Pearson Edexcel

INTERNATIONAL ADVANCED LEVEL

MATHEMATICS/ FURTHER MATHEMATICS/ PURE MATHEMATICS

EXEMPLARS

In this document you will find International A Level Mathematics Exemplars produced, Pre-First-Assessment, from the Sample Assessment Materials.

Highlighted in this document is the Pure Mathematics 1 exam paper.

As there is a lot of content overlap in content between the Legacy and Redeveloped Mathematics specifications, the purpose of these exemplars is to highlight the new structure of the Redeveloped Exams.
With that in mind there will not be exemplars produced Pre-First-Assessment for other units.

We hope that centres and candidates find these useful in preparing for the 2019 May/June Series.

1. Given that $y = 4x^3 - \frac{5}{x^2}$, $x \neq 0$, find in their simplest form

(a) $\frac{dy}{dx}$,

(3)

(b) $\int y \, dx$

(3)

Examiner Comment: *This question assesses the ability of the candidate to differentiate and integrate. Care needs to be taken in converting* $\frac{5}{x^2}$ *into index form and many candidates forget to add the constant in part (b).*

Question	Scheme	Marks
1(a)	$y = 4x^3 - \frac{5}{x^2}$	
	$x^n \to x^{n-1}$ e.g. sight of x^2 **or** x^{-3} **or** $\frac{1}{x^3}$	M1
	$3 \times 4x^2$ **or** $-5 \times -2x^{-3}$ (o.e.) (Ignore + c for this mark)	A1
	$12x^2 + \frac{10}{x^3}$ **or** $12x^2 + 10x^{-3}$ <u>all on one line</u> and no + c	A1
		(3)
(b)	$x^n \to x^{n+1}$ e.g. sight of x^4 **or** x^{-1} **or** $\frac{1}{x^1}$	M1
	Do <u>not</u> award for integrating their answer to part (a) $4\frac{x^4}{4}$ **or** $-5 \times \frac{x^{-1}}{-1}$	A1
	For fully correct and simplified answer with + c <u>all on one line.</u> Allow $\Rightarrow$ Allow $x^4 + 5 \times \frac{1}{x} + c$ $\Rightarrow$ Allow $1x^4$ for x^4	A1
		(3)
		(6 marks)

Response 1

a) $\frac{dy}{dx} = 12x^2 + 10x^{-3}$

b) $\int y = x^4 + 5x^{-1} + c$

5/5

A completely correct solution. Both parts are simplified and the candidate has remembered the constant of integration in part (b).

Response 2

1.(a) $\frac{dy}{dx} = 12x^2 + \frac{10}{x^3}$

(b) $\int y\,dx = x^4 + \frac{5}{x}$

4/5

Part (a) is completely correct. Part (b) is missing the constant of integration therefore losing the final mark.

Response 3

$$y = 4x^3 - 5x^{1/2}$$

a) $\frac{dy}{dx} = 9x^2 + 2\frac{1}{2}x^{-1/2}$

b) $\int = \frac{4x^4}{3} - \frac{5x^{1\frac{1}{2}}}{\frac{1}{2}}$

$= \frac{4}{3}x^4 - 10x^{1\frac{1}{2}} + c$

2/5

This candidate incorrectly converts $\frac{5}{x^2}$ *into* $5x^{\frac{1}{2}}$*. In part (a) they score the method mark for differentiating* x^3 *to* x^2*. Neither term is correct so only one mark is scored. Similarly, in part (b), they score the method mark for integrating* x^3 *to* x^4 *but neither term is correct.*

2. (a) Given that $3^{-1.5} = a\sqrt{3}$ find the exact value of a

(2)

(b) Simplify fully $\dfrac{(2x^{\frac{1}{2}})^3}{4x^2}$

(3)

Examiner comment: *This question assesses the relationship between surds and indices. Part (a) may be tackled in a variety of ways. A candidate may attempt to re-write* $3^{-1.5}$ *in the form of the right hand side, or perhaps, a more straightforward method is to treat* $3^{-1.5} = a\sqrt{3}$ *as an equation and solve for a. Mistakes are often made in questions like 2(b). Candidates often fail to cube all of the terms on the numerator and make slips when dividing, especially if the coefficient of the denominator is larger than that of the numerator.*

Question	Scheme		Marks
2(a)	$3^{-1.5} = \dfrac{1}{3\sqrt{3}}\left(\dfrac{\times\sqrt{3}}{\times\sqrt{3}}\right)$		M1
	$= \dfrac{\sqrt{3}}{9}$ so $a = \dfrac{1}{9}$		A1
			(2)
	Alternative		
	$3^{-1.5} = a\sqrt{3} \Rightarrow a = \dfrac{3^{-1.5}}{3^{0.5}} = 3^{-1.5-0.5}$		M1
	$\Rightarrow a = 3^{-2} = \dfrac{1}{9}$		A1
(b)	$\left(2x^{\frac{1}{2}}\right)^3 = 2^3 x^{\frac{3}{2}}$	One correct power either 2^3 or $x^{\frac{3}{2}}$.	M1
	$\dfrac{8x^{\frac{3}{2}}}{4x^2} = 2x^{-\frac{1}{2}}$ or $\dfrac{2}{\sqrt{x}}$		dM1 A1
			(3)
			(5 marks)

Response 1

2. (a) Given that $3^{-1.5} = a\sqrt{3}$ find the exact value of a

(2)

(b) Simplify fully $\dfrac{(2x^{\frac{1}{2}})^3}{4x^2}$

(3)

a) $3^{-\frac{3}{2}} = \dfrac{1}{3^{\frac{3}{2}}} = \dfrac{1}{(\sqrt{3})^3} = \dfrac{1}{3\sqrt{3}} = \dfrac{1}{9}\sqrt{3}$

$a = \dfrac{1}{9}$

b) $\dfrac{8x^{\frac{3}{2}}}{4x^2}$ $\quad 2x^{-\frac{1}{2}}$

5/5

A completely correct solution. In part (a) the candidate uses correct index laws on $3^{-\frac{3}{2}}$ *and then rationalises the denominator to correctly deduce that* $a = \frac{1}{9}$. *In part (b) both terms of the numerator are cubed before the expression is correctly simplified.*

Response 2

(3)

2(a) $3^{-\frac{3}{2}} = a\sqrt{3}$ $\quad a = \frac{1}{9}$

$\frac{1}{\sqrt{3^3}} = a\sqrt{3}$

$\frac{1}{\sqrt{27}} = a\sqrt{3}$

$\frac{1}{3\sqrt{3}} = a\sqrt{3}$

$\frac{\sqrt{3}}{9} \cancel{a} = a\sqrt{3}$

(b) $\frac{(2x^{\frac{1}{2}})^3}{4x^2}$

$= \frac{(2\sqrt{x})^3}{4x^2}$

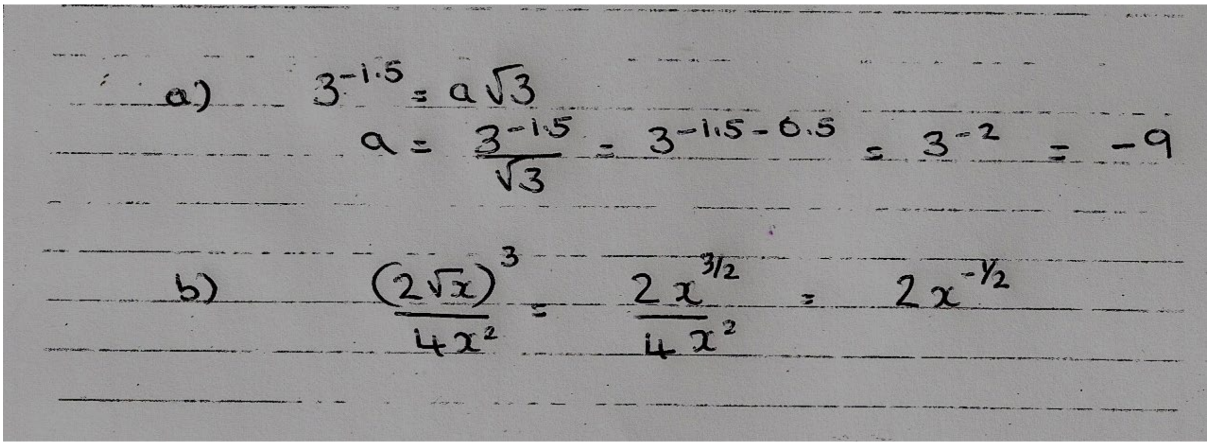
$= \frac{8x\sqrt{x}}{4x^2}$

$= \frac{2\sqrt{x}}{x}$

Part (a) is completely correct with the candidate deducing that $a = \frac{1}{9}$.

