Pearson
Edexcel

Examiners' Report
Principal Examiner Feedback

Summer 2023

Pearson Edexcel International A Level
In Pure Mathematics (WMA12/01)

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational, and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere.

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your candidates at: www.pearson.com/uk

Summer 2023
Publications Code WMA12_01_2306_ER

General

This paper proved to be a demanding but fair paper on the WMA12 content, and it was pleasing to see candidates were able to make attempts at all of the questions. Overall, marks were available to candidates of all abilities and the questions which proved to be most challenging were 7 and 8. The topic of proof continues to be an area which candidates find difficult, whilst the question on calculus in context was an area that candidates really struggled to demonstrate that they understand the techniques they are applying. Time did not appear to be an issue for candidates.

Report on individual questions

<u>Question 1</u>

A beautifully simple and extremely accessible question to start off the paper on the trapezium rule. Even those candidates who did not achieve full marks were at least able to make some attempt.

The large majority of candidates achieved full marks on this question with occasional correct methods losing the A mark for not acknowledging the requirement on the accuracy, something all candidates should be well trained to look for. The main error seen was an incorrect structure with the brackets which was very rarely recovered. The B mark was not lost as often, but a surprising number were unable to identify the value of *h,* which was disappointing, given that the table explicitly showed consecutive values of *x* and thus by deduction, *h*.

Very rarely did we see candidates work on calculating all trapezia separately, although when we did, it was generally correct, albeit via a less efficient method.

<u>Question 2</u>

This question enabled candidates to demonstrate their understanding of the factor theorem, as well as being able to solve a cubic. There was a clear warning at the top of the question that all stages of working should be shown and that solutions relying on calculator technology were not acceptable. This question proved to be more challenging than anticipated, particularly given that similar questions have been asked before.

Part (a) was well attempted by most candidates. Most knew to substitute $x=\frac{3}{2}$ into the expression for $\mathrm{f}(x)$, set this equal to zero and solve to find the value for *a*. A significant number of candidates failed to show sufficient working to gain both marks. Some candidates approached the question through long division rather than the required use of the factor theorem thus losing both marks. Candidates should be encouraged to show intermediate stages of working rather than simply stating a given answer after their initial statement if they want to secure full marks.

In part (b), most candidates were able to make a pleasing attempt and were able to score some of the available marks. Most carried out long division to obtain the quadratic factor, then attempted to make use of the discriminant of the quadratic factor. Many of those who got this far then failed to obtain the final mark. There were a variety of reasons referring to the quadratic factor which did not score the final mark such as failing to state that the value of the discriminant was less than zero, stating a "math error," stating "this does not factorise" or "no solution." A few candidates tried to find the quadratic factor using the linear factor $\left(x-\frac{3}{2}\right)$ in their working, but usually then incorrectly thought that the factors of $\mathrm{f}(x)$ were $(2x-3)(4x^2-2x+2)$. Additionally, this mark was lost due to candidates failing to provide any reference to the root $x=\frac{3}{2}$ with some even stating that the only real root was the factor $(2x-3)$. Candidates should therefore be reminded to provide sufficient and accurate detail when trying to deduce certain given properties.

Question 3

This question provided candidates an opportunity to demonstrate their understanding of circle geometry and use coordinate geometry. Candidate were usually able to make good progress on this question by scoring on average two thirds of the marks.

In part (a)(i) the majority of candidates knew how to apply the distance formula correctly and the correct answer was usually obtained. A few candidates however thought that this was the diameter, and they went on to halve it; this gave them no marks as this was an incorrect method (which also precluded them from the A1ft for the equation of the circle in part (a)(ii)). Other incorrect methods seen included adding the coordinates instead of subtracting them or mixing up their *x* and *y* coordinates which also resulted in no marks.

In part (a)(ii) those who gained the first method mark generally went on to give a correct equation of the circle. Common errors included forgetting one or both squared symbols, adding instead of subtracting the coordinates of the centre, or mixing up the *x* and *y* coordinates. A few candidates seemed not to grasp that "*C*" mentioned in the question referred to the circle itself and instead they attempted to find the equation of the line segment from the centre to the point on the circumference.

