Junior Mathematical Olympiad 2021

Marker's Report

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Comments from the Marking Team

Overall, there was excellent engagement with this year's paper and perfect solutions were submitted to every question. For those interested in the details the paper had a mean score of 29 marks out of a possible 70 marks, which suggests that it was slightly more challenging than the previous 2019 paper but less challenging than the 2018 paper. It was pleasing to see that Section A was well attempted, with a mean score of 7 marks out of a possible 10 marks. Section B had a mean score of 22 marks out of a possible 60 marks. The drop off in performance from Section A to Section B is no surprise, as the latter required full written solutions rather than just a final answer.

On many occasions candidates provided correct answers to Section B questions but they received very few marks because there was little or no justification as to why their answer was correct. It is important for candidates to explain their reasoning carefully and to justify any claims they make when attempting Section B questions. Often algebra may be needed, labelled diagrams or extended prose. It is clear to the markers that some candidates sitting the JMO are unfamiliar with the competition and the requirements of Section B in particular. We would recommend that teachers encourage candidates to look at previous papers, the official solutions and the sample student solutions which can be found at:

https://www.ukmt.org.uk/competitions/solo/junior-mathematical-olympiad/archive.

Question B1

This question was well completed by many candidates. There were three common approaches. The first was to show that 2 down started with an odd number. Then a few considered the eight prime numbers with units digit 1 or 3 for 1 across and tens digit 1, 2, 3, 4, 6 or 8 since 1 down is a square. Others showed that 2 down was 16 or 36 and progressed from there. The third approach was to place each of the six possible squares in 1 down, then look at the options for the other two squares.

Of those who took the first approach, several did not use 11 as a possible prime. For the second approach, many assumed that 2 down was 16 or 36 without justification and so scored very few marks. Also, many candidates did not consider the possibility of a square being repeated and this also led to a low mark.

Those who tried to work with the 21 possible prime numbers for 1 across often came to a sticky end since there were so many possibilities to consider and the candidates did not always get all of the prime numbers right in the first place.

It was pleasing to see so many candidates were able to provide a complete solution showing that there is excellent work going on in schools and that there are many promising young mathematicians with bright prospects for the future.

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Question B2

This question proved to be very accessible and was well answered by most candidates. There were over 240 perfect solutions and the average mark was the highest seen in Section B. It was pleasing to see that so many candidates at this young age could form and manipulate a system of simultaneous equations containing three variables.

There was only one main method for solving this problem, which consisted of forming three simultaneous equations and then eliminating one of the variables to find the difference between the other two. With this kind of question, the algebra itself will implicitly provide much of the explanation and so we were not expecting much prose to accompany it. However, some candidates interpreted too well the need to explain their working and gave essay style solutions rather than realising that clear algebraic steps were better in showing what they were doing. Although explanations lacking algebra were accepted (if deemed equivalent), the use of algebra should be encouraged because it is far easier to interpret than lengthy prose describing algebraic relationships between variables.

A one-mark penalty was frequently applied when candidates failed to define their variables correctly. For example, it is important to note that when defining variables "c" is not chilli but the "cost of a chilli". Once equations were set up, the challenge was how to deal with the simultaneous equations and 'guess and check' or 'trial and error' type solutions were penalised heavily.

Question B3

This question was one of the least attempted by candidates. For those that did attempt it, there could have been many more receiving full marks if geometrical justifications were given in full. It was far too common for candidates to show the algebraic proof with no reference to the geometrical justifications for each step. Candidates who gave no geometrical justification at all were capped at 4 marks, even when they had a fully correct algebraic proof.

A few candidates lost marks for providing partial reasoning. This may have meant that certain justifications were missing, or that those written could be expanded upon. For example, "Angles on a line" in itself was not enough to warrant marks for justification, reference to the sum of angles was also required, "Angles on a line sum to 180°" for instance.

Where candidates struggled to get started on the question, they had failed to spot that the overlapping triangles were isosceles, since they were inscribed within a semi-circle meaning that two of their sides were radii of the semi- circle. It is important for candidates to consider the starting configuration carefully and think about how to utilise the fact that the semi-circle is involved. If a circle (or part thereof) is included in a geometrical problem, then the radius is likely to play an important role.

Another common source of lost marks was for candidates to begin with what they were trying to prove and to work out other angles in terms of from there. This is only valid if a candidate goes on to show that every step in their argument is reversible.

