E 11

Solution **D**

We will consider, in turn, the possible arrangements of kangaroos if the first kangaroo from the left is in successive cells from the left.
Number the cells 1 to 7, as shown in the diagram.

1	2	3	4	5	6	7

Consider first the cases where the first kangaroo is in cell 1. Then, since no two adjacent cells can both contain a kangaroo, the possible places for the subsequent kangaroos are 3 and 5; 3 and 6; 3 and 7; 4 and 6; 4 and 7; 5 and 7.
Now consider the cases where the first kangaroo is in cell 2. The possible places for the subsequent kangaroos are 4 and 6; 4 and 7; 5 and 7.
Next consider the cases where the first kangaroo is in cell 3. The possible places for subsequent kangaroos are only 5 and 7.
No further cases need to be considered since the smallest number of cells that can hold three kangaroos with no two in adjacent cells is five. Therefore, the total number of ways the three kangaroos can be placed is $6 + 3 + 1 = 10$.

23. Jo, Flo and Mo divided up a sum of money. Jo took £10 plus a quarter of what was then left.
Next Flo took £40 plus a quarter of what was then left.
Finally Mo took what was then left.
Jo and Flo took the same amount of money.
How much did Mo take?

A £100 B £120 C £150 D £180 E £350

Solution **C**

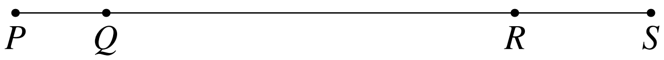
Let the initial amount of money they shared be £$(4X + 10)$. Therefore, since Jo took £10 plus a quarter of what was left, Jo took £(X + 10), leaving £3X for Flo to take her share. Since Flo took £40 plus a quarter of what was then left, she took £$(40 + \frac{1}{4}(3X - 40))$. We are told that Jo and Flo took the same amounts and so $X + 10 = 40 + \frac{1}{4}(3X - 40)$. Therefore $X - 30 = \frac{1}{4}(3X - 40)$ and so $4(X - 30) = 3X - 40$. Hence $4X - 120 = 3X - 40$ which has solution $X = 80$. Therefore the original sum of money they shared was £$(4 \times 80 + 10)$ = £330. Therefore Jo took £$(80 + 10)$ = £90, as did Flo, which left £$(330 - 2 \times 90)$, that is £150, for Mo.
Note: the initial amount of $4X + 10$ was chosen to leave an expression divisible by 4 after Jo took her initial £10.

24. Four points lie on a straight line. The distances between pairs of points are, in increasing order, 2, 3, k, 11, 12 and 14.
What is the value of k?

A 9 B 8 C 7 D 6 E 5

SOLUTION **A**

Consider the line with four points labelled P, Q, R and S, in that order, as shown in the diagram.



The distance PS must be 14 as it is the longest distance. It can be seen that the sum of the pairs of distances PQ and QS, and PR and RS will also be 14. These pairings are $2 + 12$ and $3 + 11$ in some order. This only leaves the distance QR to find.

Suppose that the distances PQ and PR are 2 and 3. Then the distance QR would be 1 which is impossible since we are told the distances are listed in increasing order. Hence we would then have the distances RS and PR as 3 and 11 and then the distance QR would be $11 - 2 = 9$.

A similar argument can be applied if the distances RS and QS were 2 and 3, leading to the conclusion that this arrangement is not possible either and that the distance QR is 9. In either case, the missing distance is 9.

25. Hayden used small cubes of side 1 to build a large cube with side 4. Then he painted three of the faces of his large cube red and the other three faces blue. When he finished, there was no small cube that had three faces painted red.
How many small cubes had both red and blue faces?

A 18 B 20 C 22 D 24 E 26

SOLUTION **D**

Since there was no small cube that had three faces painted red, we can deduce that there were not three faces that meet at a vertex painted red. The only way this is possible is to have two opposite faces and one further face painted red. This further red face would then have a common edge with each of the two opposite red faces. The cubes painted both red and blue would be the cubes on the edges of the large cube where a red and a blue face meet. These would consist of a cube at each vertex of the large cube plus two further cubes on each edge of the large cube apart from the two edges where two red faces meet and the two edges where two blue faces meet. Since a cube has 8 vertices and 12 edges, the total number of cubes painted both red and blue is $8 + 2 \times (12 - 2 - 2) = 8 + 16 = 24$.