Part (b) The first mark can be awarded for 2^3 *which is implied by the "8" in the numerator. To score the second mark there needs to be a correct attempt to combine the terms in x. This is not done so only 1 out of 3 is scored in part (b).*

Response 3



a) $3^{-1.5} = a\sqrt{3}$

$a = \frac{3^{-1.5}}{\sqrt{3}} = 3^{-1.5-0.5} = 3^{-2} = -9$

b) $\frac{(2\sqrt{x})^3}{4x^2} = \frac{2x^{3/2}}{4x^2} = 2x^{-1/2}$

3/5

Part (a) scores 1 out of 2. This candidate attempts to solve the equation in a and correctly achieves $a = 3^{-2}$. *The final A mark is not awarded as* 3^{-2} *is not processed correctly.*

Part (b) scores 2 out of 3. This is an example where the candidate has only cubed the term in x. So M1 is scored for $x^{\frac{3}{2}}$ *and the dM1 may be awarded for correctly combining their x terms to produce* $x^{-\frac{1}{2}}$. *Whilst the A1 could not be achieved due to the initial error, note that this candidate also fails to process the coefficients correctly.*

3. Solve the simultaneous equations

$$y + 4x + 1 = 0$$

$$y^2 + 5x^2 + 2x = 0$$

(6)

Examiner comment: *It is always a wise first move to make x or y the subject of the linear equation. From there a quadratic equation in x or y can be produced and then solved. Candidates should always remember to find both the x and the y values, and correctly pair them up. A common reason for losing marks in such a question is that often only the x values are found.*

Question	Scheme		Marks
3	$y = -4x - 1$ $\Rightarrow (-4x-1)^2 + 5x^2 + 2x = 0$	Attempts to makes y the subject of the linear equation and substitutes into the other equation.	M1
	$21x^2 + 10x + 1 = 0$	Correct 3 term quadratic	A1
	$(7x+1)(3x+1) = 0 \Rightarrow (x =) -\frac{1}{7}, -\frac{1}{3}$	dM1: Solves a **3 term** quadratic by the usual rules A1: $(x =)\ -\frac{1}{7}, -\frac{1}{3}$	dM1A1
	$y = -\frac{3}{7}, \frac{1}{3}$	M1: Substitutes to find at least one y value A1: $y = -\frac{3}{7}, \frac{1}{3}$	M1 A1
			(6)

Response 1

3. Solve the simultaneous equations

$$y + 4x + 1 = 0$$

$$y^2 + 5x^2 + 2x = 0$$

(6)

$[-(1+4x)]^2$

~~(...)(x+1)~~

$(7x+1)(3x+1)$

According to $y+4x+1=0$

$y=-1-4x$

$(-1-4x)^2+5x^2+2x=0$

$1+8x+16x^2+5x^2+2x=0$

$21x^2+10x+1=0$

$(7x+1)(3x+1)=0$

$\begin{cases} x_1 = -\frac{1}{7} \\ y_1 = -\frac{3}{7} \end{cases}$ $\begin{cases} x_2 = -\frac{1}{3} \\ y_2 = \frac{1}{3} \end{cases}$

6/6

This is a completely correct solution. All steps of the process are shown and the final answers to the equations are correctly paired up.

Response 2

$$y = (-4x-1)$$
$$(-4x-1)^2 + 5x^2 + 2x = 0.$$
$$16x^2 + 8x + 1 + 5x^2 + 2x = 0$$
$$21x^2 + 10x + 1 = 0.$$
$$3x \qquad 1$$
$$7x \qquad 1$$
$$(3x+1)(7x+1) = 0$$
$$x_1 = -\frac{1}{3}$$
$$x_2 = -\frac{1}{7}$$

4/6

The first 4 marks of the question are achieved as the candidate combines the two given equations to firstly form and then correctly solve the quadratic equation $21x^2 + 10x + 1 = 0$
The omission of the corresponding values of y is surprisingly common and loses the final two marks in the question.

Response 3

$y = -4x + 1 \qquad (-4x+1)^2 + 5x^2 + 2x = 0$

$16x^2 - 8x + 1 + 5x^2 + 2x = 0$

$21x^2 - 6x + 1 = 0$

$(7x-1)(3x-1) = 0$

$x = 1/7, \ 1/3$

$x = 1/7 \quad y = 3/7 \qquad x = 1/3, \ y = -1/3$

3/6

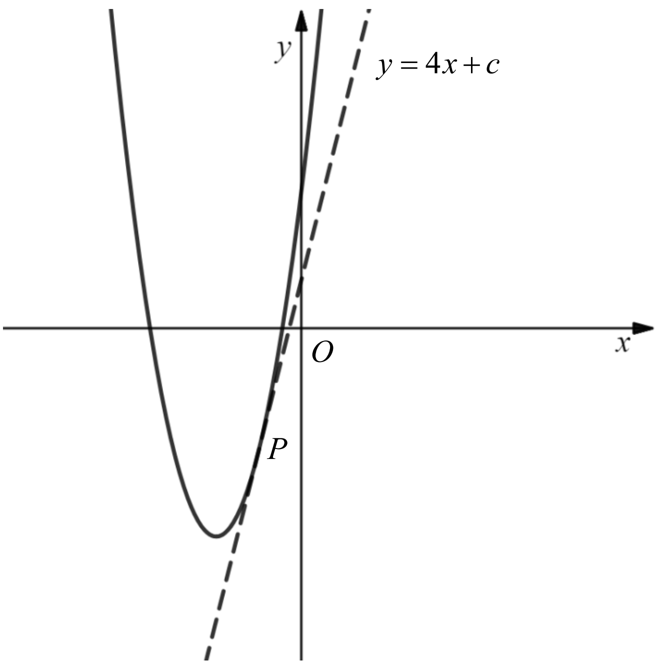
The candidate is awarded the three method marks in this question. The first M mark may be awarded as there is an attempt to make y the subject of $y + 4x + 1 = 0$ *and substitute into the second equation. The dM1 is awarded as it satisfies the rules for an attempt at factorisation. The final M is awarded as the y values are found from their x values using* $y = -4x + 1$. *No accuracy marks can be awarded in this question as all equations and values are incorrect.*

4. The straight line with equation $y = 4x + c$, where c is a constant, is a tangent to the curve with equation $y = 2x^2 + 8x + 3$

Calculate the value of c

(5)

Examiner comment: *This question may be attempted via two different methods. One way would be to combine the two equations to form an equation in x and then use the discriminant condition* $b^2 - 4ac = 0$ *as the resulting equation would only have one root. An alternative would be to find the point P on the curve* $y = 2x^2 + 8x + 3$ *with gradient 4 using differentiation. Sketching a graph like the one below will aid a candidates understanding of what is required.*



Question	Scheme	Marks
4	Sets $2x^2 + 8x + 3 = 4x + c$ and collects x terms together	M1
	Obtains $2x^2 + 4x + 3 - c = 0$ o.e.	A1
	States that $b^2 - 4ac = 0$	dM1
	$4^2 - 4\times2\times(3 - c) = 0$ and so $c =$	dM1
	$c = 1$ **cso**	A1
		(5)
	Alternative 1A	
	Sets derivative "$4x + 8$" $= 4 \Rightarrow x =$	M1
	$x = -1$	A1
	Substitute $x = -1$ in $y = 2x^2 + 8x + 3$ $(\Rightarrow y = -3)$	dM1
	Substitute $x = -1$ and $y = -3$ in $y = 4x + c$ or into $(y + 3)=4(x + 1)$ and expand	dM1
	$c = 1$ or writing $y = 4x + 1$ **cso**	A1
		(5)

Response 1

$4x+c = 2x^2+8x+3$

$-2x^2 -4x -3+c = 0.$

$2x^2+4x+3-c=0.$

$b^2-4ac=0.$

$16-4\times2\times(3-c)=0$

$16-24+8c=0$

$8c=+8$

$c=+1$

5/5

A completely correct solution via the "discriminant" method.