In part (b), most candidates successfully attempted the gradient of the radius and gained the first mark for their correct value. The majority then went on to find the negative reciprocal and attempted the equation of the tangent. Those candidates using the $y=mx+c$ approach were more prone to making errors in their calculations and were also less likely to rearrange their equation to the required form with integer coefficients. The majority of candidates used the more conventional formula for the equation of a line and most carefully rearranged their equation to the required form to gain full marks. Common errors included mixing up *x* and *y* coordinates when finding the gradient of the radius or the equation of the tangent itself. A few candidates erroneously used the coordinates of the centre of the circle instead of the point on the circumference when attempting the equation of the tangent. A small minority attempted to find the gradient of the tangent using implicit differentiation, but attempts seen were generally weak and scored very few marks.

A good range of mark traits was seen, but full marks was the most common. Many candidates who had been unable to achieve any marks in part (a) still successfully accessed this part of the question.

Question 4

Overall, this question on binomial expansion was answered very well with the majority of candidates achieving at least 6 or 7 marks. It was extremely common for candidates to use the binomial expansion to find at least the first four terms before explicitly answering any part of the question. If done well, this achieved the first 3 marks before they had potentially even read the question.

Part(a) was answered well by the candidates. The vast majority gained the first mark by writing 243 or 3^5. Occasionally there were errors in the expansion resulting in 243 not being the only constant, thus losing the B mark.

Very few candidates scored no marks in (b)(i). Generally M1A1 was awarded for correct coefficients of $B = 405p$ and $D = 90p^3$. For those who did not achieve the A mark it was rarely down to errors in the binomial coefficient, occasionally it was incorrect powers of 3 but the most common cause of inaccuracy was removing the brackets around $(px)^3$ to incorrectly achieve px^3. The large majority of candidates substituted into $B = 18D$, however those with the bracketing errors on the *D* term were unable to reach $p^2 = ...$ If the candidate found *B* and *D* correctly, they almost always obtained $p = \pm\frac{1}{2}$. Sometimes the final A1 mark was lost because the negative value of *p* was not chosen, despite the given information that $p < 0$.

In part (c) nearly all candidates were able to gain the M1 and A1 marks by finding $C = \frac{135}{2}$, as the majority had found *p* to be either 0.5 or -0.5 in (b)(i). Although slips in calculating the binomial coefficient were condoned, those that made errors dealing with brackets or powers of 3 scored no marks here as they did not reach an acceptable expression for *C* to substitute into. It was pleasing to see that the candidates generally understood the term coefficient and knew that it had to be isolated from the x^2 to gain full marks, but this was not the case for all. There was a number of candidates who had *x* or x^2 as part of their value for *p* in (b), which was then being used within their method to find *C*. If a value was extracted, this was condoned, however candidates should really appreciate which part of a term is a coefficient, if they are not to potentially be penalised in future examinations.

Question 5

This question was examining general rules and understanding of logarithms. Nearly all candidates attempted the question with the majority demonstrating a high knowledge of logarithms and gaining nearly all or full marks.

The most common approach to the question was to combine all the logarithms on one side and equate to 3. Another approach used was to write 3 as $\log_2 8$ and combine the logarithms separately on the left- and right-hand sides. There were occasional non-creditworthy attempts where logarithms were not combined correctly e.g. by adding instead of multiplying and some limited attempts where candidates made little or no progress.

The B mark was normally gained for $\log_2\left(\frac{16x(x+1)}{x+6}\right)=3 \Rightarrow \frac{16x(x+1)}{x+6}=2^3$ or $3=\log_2 8$. There were some candidates who wrote 3^2 instead of 2^3 and these did not gain the B mark.
Most candidates successfully applied the addition or subtraction laws of logarithms at least once correctly and were awarded the first M mark. This was normally for $\log_2 16x+\log_2(x+1)=\log_2\left(16x(x+1)\right)$. The A mark was for achieving the correct 3TQ with terms collected, but not necessarily on the same side. The most common error here was poor multiplication of brackets with $16x(x+1)$ often incorrectly leading to $16x^2+16$. Some candidates did not achieve a three-term quadratic as they did not simplify their equation far enough. Candidates who had poor notation such as $\frac{\log\ldots}{\log\ldots}$ instead of $\log\left(\frac{\ldots}{\ldots}\right)$ also did not gain this mark. The dM1 mark could only be achieved if the previous M mark had been awarded and the candidate had achieved a three-term quadratic. Most candidates were able to solve their three-term quadratic with a mixture of calculator use, quadratic formula, and factorising. The use of calculator to solve the three-term quadratic seemed to be more prevalent than other methods. Some candidates solved a four-term quadratic using their calculator and even though this led to the correct answer it gained no credit as the candidate had not progressed sufficiently with the algebra. The final A mark could only be awarded for the answer $\frac{3}{2}$ provided all previous marks had been awarded. A significant number of candidates failed to reject the negative solution -2 and did not achieve this mark. Many of those scoring 4 or 5 marks in this question did so with clearly laid out work and good notation. Candidates and centres should continue to be aware of the requirement to show their working on all questions. This question specifically asked candidates to use the laws of logarithms so evidence of that was required. Methods which appear to arise from using the equation solver on a calculator are unlikely to be rewarded as highly as those who show more stages of their working, such as achieving three-term quadratic, which they then can go on and solve via any suitable method, including a calculator.

