United Kingdom
Mathematics Trust

Junior Mathematical Challenge

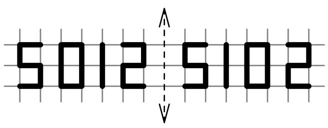
Solutions 2022

For reasons of space, these solutions are necessarily brief.

There are more in-depth, extended solutions available on the UKMT website, which include some exercises for further investigation:
www.ukmt.org.uk

1. E The values of the expressions are A 42; B 204; C 404; D 0; E 440.
So 20×22 has the greatest value.

2. A The diagram shows that 5012 is reflected onto 5102.



3. D Let the number first thought of be x. Adding five gives $x + 5$; multiplying by two then gives $2x + 10$; adding ten now gives $2x + 20$; dividing by two leaves us with $x + 10$ before subtracting the original number gives 10. Finally, adding three means that the final result is 13, whichever number is first thought of.

4. D The value of $0.6 + \frac{2}{5}$ is $\frac{3}{5} + \frac{2}{5} = \frac{5}{5} = 1$.

5. C The first and third fractions take integer values: $\frac{1}{1} = 1$; $\frac{111}{1+1+1} = \frac{111}{3} = 37$.
Now consider the other three fractions: 11 is odd, so is not divisible by 2; 1111 is also odd and hence is not divisible by 4 and 11 111 does not have a unit digit of 0 or 5 and therefore is not divisible by 5. So exactly two of the given fractions take integer values.

6. B The interior angles of a square and of an equilateral triangle are 90° and 60° respectively. Therefore, as the angles at a point sum to 360°, $\angle QUP = (360 - 90 - 2 \times 60)° = 150°$.
As square $RSTU$ has sides in common with both of the equilateral triangles, PUT and QRU, the side-lengths of these triangles are equal.
Therefore $QU = UP$, so triangle QUP is isosceles with $\angle QPU = \angle PQU = (180-150)° \div 2 = 15°$.

7. D The weight of kiwi fruit which contains approximately the same amount of vitamin C as 1 kg of oranges is $(1000 \div 2\frac{1}{2})$ g $=(1000 \times \frac{2}{5})$ g $= 400$ g.

8. E Note that $100 \div 7 = 14$ remainder 2, so a period of 100 days is equal to 14 weeks and 2 days. Therefore, in 100 days' time it will be Saturday.

9. B Let the side-length of the large square be 9. Then the diagram also contains nine squares of side-length 3, four squares of side-length 6, nine squares of side-length 1 and four squares of side-length 2. Therefore the total number of squares in the diagram is $1 + 9 + 4 + 9 + 4 = 27$.

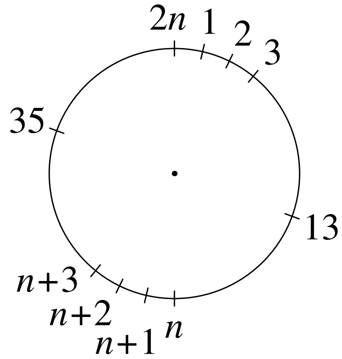
10. D Let the number be x. Then $\left(\frac{1}{2} \times \frac{1}{4} \times \frac{1}{8}\right)x = \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$. Therefore $\frac{x}{64} = \frac{7}{8}$. So $x = \frac{64 \times 7}{8} = 56$.

11. E The sum of the ten numbers is 55. For the numbers in the two groups to sum to multiples of 4, the total of the nine remaining numbers must itself be a multiple of 4. So either 3, leaving a total of 52, or 7, leaving a total of 48, could be left out. Hence the largest number which could be left out is 7. The remaining numbers then may be placed in the required two groups in a number of ways, for example 1, 2, 5 (total 8) and 3, 4, 6, 8, 9, 10 (total 40).

12. B From the information given, it may be deduced that the weight of one quarter of the paint is $(5.8 - 3.1)\,\text{kg} = 2.7\,\text{kg}$. So the weight of the empty paint pot is $(3.1 - 2.7)\,\text{kg} = 0.4\,\text{kg}$.
Hence the weight, in kg, of the full pot of paint is $0.4 + 4 \times 2.7 = 0.4 + 10.8 = 11.2$.

13. D The inner of the two shaded areas is the difference in area between a square of side 2 cm and a square of side 1 cm. The outer of the two shaded areas is the difference in area between a square of side 4 cm and a square of side 3 cm.
So the total shaded area, in cm^2, is $2^2 - 1^2 + 4^2 - 3^2 = 4 - 1 + 16 - 9 = 10$.
The area of the outer square is $25\,\text{cm}^2$.
Therefore the percentage of the area of the outer square which is shaded is $\frac{10}{25} \times 100\% = 40\%$.

14. B As the children are evenly spaced, there must be an even number of children in the ring for one child to be standing directly opposite another child. Let this number be $2n$.
The diagram shows that child 1 is standing opposite child $n + 1$, child 2 is standing opposite child $n + 2$ etc. Therefore, as child 13 is standing opposite child 35, we have $n + 13 = 35$, so $n = 22$.
Hence there are 44 children in the ring.