Response 2

$\frac{dy}{dx} = 4x+8 \qquad 4x+8=4 \Rightarrow 4x=12$

$\Rightarrow x=3$

When $x=3$ $\quad y = 18+24+3$

$= 45$

Sub $(3,45)$ into $y = 4x+c$

$45 = 12+c$

$c = 33$

3/5

This is an attempt using differentiation scoring 3 out of 5. The first M is scored for setting the gradient equal to 4 but a slip means that the accuracy mark is not achieved. The next two M's may be awarded as the method to find the value of c is correct. The error in finding the value of x means that the value of c is incorrect. Note: A decent sketch graph would have notified a candidate that this value of c is incorrect.

Response 3

$y = 4x+c.$

$\cancel{4x+c}$

$\frac{dy}{dx} = 4x+8. = 4x+c$

$c=8.$

0/5

This response scores 0 marks. Although the candidate differentiates correctly they equate the gradient function $4x+8$ *with* $4x+c$ *rather than 4.*

5. (a) On the same axes, sketch the graphs of $y = x + 2$ and $y = x^2 - x - 6$ showing the coordinates of all points at which each graph crosses the coordinate axes.

(4)

(b) On your sketch, show, by shading, the region R defined by the inequalities

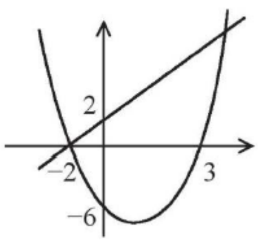
$$y < x + 2 \quad \textbf{and} \quad y > x^2 - x - 6$$

(1)

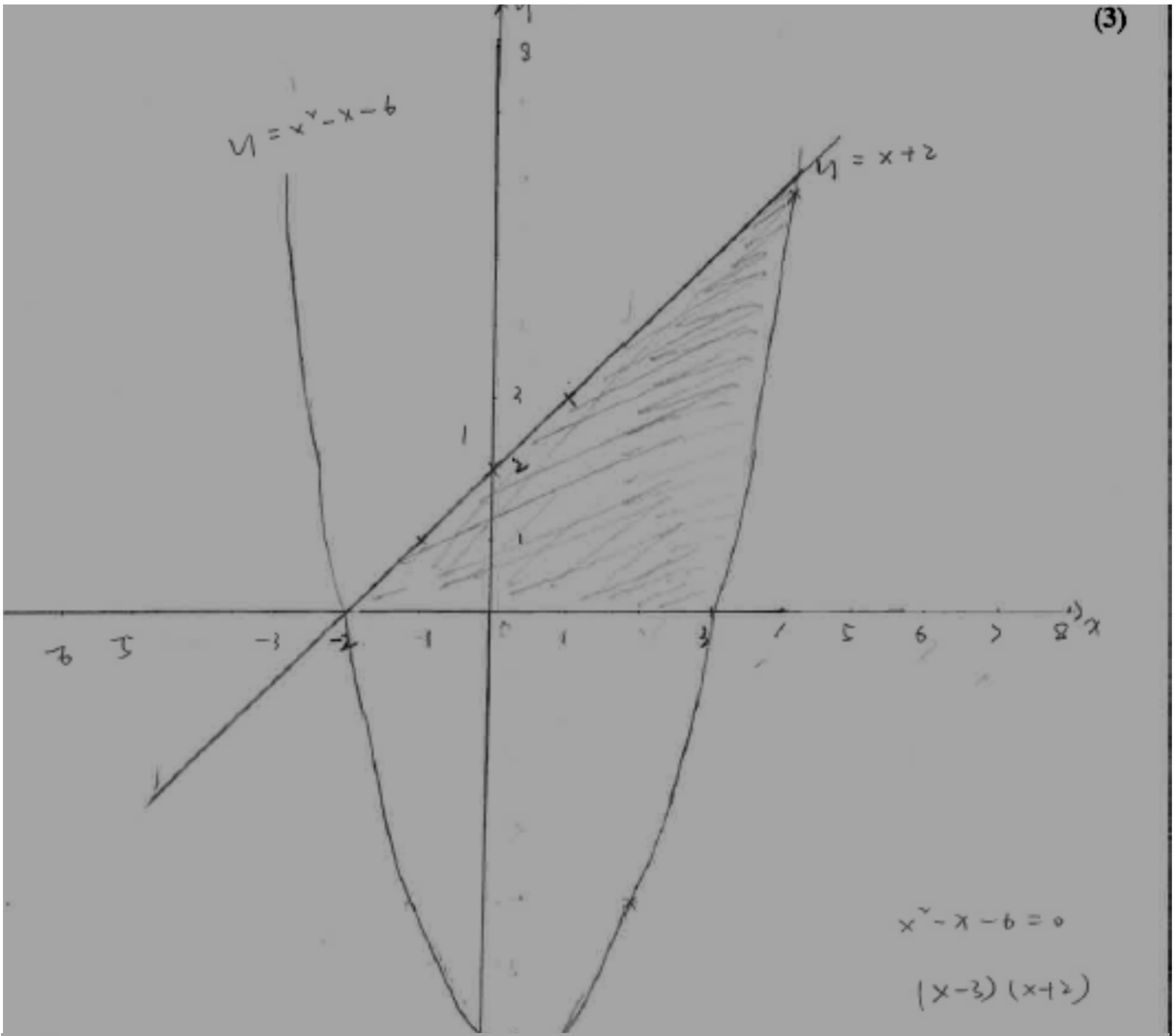
(c) Hence, or otherwise, find the set of values of x for which $x^2 - 2x - 8 < 0$

(3)

Examiner comment: *In part (a) care needs to be taken to ensure that all points of intersection are included on the sketch. Part (b) is a new topic to the course. In this case we are looking for the region below the line and above the curve. Part (c) was misinterpreted by a number of candidates so it is important to read the question very carefully.*

Question			Marks
5(a)		Straight line, positive gradient positive intercept	B1
		Curve 'U' shape anywhere	B1
		Correct y intercepts 2, -6	B1
		Correct x-intercepts of -2 and 3 with intersection shown at $(-2, 0)$	B1
			(4)
(b)	Finite region between line and curve shaded		B1
			(1)
(c)	$(x^2 - x - 6 < x + 2) \Rightarrow x^2 - 2x - 8 < 0$		
	$(x-4)(x+2) < 0 \Rightarrow$ Line and curve intersect at $x = 4$ and $x = -2$		M1 A1
	$-2 < x < 4$		A1
			(3)
			(8 marks)

Response 1



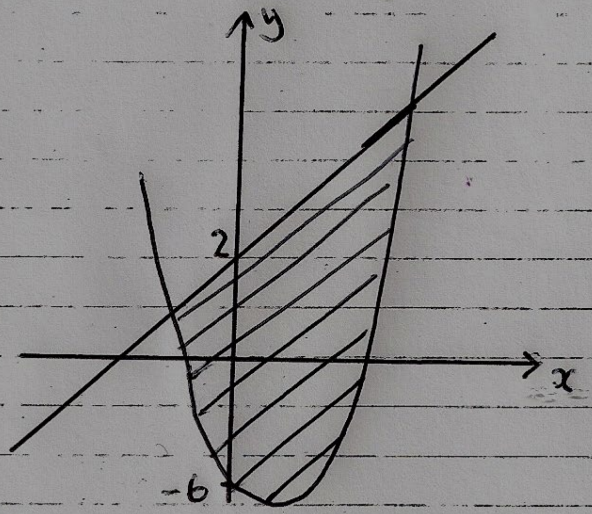
c) $-2 < x < 4$

7/8

Part (a) and part (c) are completely correct. In part (b) the candidate does not shade the correct region.