Question 6

This proved to be a particularly challenging question with around a quarter of candidates failing to score marks.

Part (a) was generally answered well, with most candidates correctly achieving the given answer with no incorrect working seen. A few errors in their algebraic manipulation cost some candidates, whilst those who formed a cubic equation often lost themselves in their algebra, unable to gain credit as they did not reach any quadratic equation.

In part (b), many candidates were able to identify $k=12$, and most were able to carry forward this value into the term equation. However, a lot of candidates did not then realise their answer should be in thousands, and incorrectly gave $\frac{64}{3}$. Some candidates did round to a decimal answer but again failed to realise the need to convert to thousands.

Candidates generally found part (c) the most challenging. A pleasing number of candidates were able to set up an acceptable inequality / equation, but there were issues with understanding the size of the number to use given that the model was already in thousands so 41 ended up being a final answer seen regularly. Manipulation of the initial equation was generally done well with only a few ending up with incorrect values such as 12^n. The use of logarithms was generally well executed with a mixture of successful approaches (typically taking logs of both sides but also $\log_{\frac{4}{3}}\left(\frac{1000}{9}\right)$ was seen). Only a small number failed to show sufficient workings and the vast majority worked in a base of 10.

Question 7

The question tested candidate skills in algebraic manipulation, their knowledge and understanding of calculus and, in particular, the nature of turning points. There were some excellent and succinct fully correct solutions. However, these were few and far between.

Many candidates struggled with fractional indices. Of the candidates who were successful in expanding the bracket in part (a), most were then able to correctly differentiate the formula for H in terms of t. A very common issue here was the relatively large number of candidates immediately replacing t with α and then differentiating with respect to α. A number of candidates attempted to differentiate using the product rule but were rarely fully successful. Thus part (a) was quite rarely completed satisfactorily. However, if they differentiated H correctly and set this equal to zero, most were able to carry out the necessary manipulation to arrive at the given equation. Some candidates did not realise the need to differentiate despite being asked to use calculus in the question.

Part (b) was generally answered correctly by most candidates as they were successful in solving a quadratic. A small number failed to reject the negative result and did not necessarily recover by picking the positive solution for use in part (c).

The most common error in part (c) was that candidates differentiated the equation in α, found from finding the stationary points, rather than finding the second derivative of H. Those candidates who correctly differentiated $\frac{\mathrm{d}H}{\mathrm{d}t}$ generally went on to substitute $\alpha = 4.491$ into the expression and gave an acceptable conclusion. There was a small number of candidates who looked at the derivative either side of the stationary point, but these candidates gained no credit as the question specified that further calculus work was needed.

Question 8

Proof again seemed to be a particular area that candidates struggled with on this paper. Whilst there were only 5 marks attributed to the question, which was split into two different questions, candidates rarely scored more than 2 marks in total.

In part (i), candidates needed to identify a pair of consecutive prime numbers whose squares when added were a number that was divisible by 10. This would then provide the necessary contradiction to the given statement. The majority of candidates could find an appropriate pair of primes, with 7 and 11 being the most often seen, but other larger pairs were also seen.

Unsuccessful attempts often used 1 as a prime number. Other unsuccessful attempts either did not use consecutive prime numbers or used one prime number with a non-prime number (9 was another common value seen). Occasionally there were errors in evaluation and candidates often lost the second mark because they were unable to give a suitable conclusion or gave no conclusion at all.

In part (ii) candidates needed to identify the four combinations of values to use to complete the proof by exhaustion. Many candidates could identify two combinations and correctly evaluate the given expression, however seeing all four combinations was much less often seen. Without four combinations evaluated, a maximum of one mark was available and that was the most common mark seen. A minority of candidates attempted algebraic manipulation to do this proof. This was not necessary due to the small number of possible values which could be used. This resulted in a lot of time being spent on this question with no credit worthy work seen.

