AQA
Questions matter

A-level

# Mathematics

7357/3 Paper 3

Report on the Examination

7357
June 2024

Version: 1.0

**General**

Students performed well across the paper compared to previous examination series. The paper provided opportunities for all students to achieve well. While students excelled in calculations, they noticeably struggled in questions requiring the explanation of reasoning. Many students used their calculators effectively.

**Section A Pure**

**Question 1 (multiple choice)**

Three-quarters of students answered this question correctly. The most common incorrect response was the second option.

**Question 2 (multiple choice)**

Almost 95% of students gave the correct answer.

**Question 3 (multiple choice)**

This was the most challenging multiple-choice question in the paper, with just over 50% of students answering it correctly. The most common incorrect response was the last option.

**Question 4**

There were many good responses to this question, with more than half of students gaining both marks and almost all at least one mark. Students often did not recall the derivative of $2^x$ so tried various incorrect methods to differentiate it. Other students obtained the correct answer but lost the second mark by incorrectly attempting to further simplify their expression.

**Question 5**

Two-thirds of students gave a fully correct solution, demonstrating excellent recall and use of the relevant formulae. However, a significant number of students did not fully grasp the importance of the ‘show that’ instruction, which led to a lack of clarity in their solutions. These students often failed to explicitly substitute the given values into the formulae. Simply writing the formula for the area of a sector followed by $27\pi$ and the formula for the area of a triangle followed by $81$ did not earn credit. Additionally, some students did not give their answer in the required form and lost the final mark.

**Question 6**

Most students correctly rewrote $\frac{1}{\sqrt{x}}$ as $x^{-1/2}$ in part (a). However, some struggled with the coefficient of 5, incorrectly simplifying it to $\frac{1}{5}$ or $\sqrt{5}$ in some cases. The integration process was well executed by most students. Despite this, a significant number forgot to include the constant of integration, which, although not penalised in (a), usually meant no marks were given in part (b).

For students who correctly identified the need to use the integrated function from part (a), they then used the given point to determine the constant of integration to obtain the correct equation of the curve, with very few mistakes seen. However, many students mistakenly found the equation of the tangent or normal to the curve.

**Question 7**

In part (a), few students shaded areas outside the regions labelled 1, 2, and 3 in the mark scheme. Correct shading of regions 1 and 2 was common, but some students incorrectly shaded only region 2 or included region 3. A common mistake was confusing $y \geq 0$ with $x \geq 0$, leading to shading below the $x$-axis or neglecting region 1. The best solutions used pencil to clearly shade the entire region *R*. A few students lost a mark due to vague shading that did not clearly indicate the intended region.

In part (b), most students scored full marks. Those who did not often struggled with algebraic manipulation, either incorrectly rearranging $x + y = 1$ or after elimination not rearranging into a solvable quadratic form. The most common approach involved eliminating $y$ and using a calculator to solve the resulting quadratic equation in $x$. Students used the graph well to identify that point $A$ was in the second quadrant and thus usually selected the correct value from the two possible solutions. A few students lost the final accuracy mark by giving their answers as rounded decimals, despite the instruction to provide exact values.

**Question 8**

Most students understood to substitute $t = 0$ into the model and correctly calculated the value of 20 for part (a). The most common error was omitting the units from the answer.

In part (b), many students recognised that as $t \to \infty$, $e^{-kt} \to 0$, leading to the maximum temperature being determined. Those who attempted differentiation and set the derivative equal to 0 did not proceed further unless they recognised that this implied $t \to \infty$.

Part (c)(i) was answered well, with most students correctly substituting $\theta = 86$ and $t = 1$ to form the correct equation. Common errors included mistakes in rearranging, sign errors, and a minority mistakenly substituting $t = 60$.

In part (c)(ii), a good proportion of students formed a correct equation using values from previous parts. While many went on to solve the equation correctly, others struggled with $t$ being within the exponent. Additional errors included not using $T-1$, arithmetic slips and basic rearrangement mistakes involving signs and brackets.

**Question 9**

While most students were able to complete the square twice in part (a) and obtain the required terms in brackets, some incorrectly handled the constants. Most students correctly identified $k=25$, although a few incorrectly gave the value −27, −47 or −79. Others included incorrect signs in the brackets, reversing the 4 and 6, or not halving the coefficients of $x$ and $y$.

The vast majority of students successfully used their answer from part (a) to correctly state the coordinates of the centre in part (b).

Most students knew to substitute $x = 0$ in part (c) and thus obtained a quadratic equation in $y$. Those with incorrect circle equations often used the form given in the question rather than their own, enabling them to gain both marks. A common error was ignoring the $x$ part of the standard circle equation, resulting in $(y-6)^2=25$. A successful method involved drawing a right-angled triangle with the radius as the hypotenuse, using Pythagoras' theorem to find the height, and then adding 6 to it. For those using the quadratic equation method, many found the two solutions and the majority correctly identified which was required.