Response 2

(a)



(c) $x^2-x-6<x+2 \Rightarrow x^2-2x-8<0$

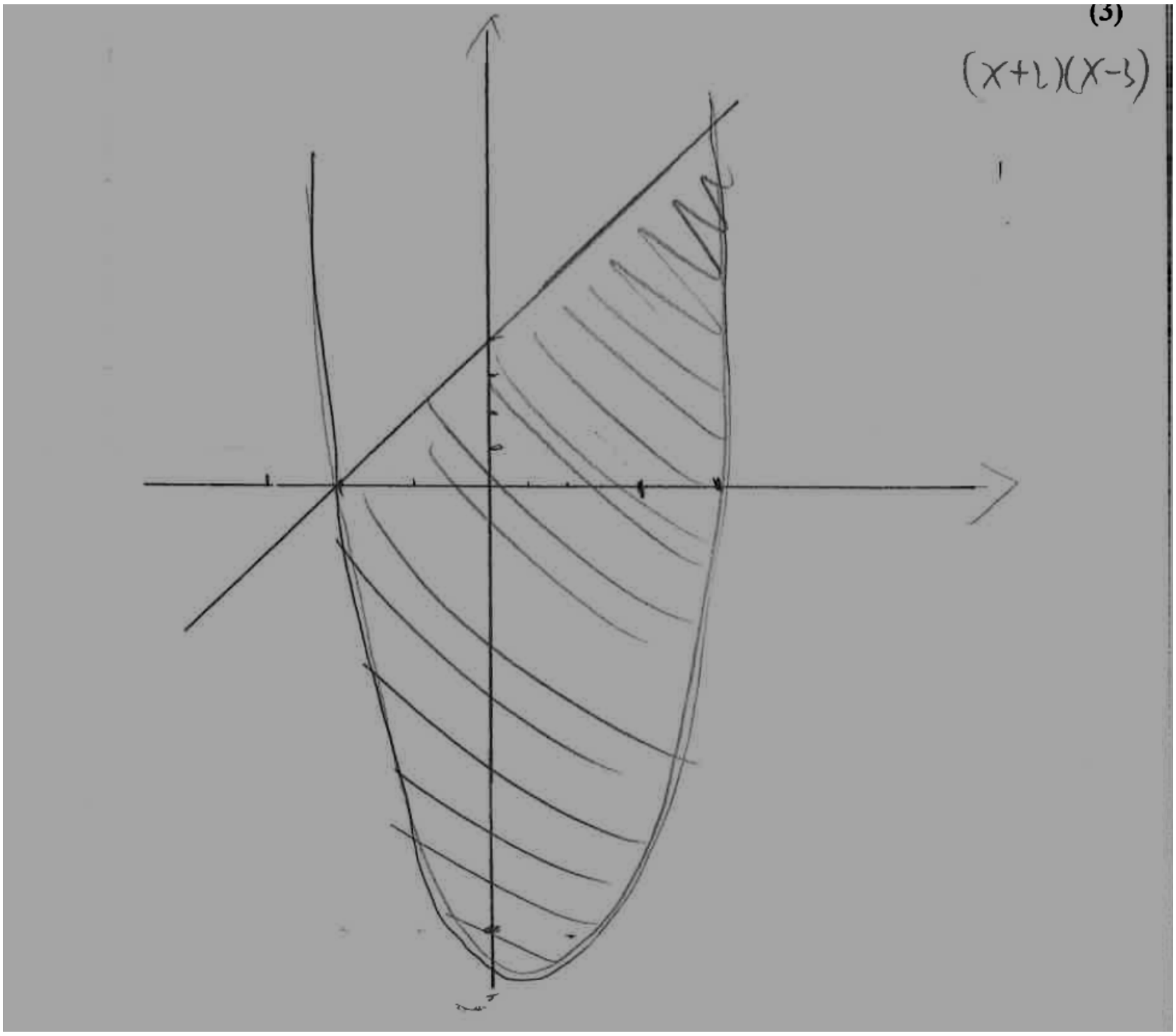
$(x-4)(x+2)<0$

$x<-2$ and $x<4$

6/8

Part (a) scores 3 out of 4. Both the "shape" of the line and the "shape" of the curve are correct. The final B1 mark is withheld as the x-intercepts are not given (and they do not intersect at $(-2,0)$). In part (b) the correct region is shaded. In part (c) 2 of the 3 marks can be awarded as the correct critical values are found but the final inequality is incorrect.

Response 3



3/8

Part (a) scores 2 out of 4. The shape for both the line and the curve are correct. No intercepts are given so neither B mark may be awarded. In part (b) the candidate shades the correct region. The response in part (c) was common. The answer $-2 < x < 3$ *is the solution to the inequality* $x^2 - x - 6 < 0$

6.

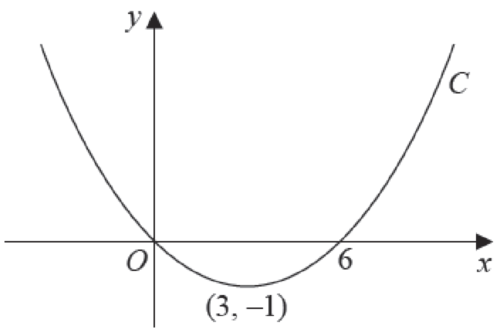


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$

The curve C passes through the origin and through (6, 0)

The curve C has a minimum at the point (3, –1)

On separate diagrams, sketch the curve with equation

(a) $y = \mathrm{f}(2x)$

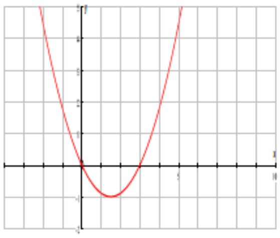
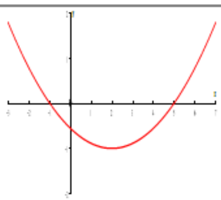
(3)

(b) $y = \mathrm{f}(x + p)$, where p is a constant and $0 < p < 3$

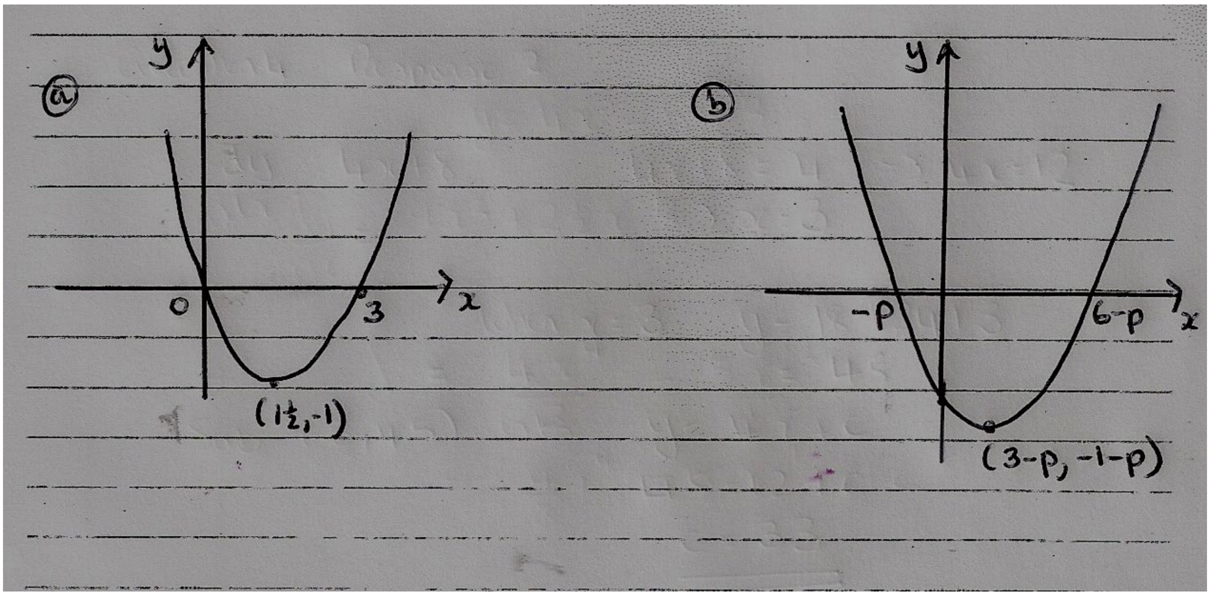
(4)

On each diagram show the coordinates of any points where the curve intersects the x-axis and of any minimum or maximum points.