15. E The value of $2 \div (4 \div (6 \div (8 \div 10)))$ is $2 \div (4 \div (6 \times \frac{10}{8})) = 2 \div (4 \div \frac{15}{2}) = 2 \div (4 \times \frac{2}{15}) = 2 \div \frac{8}{15}$ $= 2 \times \frac{15}{8} = \frac{30}{8} = \frac{15}{4}$.

16. C The perimeter of the polygon $PQRSTUV$ is equal to the combined perimeters of equilateral triangles PQW and STU minus the perimeter of the overlapping area, namely equilateral triangle RWV. Therefore the required perimeter, in cm, is $3 \times 5 + 3 \times 8 - 3 \times 2 = 3 \times 11 = 33$.

17. C After each round of the game, the total number of counters held by the two players increased by two. As they started with a total of 20 counters and finished with a total of 56 counters, the number of rounds played was $(56 - 20) \div 2 = 18$.
Let the number of rounds won by Amrita be n. Then she lost $18 - n$ rounds.
Therefore $10 + 3 \times n - 1 \times (18 - n) = 40$. Hence $10 + 3n - 18 + n = 40$, that is $4n = 48$.
So Amrita won 12 rounds of the game.

18. E As the figure is a parallelogram, the sum of the two angles on the left of the figure is 180°.
So $3x - 40 + 2x - 30 = 180$. Hence $5x = 250$ and therefore $x = 50$. Diagonally opposite angles in a parallelogram are equal. So $4y - 50 = 2x - 30 = 70$. Hence $4y = 120$ and therefore $y = 30$.

19. B Let the number of apples and pears at the start of the day be $3n$ and n respectively.
Then $n = 2(3n - 5)$. Hence $5n = 10$, so $n = 2$.

So the number of pieces of fruit at the start of the day was $4n = 4 \times 2 = 8$.

20. **B** First note that exactly one of Pam and Quentin is telling the truth. Though it is not possible to tell who it is, it means that none of Roger, Susan and Terry is telling the truth.
So just one of the five students is telling the truth.

21. **C** The sum of the numbers in List S is 55 and the corresponding number for List T is 59. Therefore, for the sum of the numbers in List S to equal the sum of the numbers in List T it is necessary that the number which Jenny moves from List S to List T is smaller by 2 than the number which moves from List T to List S. She may achieve this in three ways: by exchanging the 3 in List S with the 5 in list T; by exchanging 8 and 10 or by exchanging 11 and 13.

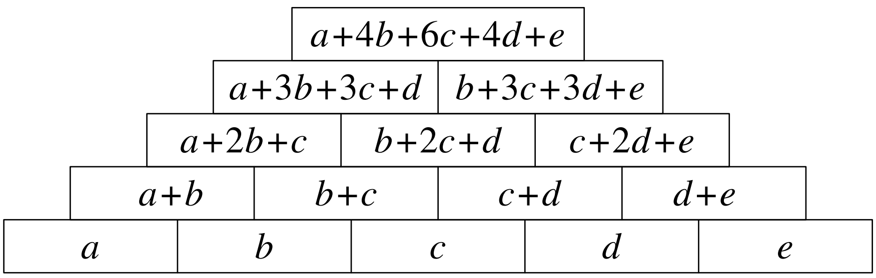
22. **E** When the pyramid is removed from the cube, the solid loses the edges TU, UQ and UV. However, it also gains the three edges of triangle TQV. So the remaining solid has the same number of edges as the original cube, that is 12.

23. **D** Let the original price of the ticket be £P.
Then $P \times 1.05 \times 0.8 = P - 4$. So $P \times 0.84 = P - 4$. Hence $P = \dfrac{4}{0.16} = \dfrac{400}{16} = 25$.
Therefore the original cost of the ticket was £25.

24. **A** Let the number of purple flowers be n Then there are $2n$ yellow flowers.
Therefore the number of red flowers is $2n \times \dfrac{4}{3} = \dfrac{8n}{3}$ and the number of white flowers is $\dfrac{8n}{3} \times \dfrac{5}{6} = \dfrac{48n}{15} = \dfrac{16n}{5}$.
As both $\dfrac{8n}{3}$ and $\dfrac{16n}{5}$ are positive integers, we may deduce that n is a multiple of 15.

Let $n = 15k$. Then the total number of flowers is $15k + 30k + 40k + 48k = 133k$.
We are told that there are fewer than 150 flowers in the shop, so $k = 1$ and there are 133 flowers in total.

25. **C** Let the numbers on the bottom row of the number pyramid be a, b, c, d, e.
The diagram shows the contents of the other cells in terms of those variables.



So $a + b + c + d + e = 17$ ... [1]; $b + 2c + d = 16$ ... [2]; $a + 4b + 6c + 4d + e = 61$ ... [3].
[3] - [1]: $3b + 5c + 3d = 44$ ... [4]; [2] × 3: $3b + 6c + 3d = 48$ ... [5]; [5] - [4]: $c = 4$.
So the central number of the bottom row is 4.
(*It is not possible to determine the values of* a, b, d *and* e*, but we can deduce that* $a + e = 5$ *and* $b + c = 8$.)