Junior Mathematical Olympiad 2023 Markers Report

Comments from the Marking Team

This year's paper was found to be accessible; with the average marks for Section A and Section B being 6.8 out of 10 and 22 out of 60 respectively. Whilst the Section B average may appear relatively low it is worth remembering that providing a full written solution to an Olympiad problem is something many candidates are relatively unfamiliar with. Any candidate fully solving two or more problems should be proud of themselves.

It was pleasing to see so many attempts to the six Section B problems achieving at least a mark; this was more than 1000 for each of the first three problems and approximately 600, 700 and 800 for the fourth, fifth and six problems respectively. The increasing number of scoring attempts from problems three to six will give the Problems Group pause for thought and they will certainly reflect on the difficulty of the problems versus their position on the paper.

Every solution submitted for a given problem was marked according to one of two separate mark schemes, which we refer to as 0+ and 10-. Before a solution was marked, a marker will have carefully read it and decided if the attempt had met the relevant 10- criterion. This is a threshold that must be crossed for the 10- mark scheme to be applied. If this threshold was not met then the attempt was marked according to the 0+ mark scheme. Typically, 0+ mark schemes can award 0 to 3 marks and 10- mark schemes can award 7 to 10 marks.

There were close to 2250 solutions submitted which were awarded 7 marks or more. There were over 1200 perfect solutions submitted for Section B, two candidates achieved full marks in Section B and one candidate achieved full marks across the whole paper.

Usually we would see many candidates attempt Section B in much the same way they attempt Section A, by this we mean that they find a correct answer but do not provide any written explanation to accompany it. For this paper this wasn't generally the case and it appears that candidates are taking notice of the instructions on the front of the paper. A constant source of lost marks was not a complete lack of reasoning but rather gaps in reasoning. There were many occasions where a candidate likely used a correct mathematical result but they didn't indicate the result in their written solution and assumed the marker would fill in the gaps. Unfortunately, it is not for the marker to assume that they know what a candidate is thinking and results such as divisibility rules must be explicitly stated if used in a solution.

Question 1

There was excellent engagement with this question with 942 candidates achieving 7 or more marks. The main source of lost marks was a lack of explanation with regards to why n must be larger than 15. Many candidates solved an equation to show that the four fractions summed to 1 when $n = 15$. From here some claimed the end result without justification. It is not expected that JMO candidates be able to formally solve an inequality and so a statement saying that as n increases the fraction $\frac{1}{n}$ decreases was all that was needed to achieve full marks with this method. Candidates who formed and solved an inequality achieved full marks when their conclusion was correct, often despite the brevity of their attempt.

Question 2

Whilst the majority of candidates attempted this question, the number of candidates achieving 7 or more marks fell drastically to only 396. Candidates who assumed numerical values for the total distance travelled and the total time taken were heavily penalised even though this invariably led to the correct ratio. It is vital that if a candidate believes a final answer is independent of any variables in the question, they must demonstrate why this is the case. Often the easiest way to do that is to take an algebraic approach, as these variables will cancel during calculation.

The vast majority of those who achieved 7 or more marks did take an algebraic approach to the problem. This meant that it was clear their final ratio was independent of the distance travelled and the time spent travelling.

An appropriate and rather elegant non-algebraic method, which a small number of candidates employed successfully, was to calculate the ratio of the speeds by comparing the distances travelled in the same time interval for the two sections of the ride.

Unfortunately, many candidates incorrectly stated that riding 10% of the total distance in 20% of the total time equated to an average speed of $\frac{1}{2}$, whereas the significance of that fraction is that the average speed for the first section of the ride was one half of the overall average speed. Doing the same thing for the second section of the ride did lead to the correct ratio of speeds, but this fundamental lack of understanding was rewarded with few marks.

Question 3

Interestingly, 1020 candidates achieved at least a mark for this question yet only 203 achieved 7 or more marks. Many of the good solutions lost one mark for not explaining why they could divide by $(x - 1)$. It is important to take note of conditions given in the question and refer to them when they are needed.

To solve this problem, candidates were required to do two things:

(1) Derive an equation, such as $k + 5(x - 1) = kx$, from the information given.

(2) Show regardless of the unknown constant x, which is greater than 1, k is always 5.

Close to 600 candidates were able to derive a correct equation but only a small proportion presented a robust and rigorous argument to show that k is always 5, regardless of the value of x.

Many candidates rewrote the equation as $kx - k = 5x - 5$ and then claimed that this implied that k could only equal 5. While this is true, it is not for the reason stated. In particular, if x is 1, then k could be any real number; and it is not obvious that there are no other values of x which allow for multiple values of k.

The cleanest and most straightforward approach to (2) is to first factorise both sides, obtaining $k(x - 1) = 5(x - 1)$. The conditions of the question then imply that x is not 1, so both sides can be divided by $(x - 1)$ to obtain $k = 5$ as the only possible value.

It is possible that candidates assumed that the equation above was an identity (something that is true for all values of x), even if they were not aware that they were making this assumption. While it is true that, when $k = 5$, x can take any value, what was needed was a proof that k cannot be anything else.