Examiner Comment: *Part (a) is a fairly standard question testing transformations. In this case a stretch* $\times\frac{1}{2}$ *in the* x *direction. Candidates must be remember to halve all* x *coordinates but keep the y coordinates the same. Part (b) is more demanding due to the use of a constant p. The transformation is a translation to the left of p units (as* $p > 0$*), and since* $p < 3$, *the minimum point will remain in quadrant 4.*

Question	Scheme		Marks
6(a)		Shape $\cup$ through (0, 0)	B1
		(3, 0)	B1
		(1.5, –1)	B1
			(3)
(b)		Shape $\cup$, <u>not</u> through (0, 0)	B1
		Minimum in 4th quadrant	B1
		$(-p, 0)$ and $(6-p, 0)$	B1
		$(3-p, -1)$	B1
			(4)
			(7 marks)

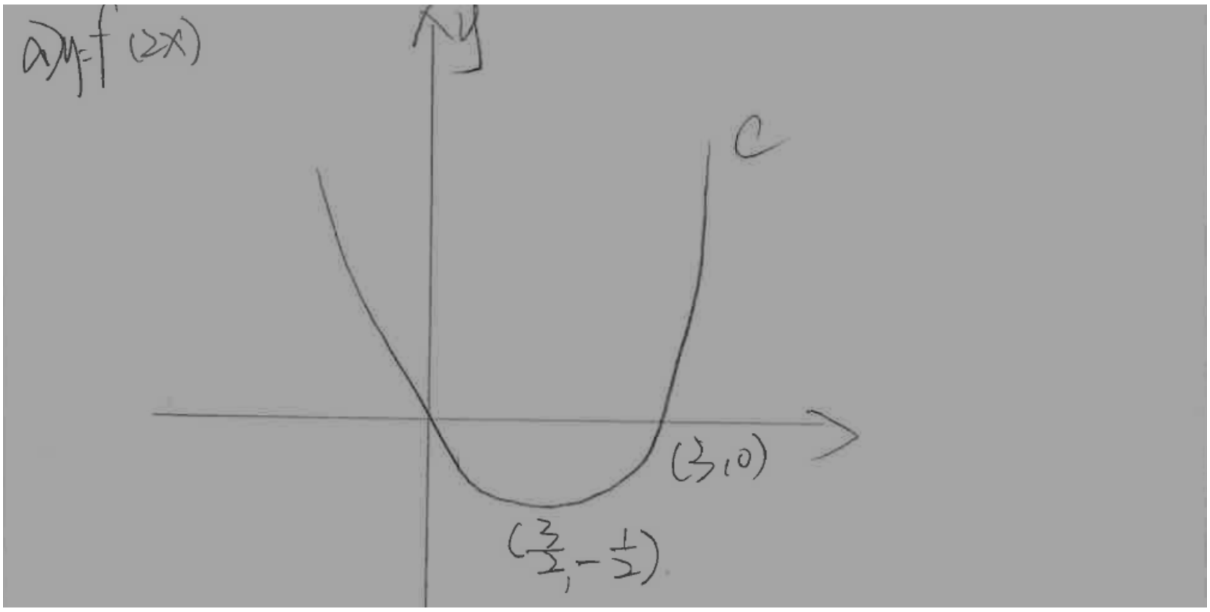
Response 1

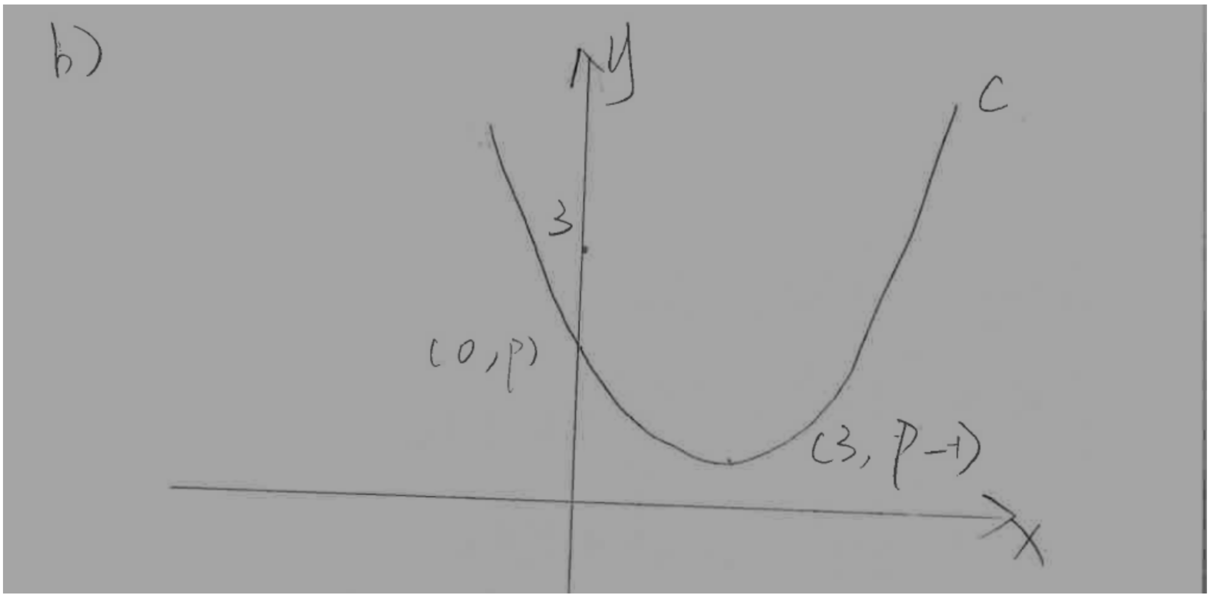


6/7

Part (a) is completely correct. Part (b) scores 3 of the 4 marks. A mark is withheld as the y coordinate of the minimum point is incorrect.

Response 2



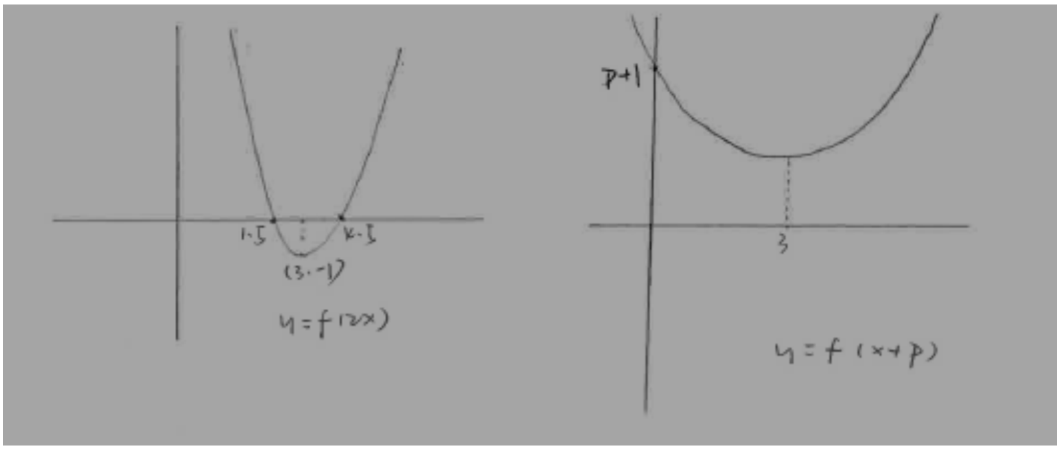


3/7

In part (a) the first two marks may be awarded for a correct shape through $(0,0)$ *and the correct x - intercept of* $(3,0)$ *is given. The last mark in this part could not be awarded as the coordinates of the minimum point is incorrect.*

In part (b) only the first mark may be awarded for a U-shaped curve not passing through $(0,0)$.

Response 3



1/7

Part (a) is completely incorrect and cannot be awarded any marks. The candidate appears to have stretched the graph by a scale factor of $\frac{1}{2}$ *but from the line* $x = 3$

In part (b) only the first B mark may be awarded as a U-shaped curve is drawn that does not pass through $(0,0)$.