Many solutions lacked a diagram which resulted in some solutions becoming confused, especially when candidates were not following a common angle-naming convention. We would encourage all candidates to draw a clear diagram at the start of any solution to a geometry problem.

Question B4

This proved to be a popular question, most likely because this type of word sum will have been encountered before, and many candidates found the correct answer. However, finding the correct answer and providing a full solution are two very different things. There were relatively few full mark solutions to this question and there were many candidates who obtained the correct answer but scored relatively few marks because their logical argument contained gaps.

The most common successful approach was to use the right-hand column to observe that A+B must equal 10, and then go on to deduce that the next column was the same as the first with 1 carried to give A+C=9 or C=B-1. Most candidates who did this failed to adequately explain why the carry from each of these columns could not be 2 and so marks were deducted. Finally, it is simple to observe that the hundreds column is the same as the tens, giving A=1 from which the correct result follows.

Many candidates noticed that A had to be 1 or 2, because it is the thousands digit of the sum of three numbers with three digits each, and then went on to check that A=2 was not possible. The logical arguments for this were often flawed because again candidates did not consider whether some of the earlier carries could have been 2. Some concluded that A=2 without considering any carry from the hundreds column, which was seen as a significant flaw in the solution.

A small number of candidates turned the initial sum into a single equation, B + 110C = 889A. From this and the fact that A, B and C are single, non-zero digits, it was easy to show that A must be 1 and find B and C from that. This was, perhaps, the most elegant method.

A number of candidates found additional, false, solutions which meant they could score a maximum of 4 marks. This highlights the importance of candidates checking their solutions when they believe them to be complete.

Question B5

This question turned out to be the most challenging on the paper, yielding the fewest full mark solutions and the lowest average score. Having said that, there were a relatively large number of attempts and many of those scored marks.

Almost all successful attempts at this question made use of algebra to write down each of the first eight terms of the sequence. Some candidates chose to define variables for the first two terms and work forwards through the sequence, others decided to define variables for the sixth and seventh terms and work backwards through the sequence. There were then two main strategies for dealing with the eighth term, which was known to be 400. The first was for candidates to consider all possible solutions to the equation, using the positivity of the terms as a constraint and in some cases considering divisibility arguments to reduce the options, and test which yielded the smallest third term. The second was to rewrite the eighth term in terms of the third, which showed that to minimise the third term one must maximise the second term. A very small number of candidates showed remarkable ingenuity and introduced only a single variable for the seventh term, worked backwards to derive expressions for the first three terms and then set each of these to be greater than zero.

There were two one-mark penalties that were frequently applied. The first was if a candidate stated that the second term must be divisible by eight but failed to explain why. The second was if a candidate assumed that the first or second terms were bounded but failed to explain why. It is important that candidates do not expect the markers to fill in gaps in their reasoning.

There were some candidates who attempted this question without the use of algebra and unfortunately their arguments were rarely rigorous. It is important that any claims made are

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justified, and in a question like this the use of algebra is quite simply the best and easiest way to do this!

Question B6

The final question at this year's JMO was found accessible by a relatively large number of candidates compared to previous years. There were over 350 scripts with a positive score, with as many as 79 candidates achieving full marks.

The key step was to recognise that an integer greater than or equal to 20 must go in a square that is adjacent to one of the squares marked 1-4. This recognition could come directly as in the official solution or by placing 36 in one of the diagonal squares and realising that an integer less than or equal to 20 will then have to be adjacent to 36.

Candidates who got the key idea but did not achieve full marks ended with that outcome due to penalties for either careless assertions or poorly written solutions.

A large number of candidates claimed that they had concluded the proof because they showed a specific arrangement worked. The question required a general argument that worked for all possible placements of which there were many (literally billions, as one marker pointed out). Therefore, a demonstration that one specific arrangement worked did not constitute a valid proof as it omitted dealing with the other possible placements. These candidates did not score many marks.

Many candidates also lost marks because they asserted that 20 must be placed adjacent to one of 1-4, and in many cases even a stronger assertion that 20 must be placed adjacent to 4. This was not correct. The key was that because there are only 15 integers with a difference of less than 16 to all of 1-4, it meant that at least one integer greater than or equal to 20 (so at least one integer from 20-36) needed to be adjacent to 1-4, rather than there being any requirement for 20 to be adjacent to one of 1-4.