Alternative approaches were seen but were rarely successful. These were contradiction (assume k is not 5, arrive at a contradiction and then check $k = 5$ works), induction on x, reference to linearity of the equation with respect to x and divisibility claims regarding k. These usually failed because of an implicit assumption that the equation derived was an identity, as mentioned above.

Question 4

This proved to be the most unpopular and unsuccessful question on the paper, with only 601 candidates achieving at least a mark and just over 107 candidates achieving 7 to 10 marks. This is no surprise; geometry is often a topic candidates avoid. For anyone wanting to be successful in an Olympiad competition, it is clear that they will get a significant boost if they give some attention to geometry. Geometry at this junior level often requires no more than finding angles using the basic theorems learned in the classroom. The low number of attempts is likely down to a lack of practice in this area and could be easily remedied. Of those candidates who attempted the problem, it was pleasing to see that many of them were familiar with angle facts about polygons but it is important that they make clear in their solution how they know the value of a particular angle.

The task in this question was to prove that a point M was on the line HD. Many candidates failed to answer this question correctly because they used a circular argument, that is they implicitly assumed that M was on HD in the course of arguing that M was on HD. This mistake is encouraged if using a single diagram where M is marked on the line HD. It would have been helpful if candidates had drawn two diagrams: one not involving M and the other not involving D. Then it would have been clearer what to do. For example, you could prove that $\angle DHI = 80°$ and $\angle MHI = 80°$, from which it follows that HM and HD are the same line.

Consider the circumcircle of a regular nonagon. A standard arc (say IA) will subtend an angle of $20°$ at any point of the circumcircle, and so at all the other vertices of the nonagon. This very useful fact can open up the problem very quickly, by regarding longer arcs as unions of non-overlapping standard arcs. A small number of candidates used this fact without proof and the markers decided that this was a serious omission, but still allowed such attempts to be marked using the 10− mark scheme.

Question 5

For a late question, this attracted a large number of attempts with over 727 candidates achieving at least a mark and 239 achieving 7 or more marks.

Unfortunately, a significant number of candidates did not read the question carefully enough and thought that the eleven-digit number was divisible by all of the numbers 1 to 9. Candidates should be encouraged to read each question at least twice to ensure they have understood what is being asked.

It was clear that many more than the 239 candidates who were awarded a high mark for this question understood exactly what was going on. The majority of lost marks came from lack of explanation. This may have been a reluctance to explicitly refer to relevant divisibility rules or lack of explanation as to why the eleven-digit number must not be a multiple of 5. There will no doubt be candidates who are disappointed with their mark for this question, but it is important that they take the positives from this experience and look to become even better mathematicians in the future. When a mathematician writes a proof, it is essential that they provide the necessary details for others to follow along and there shouldn't be gaps for the reader to fill in for themselves. Simply referring to known theorems is often enough and in this case that would have be the divisibility rules for at least 2 or 4 and 5 and 8.

Some candidates appeared to have carried out their thinking on other sheets and their submitted solution was more of a summary. Unfortunately, in many of these cases the summaries were too brief and lacked the necessary detail for a high mark. Candidates do not need not to summarise and would do better handing in their detailed thinking instead.

Some candidates started their answer with the correct values of A and B and then demonstrated why this was a valid solution. We were looking for reasoning to explain how they arrived at their solution and not just a defence of their claimed answer. We suspect that the students had reasoned in some rough working and perhaps did not understand the nature of the explanations we were looking for.

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Question 6

This question was about counting, or rather combinatorics, which is a topic often neglected in the classroom and so it was pleasing to see 830 candidates achieve at least a mark and 374 candidates achieve 7 to 10 marks.

We do not see rough working but imagine that most candidates will have started with a few diagrams on a sheet of rough paper to see what was going on in the question and where the difficulties were. The key is to spot that the four vertices round the square cause the main problem. When two opposite vertices have the same colour there are two possibilities for each of the other two vertices in the square and when they have different colours then the other two vertices are forced to be the third colour. Roughly speaking, those candidates who realised this and had a good strategy for dealing with those two cases scored well and those who either missed that point or couldn't really deal with it scored low marks.

It is always amazing to see the resourcefulness of the candidates in finding ways to approach to the problem which could lead to a solution. At one end was a list of 36 colourings, which is both time consuming and highly prone to error. At the other end were those candidates who fixed the colours of one vertex of the square and that of a neighbouring vertex in the square. They then listed or drew just three diagrams of the possibilities for the other two vertices of the square. They then argued that you could multiply those three by six to allow for the permutations of the colours and a further two to allow for the possibilities for the central circle. Most were somewhere between these two approaches.

As usual those candidates who both drew some diagrams and explained their work did the best. Those missing the diagrams made the explanations hard to follow (and more likely to contain errors). Those who missed the explanation made it hard to follow the diagrams (and they were more prone to double counting some solutions and/or missing out some).

In a few cases candidates had clearly drawn lovely multi-coloured diagrams and they had been scanned in black and white at school before uploading. Where a school has the facility to scan in colour that is worth bearing in mind and where it doesn't it is worth briefing the candidates to remember this fact and for example to use B, R and G or three different symbols to indicate the three colours. We think we were able to resurrect just about all the originals, but our apologies if we have not quite always been successful.