7. A curve with equation $y = \mathrm{f}(x)$ passes through the point (4, 25)

Given that

$$\mathrm{f}'(x) = \frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1, \qquad x > 0$$

find f(x), simplifying each term.

(5)

Examiner comment: *This is a fairly standard question on integration. Care needs to be taken with index work and candidates sometimes have problems with simplifying the coefficients, especially when fractions are involved. Another place where marks are lost is when the constant of integration is not added and the point* $(4, 25)$ *is never used.*

Question	Scheme	Marks
7	$\mathrm{f}(x) = \int \left(\frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1 \right) \mathrm{d}x$	
	$x^n \to x^{n+1} \Rightarrow \mathrm{f}(x) = \frac{3}{8} \times \frac{x^3}{3} - 10\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + x(+c)$	M1 A1 A1
	Substitute $x = 4, y = 25 \Rightarrow 25 = 8 - 40 + 4 + c$ $\Rightarrow c =$	M1
	$\mathrm{f}(x) = \frac{x^3}{8} - 20x^{\frac{1}{2}} + x + 53$	A1
		(5)
		(5 marks)

Response 1

(5)

$$f(x) = \int f'(x) + c.$$

$$f(x) = \frac{\frac{3}{8}x^3}{3} - \frac{10x^{\frac{1}{2}}}{\frac{1}{2}} + x + c.$$

$$= \frac{1}{8}x^3 - 20x^{\frac{1}{2}} + x + c$$

$$25 = \frac{1}{8} \times 4^3 - 20 \times 4^{\frac{1}{2}} + 4 + c$$

$$25 = 8 - 40 + 4 + c$$

$$c = 53.$$

$$f(x) = \frac{1}{8}x^3 - 20x^{\frac{1}{2}} + x + 53.$$

5/5

A completely correct response.

Response 2

$$f'(x) = \frac{3}{8}x^2 - 10x^{-1/2} + 1$$

$$f(x) = \frac{3}{8}\frac{x^3}{3} - 10\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$f(x) = \frac{1}{8}x^3 - 20x^{1/2} + c$$

$(4,25)$ $\quad 25 = 8 - 40 + c$

$$c = 25 + 32 = 57$$

$$\underline{f(x) = \frac{1}{8}x^3 - 20x^{1/2} + 57}$$

3/5

This solution scores 3 out of 5 marks. M1 A1 is achieved for integrating two terms correctly. The third mark cannot be awarded as 1 should be integrated to x. There is a constant of integration and the $(4,25)$ is used correctly to find a value for c.

Response 3

$$F(X) = \frac{\cancel{3}}{8}\frac{X^3}{\cancel{3}} + 20X^{\frac{1}{2}} + X$$

$$= \frac{1}{8}X^3 + 20\sqrt{X} + X$$

2/5

The term in x^3 is correct scoring M1 A1. As only two terms are correct the next A1cannot be awarded. As there is no attempt to use $(4,25)$ with a constant of integration no more marks can achieved.

8.

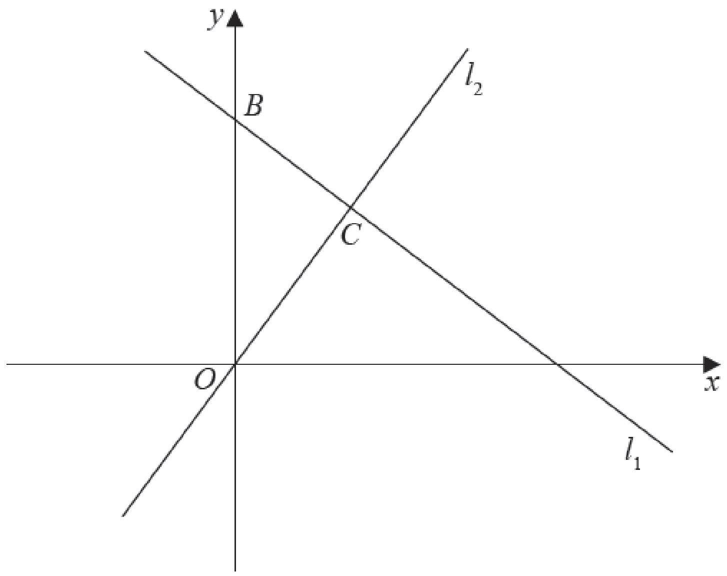


Figure 2

The line l_1, shown in Figure 2 has equation $2x + 3y = 26$

The line l_2 passes through the origin O and is perpendicular to l_1

(a) Find an equation for the line l_2

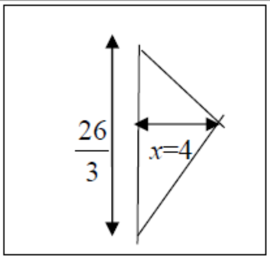
(4)

The line l_2 intersects the line l_1 at the point C. Line l_1 crosses the y-axis at the point B as shown in Figure 2.

(b) Find the area of triangle OBC. Give your answer in the form $\frac{a}{b}$, where a and b are integers to be found.

(6)

Examiner comment: *This question involves some problem solving and the use of the rule for perpendicular gradients* $m_1 \times m_2 = -1$. *It is a good idea to use the diagram as it will help the candidate appreciate what is required to solve the problem. In finding the area of the triangle it is important to use a simple method. (For example* $\frac{1}{2} \times OC \times CB$ *is unnecessarily complicated in this case.)*

Question	Scheme		Marks
8(a)	$2x+3y=26 \Rightarrow 3y=26\pm 2x$ and attempt to find m from $y=mx+c$		M1
	$\left(\Rightarrow y=\frac{26}{3}-\frac{2}{3}x\right)$ so gradient $= -\frac{2}{3}$		A1
	Gradient of perpendicular $=\frac{-1}{\text{their gradient}}$ $\left(=\frac{3}{2}\right)$		M1
	Line goes through (0, 0) so $y=\frac{3}{2}x$		A1
			(4)
(b)	Solves their $y=\frac{3}{2}x$ with their $2x+3y=26$ to form equation in x or in y		M1
	Solves their equation in x or in y to obtain $x=$ **or** $y=$		dM1
	$x=4$ or any equivalent e.g. $\frac{156}{39}$ **or** $y=6$ o.a.e		A1
	$B=(0,\frac{26}{3})$ used or stated in (b)		B1
		Area $=\frac{1}{2}\times\text{"4"}\times\frac{\text{"26"}}{3}$	dM1
		$=\frac{52}{3}$ (o.e. with integer numerator and denominator)	A1
			(6)
			(10 marks)

Response 1

(6)

a) ~~$2x+3y$~~ $=$

$2x+3y=26$

$3y=26-2x$

$y=-\frac{2}{3}x+\frac{26}{3}$

$\therefore$ l_2 tangent $=\frac{3}{2}$

$\therefore$ $y_{l_2}=\frac{3}{2}x+c$

$0=0+c$

$\therefore$ $y_{l_2}=\frac{3}{2}x$

b) $\because$ $y_{l_1}=-\frac{2}{3}x+\frac{26}{3}$

$x=0.$ $y=\frac{26}{3}$

~~$\frac{3}{2}x$~~ $\frac{3}{2}x=-\frac{2}{3}x+\frac{26}{3}$

$9x=-4x+52$

$13x=52$

$x=4$

$\therefore A_{\triangle OBC}=\frac{26}{3}\times 4\times\frac{1}{2}=\frac{52}{3}$

$a=52$ $b=3$

10/10

A completely correct solution

Response 2

A $3y = 26 - 2x$

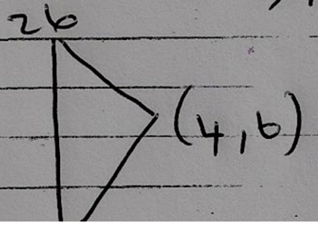
$y = \dfrac{26 - 2x}{3}$

l_2 is $+\dfrac{3}{2}x$

B $2x + \dfrac{9x}{2} = 26$

$6\frac{1}{2}x = 26$

$x = 4, y = 6$



Area $= \frac{1}{2} \times 26 \times 4 = 52$

7/10

Part (a) scores 3 out of 4. The method is correct but the equation of the line is missing the y *in* $y = \frac{3}{2}x$*. Part (b) scores the first three marks for achieving* $x = 4$ *via a correct method. The y intercept, 26, is incorrect but the method of finding the area is correct. Hence 4 marks can be awarded in part (b).*

Response 3

(a)

$2x+3y=26$

$3y=-2x+26$

$y=\frac{-2}{3}x+26$

grad l_1 is $-\frac{2}{3}$

$\therefore$ gradient l_2 is $-\frac{3}{2}$

$\therefore y=-\frac{3}{2}x$

(b)

$2x + 3x\,\frac{-3x}{2} = 26$

$4x - 9x = 26$

$-5x = 26$

$x = -5.2\ ,\ y = 7.8$

4/10

Part (a) is awarded 2 out of 4 marks. The gradient of l_1 is correct but the method of finding a perpendicular gradient is not. This candidate seems to have just found the reciprocal rather than the negative reciprocal.