Part (d) was less well answered. Most students recognised the need to find the lengths of *QR* and *PR* and used a trigonometric equation to find the angle. While basic trigonometric ratios were commonly used, some students attempted the sine or cosine rule less successfully. Many did not realise they could use the radius found in (a). Errors included incorrect versions of formulae, substituting incorrect lengths in correct formulae, and finding the wrong angle (such as angle *PRQ* instead of *QPR*). Some students tried to use gradients. There were often mistakes with the distance formula, such as adding rather than subtracting values, and a lack of labelling, which made it difficult to identify lengths. A common error was calculating the angle correctly but giving the answer in degrees instead of radians.

**Question 10**

Nearly all students attempted this question and gained at least one of the first two marks. Some students omitted the term $(x+h)$ in the expression for $\mathrm{f}(x+h)$ or forgot the brackets in $-(5x^3+h)$. Quite a few students expanded $(x+h)^3$ as $(x+h)^2(x+h)$ which was less successful than using the binomial expansion, because students did not collect like terms. Most correctly divided by $h$ to obtain the +1 term, but a few incorrectly used $h=0$ rather than $\lim\limits_{h\to 0}$.

**Question 11**

Most students made a substantial attempt at answering this question, with many gaining method marks, even if their final answer was incorrect. Most correctly identified the need to integrate the given equation and found the correct limits for the definite integral very efficiently. Some obtained incorrect limits of 0 and 8 by solving $x^2 - 8x = 0$ and missed the possibility that $\ln x = 0$, not recognising that two positive limits were required, as indicated by the graph. Many students correctly identified $u = \ln x$ and $\frac{dv}{dx} = x^2 - 8x$ for integration by parts. Integration was then usually done well but omitting brackets in the final integrated expression led to sign errors, causing the loss of accuracy marks. Marks were awarded for correctly applying the limits and completing the argument to obtain a fully correct answer in the required form.

Students should explicitly substitute their limits into the integrated function and subtract, as those who skipped this step were more likely to make slips and lose marks. Students who correctly integrated the function generally arrived at the value $\frac{623}{9} - 256\ln 2$ for the definite integral. However, fewer students recognised that this was negative, and its absolute value was needed for the correct area.

**Section B (Statistics)**

**Question 12 (multiple choice)**

This question was answered correctly by about 60% of students. The most common incorrect response was the second option, 4.

**Question 13 (multiple choice)**

This question was answered correctly by just over half of students. The most common incorrect response was selecting the top left diagram.

**Question 14**

Over 90% of students gained the mark in part (a).

Part (b) was also well done with over 80% of students getting both marks. The most common errors in calculating the standard deviation were attempting to use $\sqrt{\frac{\sum(x-\bar{x})^2}{n}}$ or $\sqrt{\frac{\sum x^2}{n}}$.

Part (c), however, was very poorly answered, with many students unable to score any marks. Many simply wrote statements like, ‘Village A had a lower average/mean than Village B, but Village B had a higher standard deviation,’ which is not a comparison in context. When comparing the means, students should use a phrase such as ‘on average’ and refer specifically to the annual cost, not the cost of a unit of energy. For example, 'On average, the annual cost in Village A is lower than in Village B.' When comparing the standard deviations, students should use terms such as 'variable,' 'consistency,' 'varies,' or 'variation,' and avoid terms like 'variance,' 'range,' 'variety,' or 'similar' which will not be given credit. For instance, 'The annual costs in Village B show greater variation, indicating less consistency than in Village A.' It is incorrect to refer to Village A having a 'greater range,' as the range is a specific statistical measure not provided in the question. Some students did not provide any context and merely compared the numerical values of the mean and standard deviation, which resulted in no marks.

**Question 15**

A large majority of students correctly calculated the mean in part (a).

In part (b) most students were able to clearly show the variance of $X$ as 6.93 by explicitly showing the substitution of values into $np(1-p)$. There was evidence that some students did not know that the variance formula is in the formulae booklet.

Most students answered part (c) correctly, although fewer than did so for part (d). Common errors included finding $\mathrm{P}(X \leq 10)$ instead of $\mathrm{P}(X < 10)$ or failing to provide the answer with sufficient accuracy.

Many students simply wrote down the answer to part (d). Some used $\mathrm{P}(X < 5)$ instead of $\mathrm{P}(X \leq 5)$ or did not state the answer with sufficient accuracy.

Similarly in part (e), students often just wrote the correct probability. Those who broke the calculation into steps usually found $\mathrm{P}(X \leq 15)$ correctly, which earned the method mark. However, many then incorrectly subtracted $\mathrm{P}(X \leq 9)$.

Students found part (f) most challenging, and scoring both marks was rare. The main issue was a lack of context in answers. Many students knew the conditions for a binomial distribution to be appropriate but did not achieve marks because they did not relate these conditions to the context of the question. For example, responses like 'the events might not be independent' or 'probability might not be fixed' were insufficient without explaining why this might be the case. Students needed to be clear about whether their reasons supported the model's appropriateness in the given context. Some responses veered out of context, discussing different routes or business versus economy seats. Others mentioned probabilities being independent, which is incorrect.