Question 9

This question on trigonometric identities and equation solving was a good source of marks in both parts and a majority of candidates gained 5 out of 7 marks.

In part (a), the majority of candidates identified the need to use the identity $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ but a number of candidates were less successful in multiplying the expression $\dfrac{\sin\theta}{\cos\theta}\sin\theta$ by $3\cos\theta$ and achieved $3\cos\theta\dfrac{\sin\theta}{\cos\theta}3\cos\theta\sin\theta$. Most of the candidates who managed to reach $3\cos\theta\left(\dfrac{\sin^2\theta}{\cos\theta}+3\right)=...$ realised that they must use the other identity $\sin^2\theta+\cos^2\theta=1$ in order to get an equation in $\cos\theta$ only, but at this stage of their work there was some evidence of disappointing mistakes, one of which was the use of the incorrect identity $\sin\theta = 1-\cos\theta$. There were a considerable number of poor notation errors most notably a lack of understanding that $\cos^2\theta \neq \cos\theta^2$ or some just wrote sin or cos without θ, which led to the loss of the final mark.

In part (b), although a small number of candidates did not use the printed result in part (a) and unnecessarily attempted to restart, the majority could apply the result from (a) and solve the resulting quadratic in $\cos 2x$, usually using their calculators or by factorising. A small number of candidates solved the quadratic using the formula or by completing square. Most candidates directly rejected the other root whilst others tried to find an angle and indicated there was an error. A lot of candidates failed to find an angle for x as they had not appreciated that they needed to divide their value(s) by 2. Many who achieved $x = 48.2$ and $x = 311.8$ as their answer lost 3 marks. A number of candidates thought that solving $\cos 2x = \dfrac{2}{3}$ was the same as solving $\cos x = \dfrac{1}{3}$ and lost the 3 marks. Most candidates who managed to calculate the first solution $x = 24.1$, calculated the second solution $x = 155.9$ but did not continue to find the other two solutions as they did not appreciate that the range of $2x$ is between 0 and 720. The candidates who reached all 4 solutions mostly used the CAST diagram and a few sketched useful graphs.

Question 10

There were some efficient and completely correct solutions for this problem, but they were a minority. The question was not well answered by the majority of candidates, with a significant number not attempting any part. Full marks were rarely seen. Many candidates struggled with part (a), then seemed to give up, when they might have had some success with the rest had they persevered.

Overall, there were a lot of errors in part (a) meaning that few got the accuracy marks but many got the method marks. For the first M mark, most candidates correctly multiplied out the brackets, but a surprisingly large number of candidates struggled to correctly simplify the expression before integrating. There were a number of errors in multiplying out and splitting into separate terms:

- some did not combine the indices correctly on all the terms; it was quite common to see the negative power omitted and two terms in $x^{\frac{1}{2}}$
- many multiplied by $x^{\frac{1}{2}}$ rather than $x^{-\frac{1}{2}}$.
- a few missed off the k^2 at the end and just wrote k.
- some candidates added on a fourth term of $x^{-\frac{1}{2}}$, or only multiplied the final term.
- k was often 'combined' with the x terms using incorrect laws of indices. For example, $k^2x^{-\frac{1}{2}}$ sometimes became $k^{1.5}$.

Those candidates who were successful in achieving the first M mark were often able to gain at least the next M mark through one term being correctly integrated, although there were many errors including:

- a substantial minority differentiated instead of integrating.
- many mistakes were made dealing with the integration. In particular, the fractional powers caused a lot of problems.
- many raised the power by 1 on the first term, but then multiplied by $\frac{5}{2}$, for example, rather than dividing.
- a few tried to integrate the numerator and denominator separately.
- k was often treated as a variable and integration was often attempted with respect to k, for both terms involving k.
- some candidates multiplied the $x^{\frac{1}{2}}$ term by 0.5 instead of diving by 0.5.
- quite a few candidates simply forgot to integrate at all.

Generally the limits of 16 and 1 were substituted in correctly and then subtracted, but the following mistakes were common:

- many mistakes were made in the subtraction, often due to bracketing issues, so only a small number of candidates achieved the correct answer. For example, $10k^2$ was often seen instead of $6k^2$ as a result of a subtraction error.
- a small number used a lower limit of 0 instead of 1.
- several candidates put the limits into their expression for *y*, without integrating. This was sometimes after gaining the first M for combining indices etc, but sometimes immediately using the given expression for *y*.