In part (b) is also awarded 2 marks. There is a correct method to combine their two equations and an allowable attempt to find the point of intersection. The error in not multiplying all terms by 2 would be condoned for the method mark. No more marks are scored however.

9.

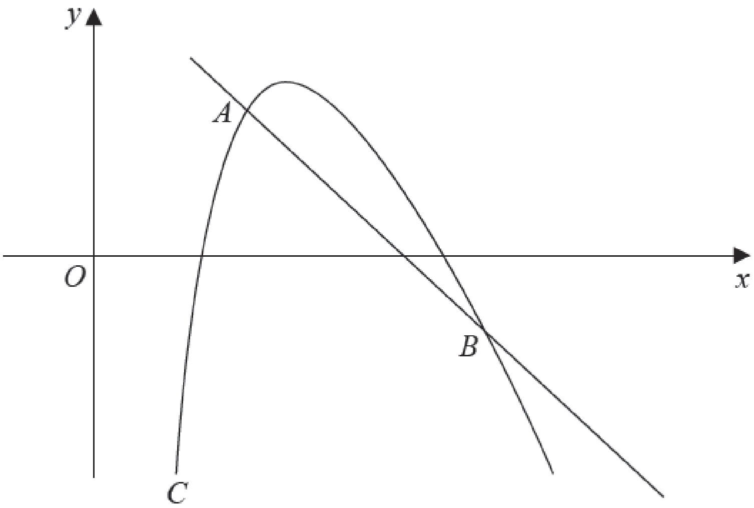


Figure 3

A sketch of part of the curve C with equation

$$y = 20 - 4x - \frac{18}{x}, \qquad x > 0$$

is shown in Figure 3.

Point A lies on C and has x coordinate equal to 2

(a) Show that the equation of the normal to C at A is $y = -2x + 7$.

(6)

The normal to C at A meets C again at the point B, as shown in Figure 3.

(b) Use algebra to find the coordinates of B.

(5)

Examiner comment: *Part (a) combines the topics of differentiation and coordinate geometry. Care needs to be taken when differentiating* $y = 20 - 4x - \frac{18}{x}$, *especially the* $\frac{18}{x}$ *term which must be written as* $18x^{-1}$ *before any differentiation is attempted. To find the equation of the normal, two pieces of information are required, in this case, its gradient and a point that lies on the line . The gradient is the negative reciprocal of the tangent whilst the y coordinate can be found by substituting* $x = 2$ *into* $y = 20 - 4x - \frac{18}{x}$. *Again, the diagram can be used by the candidate to help them process the information and form ideas as to the suitability of their answers.*

Question	Scheme	Marks
9(a)	Substitutes $x = 2$ into $y = 20 - 4\times2 - \frac{18}{2}$ and gets 3	B1
	$\frac{dy}{dx} = -4 + \frac{18}{x^2}$	M1 A1
	Substitute $x = 2 \Rightarrow \frac{dy}{dx} = \left(\frac{1}{2}\right)$ then finds negative reciprocal (-2)	dM1
	States or uses $y - 3 = -2(x - 2)$ or $y = -2x + c$ with their (2, 3)	ddM1
	to deduce that $y = -2x + 7$	A1*
		(6)
(b)	Put $20 - 4x - \frac{18}{x} = -2x + 7$ and simplify to give $2x^2 - 13x + 18 = 0$ Or put $y = 20 - 4\left(\frac{7-y}{2}\right) - \frac{18}{\left(\frac{7-y}{2}\right)}$ to give $y^2 - y - 6 = 0$	M1 A1
	$(2x - 9)(x - 2) = 0$ so $x =$ **or** $(y - 3)(y + 2) = 0$ so $y =$	dM1
	$\left(\frac{9}{2}, -2\right)$	A1 A1
		(5)
		(11 marks)

Response 1

a) $y = -4x - \frac{18}{x} + 20$

$= -4x - 18x^{-1} + 20$

$y' = -4 + 18x^{-2}$

$\because x_A = 2, \quad \therefore y' = \cancel{-4 + 18 \times \frac{1}{4}} \ \frac{1}{2}$

$\therefore y_{normal} = -2x + c$

$\because x = 2, \ y = 3$

$\therefore \ 3 = -4 + c$

$c = 7$

$\therefore \ y = -2x + 7$

b) $-2x + 7 = 20 - 4x - \frac{18}{x}$

$-2x^2 + 7x = 20x - 4x^2 - 18$

$2x^2 - 13x + 18 = 0$

$(2x - 9)(x - 2) = 0$

$x_1 = \frac{9}{2} \quad x_2 = 2$

$\cancel{y = -2x + 7}$

$\therefore x_B = \frac{9}{2}$

$\therefore y_B = -2 \times \frac{9}{2} + 7 = -2$

$\cancel{\therefore B\ (\frac{9}{2}}$ $\quad \therefore B\ (\frac{9}{2}, -2)$

11/11

This is a completely correct response.

Response 2

Question 9 Response 2

(a) $y = 20 - 4x - 18x^{-1}$

$\frac{dy}{dx} = -4 + 18x^{-2}$

at $x = 2$ $\frac{dy}{dx} = -4 + 4\frac{1}{2} = \frac{1}{2}$

Equavtion of normal is $y = -2x + 7$

(b) $-2x + 7 = 20 - 4x - \frac{18}{x}$

$\frac{18}{x} = 13 - 2x$

$18 = 13x - 2x^2$

$2x^2 - 13x - 18 = 0$

$(2x - 9)(x - 2) = 0$

$x = 4\frac{1}{2}, 2$

Meet at $(2, 3)$ $(4\frac{1}{2}, -2)$

6/11

Part (a) scores 2 out of 6. It is really important for candidates to show all the necessary working when asked to show a result. In this case the differentiation is correct but no more marks can be awarded. The y value is not found and the candidate does not show any working after finding the gradient of the curve at $x = 2$.

Part (b) scores 4 out of 5. Two values are offered at the end so the point B is not identified as having coordinates $(4.5, -2)$

Response 3

A/ $\frac{DY}{DX} = -4 + \frac{18}{1X}$

$X = 2$ GRADIENT $= -4 + 9 = 5$

POINT $= (2, 3)$

$Y - 3 = 5(X - 2) \rightarrow Y = 5X - 7$

$= -2X + 7$

B/ $Y = -2X + 7$ MEETS $Y = 20 - 4X - \frac{18}{X}$

AT ~~R7~~ $(2, 3)$ AND $\underline{(4\frac{1}{2}, -2)}$ ✓✓

1/11

Part (a) scores 1 out of 5. The coordinates of point A (2, 3) are correct but the method of differentiating here is dependent upon seeing $x^{-1} \rightarrow x^{-2}$. As the second M is dependent no more marks can be awarded in part (a).

The demand in part (b) is "Use algebra to find the coordinates of point B." There is no working shown here and the answer could have been written down from a graphical calculator so no marks are awarded.

10.

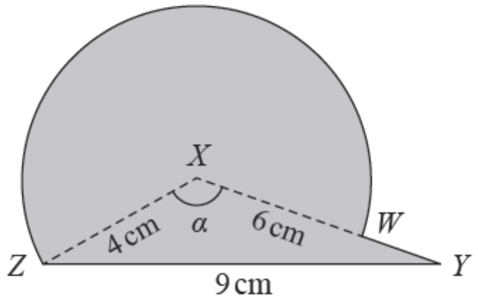


Figure 4

The triangle XYZ in Figure 4 has $XY = 6$ cm, $YZ = 9$ cm, $ZX = 4$ cm and angle $ZXY = \alpha$.

The point W lies on the line XY.

The circular arc ZW, in Figure 4, is a major arc of the circle with centre X and radius 4 cm.

(a) Show that, to 3 significant figures, $\alpha = 2.22$ radians.

(2)

(b) Find the area, in cm^2, of the major sector $XZWX$.

(3)

The region, shown shaded in Figure 4, is to be used as a design for a logo.

Calculate

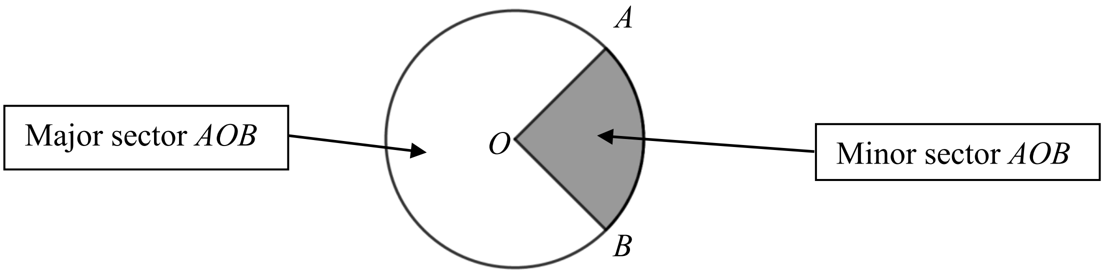
(c) the area of the logo

(3)

(d) the perimeter of the logo.

(4)

Examiner comment: *This question tests trigonometry and more specifically the two radian formulae for arc length and area of a sector. It is important that candidates remember the formulae. As part (a) is a proof, it is important that all necessary steps are shown. In part (b) the major sector is the larger of the two sectors*



In parts (c) and (d) it is important that candidates make their method clear. It is often the case that incorrect angles and lengths are used in calculations. Drawing a clear diagram would help in such cases.

Question	Scheme		Marks
10(a)	$9^2 = 4^2 + 6^2 - 2\times4\times6\cos\alpha \Rightarrow \cos\alpha = ...$	Correct use of cosine rule leading to a value for cos α	M1
	$\cos\alpha = \dfrac{4^2+6^2-9^2}{2\times4\times6}\left(= -\dfrac{29}{48} = -0.604..\right)$		
	$\alpha = 2.22$ * cso		A1
			(2)
	Alternative		
	$XY^2 = 4^2 + 6^2 - 2\times4\times6\cos 2.22 \Rightarrow XY^2 = ..$	Correct use of cosine rule leading to a value for XY^2	M1
	$XY = 9.00....$		A1
			(2)
(b)	$2\pi - 2.22(= 4.06366......)$	$2\pi - 2.22$ or $2\pi - 2.2$ or awrt 4.06 (May be implied)	B1
	$\frac{1}{2}\times4^2\times"4.06"$	Correct method for major sector area. Allow $\pi - 2.22$ for the major sector angle.	M1
	32.5	Awrt 32.5	A1
			(3)
	Alternative – Circle Minor – sector		
	$\pi\times4^2$	Correct expression for circle area	B1
	$\pi\times4^2 - \dfrac{1}{2}\times4^2\times2.22 = 32.5$	Correct method for circle - minor sector area	M1
	$= 32.5$	Awrt 32.5	A1
			(3)
(c)	Area of triangle = $\dfrac{1}{2}\times4\times6\times\sin 2.22(= 9.56)$	Correct expression for the area of triangle XYZ (allow 2.2 or awrt 2.22)	B1
	So area required = "9.56" + "32.5"	Their Triangle XYZ + part (b) or correct attempt at major sector (**Not** triangle ZXW)	M1
	Area of logo = 42.1 cm² or 42.0 cm²	Awrt 42.1 or 42.0 (or <u>just</u> 42)	A1
			(3)
(d)	Arc length $= 4\times4.06(=16.24)$ **or** $8\pi - 4\times2.22$	M1: $4\times\textit{their}(2\pi - 2.22)$ **or** circumference – minor arc A1: Correct ft expression	M1 A1ft
	Perimeter = $ZY + WY$ + Arc Length	9 + 2 + Any Arc	M1
	Perimeter of logo = 27.2 or 27.3	Awrt 27.2 **or** awrt 27.3	A1
			(4)
			(12 marks)

Response 1

(4)

(a) $9 = \sqrt{4^2+6^2-2\times4\times6\times\cos\alpha}$

$81 = 52 - 48\cos\alpha$

$48\cos\alpha = -29$

$\cos\alpha = \frac{-29}{48}$

$\alpha \approx 2.22$ radians

(b) $\frac{1}{2}\times4^2\times(2\pi-2.22)$

$= 32.5\ cm^3$

(c) $A_{\triangle}\ ZXY = \cancel{\frac{1}{2}}\ 4\times6\times\sin\alpha$

$= 19.1$

$19.1+32.5$

$= 51.6\ cm^2$

(d) $4\cdot(2\pi-2.22)$

$= 16.3$

$16.3+9+(6-4)$

$= 27.3\ cm$

Parts (a), (b) and (d) are fully correct. The candidate couldn't be awarded full marks in part (c) as they use a formula $ab\sin C$ for the area of triangle ZXY.

Response 2

(4)

a) $9^2 = 4^2 + 6^2 - 2\times 4\times 6\times \cos\alpha$

$29 = 48\cos\alpha$

$\cos\alpha = \frac{48}{29}$ $\alpha = 2.2$.

b) $A = \frac{1}{2}\times 4\times(2\pi - 2.2) = 8.166.$

c) $S = 8.166 + \frac{1}{2}\times 4\times 6\times \sin 2.2$

$= 17.87.$

d) $P = 4\times(2\pi - 2.2) + 2 + 9 = 27.33.$

8/12

Part (a) scores M1 A0. Although a correct version of the cosine rule is written down, it is not followed by a correct value for $\cos\alpha$, the $-$ sign is missing and the fraction is upside down. Additionally the answer is not shown to sufficient accuracy.

Part (b) scores B1 M0 A0. A correct angle is found $(2\pi - 2.2)$ but the expression written down $\frac{1}{2}\times 4\times(2\pi - 2.2)$ implies a formula $\frac{1}{2}r\theta$

Part (c) scores B1 M1 A0. The expression for the area of the triangle is correct and the candidate adds this to part (b).

Part (d) is correct and of an allowable accuracy.

Response 3

(4)

a) $9^2 = 6^2 + 4^2 - 2\times 6\times 4\times \cos A.$

$A = 2.219$

$= 2.22.$

b) $\pi \times 4^2 \times \frac{2.22}{2\pi} = 17.76\text{cm}^2$

c) $\frac{4\times \sin 2.22 \times 6}{2} = \frac{19.12}{2}.\text{cm}^2 = 9.56.\text{cm}^2$

~~19.12~~ $9.56 + 17.76 = 27.32.$

d) $2\pi r \times \frac{2.22}{2\pi} = 8.88\text{cm}.$ $6-4 = 2\text{ cm}$

$8.88 + 2 + 9 = 19.88\text{ cm}.$

4/12

Part (a) scores M1 A0. Although a correct version of the cosine rule is written down, it is not followed by a correct value for $\cos A$*. This is a "show that" question and it is a necessary step.*

Part (b) is incorrect and scores 0 marks. Although a version of $\frac{1}{2}r^2\theta$ *is used, there must be an attempt to find the area of the **major** sector. The answer given is the area of the **minor** sector.*

Part (c) scores B1 M1 A0. There is a correct expression for the area of triangle XYZ and this is added to their answer for (b). As part (b) is incorrect, this means that the final mark is withheld.

Part (d) scores M0 A0 M1 A0.
M0 A0 as an incorrect angle is used to find the arc length but $9+2+$ *any arc length scores the M1 mark.*