**Question 16**

This question was not as well answered as the other hypothesis test questions, with a substantial number of students gaining no marks. Common issues included incorrect setup of hypotheses and misuse of notation. Many students incorrectly used symbols other than $\rho$ and others used 0.45 or 0.2787 in their hypotheses. Even when comparisons were made, mistakes were frequent. For example, some students compared $-0.45$ with $0.2787$ rather than using the negative critical value, leading to confusion about whether to reject $H_0$. A few students used diagrams, sometimes correctly but often incorrectly, comparing both negative or both positive values. Others correctly used modulus signs as per the mark scheme, but some compared incorrect values, such as $0.2787$ with $0.025$. Some students attempted to solve the problem using binomial or normal distributions, indicating a misunderstanding of the topic. The conclusion was often correct for those who reached this stage, but some students used definitive terms like 'show,' 'prove,' and 'conclude' or omitted important elements such as 'negative correlation' or the context of the question.

**Question 17**

Most students were able to score full marks in parts (a), (b), (c), and (d), demonstrating strong proficiency in using their calculators. However, a small proportion of students mistakenly treated the normal distribution as discrete.

Students did not do well in part (e). The most common error was using an incorrect $z$-value, such as $\pm 2$ or $1.6449$. Many students did not understand the phrase 'exceeded by 95%' often using the wrong tail of the normal distribution. Additionally, some students incorrectly applied the rule that '95% of the distribution lies within 2 standard deviations of the mean,' which did not receive credit.

The most common methods used in part (f) were calculating the probability or the test statistic with very few students using the critical region method. Students have improved in stating their hypotheses correctly and using the appropriate notation. However, some still attempted to write sentences for their hypotheses or used incorrect symbols such as $\bar{x}$ or used $51.5$ for $\mu$. Most calculated the sample mean correctly. Many students correctly modelled the distribution of the sample mean and used this model in their calculations, either for the relevant probability or the test statistic value. This typically led to accurate comparisons and correct decisions regarding $H_0$. Students have become proficient in writing their conclusions using key phrases like 'sufficient evidence' and 'to suggest' in their contextually accurate conclusions. Only a few students incorrectly concluded that the evidence 'proved' there was an increase. Another common error in the conclusion was to refer to 'length' instead of 'mean length.'

**Question 18**

Parts (a)(i) and (a)(ii) of this question were very well answered.

For part (a)(iii), students usually applied the conditional probability formula correctly, but answers were often left in an incorrect form, such as $\frac{0.07}{0.61}$. It was also common to see students getting either just the denominator or just the numerator correct, with $0.54$ frequently given as the denominator. Another common mistake was incorrectly dividing by $P(H)$ rather than $P(G')$ to find $P(H \mid G')$.

In part (b), students employed various methods, with the most successful and common approach being the comparison of $P(G) \times P(H)$ and $P(G \cap H)$. Among those using a correct method, the most frequent errors included incorrectly calculating one or more of the necessary probabilities, failing to fully compute the solution and leaving it as 0.39 × 0.28, not comparing like with like (eg comparing a decimal with a fraction), and not providing a final statement that $G$ and $H$ were not independent.

**Question 19**

In part (a)(i) most students used the correct notation to state $H_0$ and $H_1$ clearly and accurately. However, common errors included using incorrect letters such as $x$ or $\mu$, using the wrong probability such as 0.3, or using an inequality symbol. Some students described their hypotheses in words, which was not accepted.

Many students used the correct model, although some mistakenly tried using a normal distribution in part (a)(ii). Most knew to conduct a two-tailed test with the binomial model, but a few incorrectly used a one-tailed test, comparing their probabilities with 0.1. This led to the correct lower critical region, $x \le 16$ , but marks were withheld since this came from an incorrect method. While many students correctly identified $x \le 16$ , not all combined it correctly with $x \le 24$ , resulting in incorrect inequalities like $24 \le x \ge 16$ . After making a correct comparison, students were sometimes unclear whether 16 and 24 were inside or outside the critical region, leading to incorrect answers like $x \le 17$ and $x > 24$ . Additionally, some students listed various probabilities without labels, making it difficult to understand their calculations.

In part (a)(iii) most errors arose from a lack of context in answers. It was necessary to refer to cars, proportion/percentage, emissions, changed, evidence, but some otherwise correct responses were not given the credit because they used probability/number instead of proportion/percentage. Marks were also lost for making definitive statements such as ‘shows that,’ ‘proves that’ or ‘concludes that.’ A minority of students who knew how to write a conclusion in context were not given the credit because they had not provided a critical region in part (a)(ii) and thus did not have a basis for their conclusion.

Very few students scored both marks in part (b). Most students were able to score one mark for identifying limitations of the data set, such as it only representing three regions, five makes of car, or two years. Only a minority of students made specific statements such as ‘no diesel cars in the LDS with CO emissions more than 0.5g/km’ or ‘values of CO emissions for some diesel cars are missing from the LDS’.

**Mark Ranges and Award of Grades**

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.