Many candidates only attempted part (a), but for those who tried part (b), it was often carried out well. However, a considerable number of candidates did not know how to start, despite being given the coordinates of point *A* at this point in the question. Those who realised that the equation of the curve must be used generally had no problems in correctly finding the value of *k*. Overall, there were relatively few problems substituting 1 and 9 into the equation of the curve to achieve $(1-k)^2 = 9$. Subsequently, multiplying out the brackets to form a three-term quadratic was the most common and most successful method.

Common errors were:

- a few set the expression they obtained in part (a) equal to zero in their attempt to find *k*.
- those who square rooted often forgot the $\pm$ to achieve $1-k=3$, before proceeding incorrectly to the given answer of $k=4$.
- some tried to find the gradient between *A* and *B*.

The solution $k=-2$ was usually rejected correctly.

There was a limited number of attempts at part (c) of the question, with the large number of blank responses and poorly attempted efforts showing a lack of confidence dealing with brackets and fractional indices. However, for those candidates who got this far, they often realised that the required area could be found using the area under the line minus the area under the curve and some good attempts were made to do this. Very few subtracted the wrong way round. The majority who tried this part attempted to find the equation of *AB* and then attempted to use integration to find the trapezium area. There were a few efficient candidates who quoted the formula for area of a trapezium. Very few candidates used rectangles and triangles. Despite the demand in the question, some candidates did not realise that they should use their answer to part (a) and attempted to integrate the equation of the curve again using $k=4$ and so received a maximum of B1. Finding q was often not attempted. However, many candidates who did reach $q=36$, often had this hidden within their working for the equation of the straight line.

Some other errors that occurred included:

- rven for some candidates attempting to use $y=\dfrac{(x-k)^2}{\sqrt{k}}$, they were unable to proceed to the correct value for *q*, possibly due to confusion between *k* and *x*.

- many worked out the area using their calculator. However, as this often did not match with their answer to part (a), they received no credit.
- some candidates failed to complete the calculation by subtracting the area under the curve from their area of the trapezium.

Question 11

This was a very discriminating question on recurrence relationships with nearly a quarter scoring no marks, but nearly a third scoring full marks.

In part (a) candidates were required to find the second and third terms of a sequence defined by a recurrence relationship, giving the answers in terms of *a* and *b*. This was done well by the majority of candidates and both marks were usually obtained. Some candidates went on to incorrectly multiply out the brackets in the expression for u_3, but they still gained the mark as their subsequent working was ignored for this mark. There was a small, but significant number of candidates who did not seem familiar with recurrence relationships and they either did not try the question at all, or their attempt was incorrect.

Part (b) required candidates to show how a quadratic equation in terms of *a* is achieved, given the sum of the first three terms was 153, and $b = a + 9$. Many candidates attempted part (b) successfully. Most set the sum of their first three terms (which were themselves in terms of *a* and *b*) equal to 153, replaced *b* with *a* + 9 and went on to form the given quadratic equation. This approach generally led to full marks, although a sign slip when multiplying out a bracket was the main cause for those who lost the accuracy mark. Other candidates first replaced *b* with *a* + 9 in the expressions for their terms and then added them together, set this equal to 153 and proceeded to the given result. Some candidates did not seem sure how to approach the question and simply solved the given quadratic equation. A small number of candidates instead found an expression for the fourth term and set that equal to 153.

Part (c) required candidates to find the larger value of the second term. Candidates who attempted this part broadly fell into three main groups. A pleasing number of candidates were able to execute this part of the question very efficiently and gain full marks, the scaffolding to assist them having been put in place in parts (a) and (b). A few neglected to reject a second solution, thus losing them the final mark. Another large number of candidates realised that they needed to solve the given quadratic equation, usually gaining the first mark for $a = -6$ but they were unsure how to proceed. Many from this group erroneously used only their larger value for *a* which gained them no credit. Some adopted a mixed approach, using both values for *a* to generate values for *b*, but then mixing up the pairings when substituting into the expression $b - 3a$ leading to incorrect values for their third term. The third group of candidates made no attempt at part (c). Some of this group had already solved the quadratic equation in part (b), but this gained them no credit as $a = -6$ had to be seen or used in part (c).

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom