Pearson Edexcel

INTERNATIONAL ADVANCED LEVEL

PURE MATHEMATICS

Exemplars with examiner commentaries
Unit 1 - WMA11

Contents

Introduction

1.1 About this booklet

This booklet has been produced to support teachers delivering the Pearson Edexcel International Advanced Level Pure Mathematics specification. The Paper WMA11 exemplar materials will enable teachers to guide their students in the application of knowledge and skills required to successfully complete this course. The booklet looks at questions 2, 3, 4, 5, 7 and 9 from the June 2019 examination series, showing real candidate responses to questions and how examiners have applied the mark schemes to demonstrate how student responses should be marked.

1.2 How to use this booklet

Each example covered in this booklet contains:

- Question
- Mark scheme
- Example responses for the selected question
- Example of the marker grading decision based on the mark scheme, accompanied by examiner commentary including the explanation for the decision and guidance on how the answer can be improved to earn more marks.

The examples highlight the achievement of the assessment objectives at lower to higher levels of candidate responses.

Centres should use this content to support their internal assessment of students and incorporate examination skills into the delivery of the specification.

1.3 Further support

A range of materials are available from the Pearson qualifications website to support you in planning and delivering this specification.

Centres may find it beneficial to review this document in conjunction with the Examiner's Report and other assessment and support materials available on the Pearson Qualifications website.

Question 2

2. **Answer this question showing each stage of your working.**

(a) Simplify $\dfrac{1}{4 - 2\sqrt{2}}$

giving your answer in the form $a + b\sqrt{2}$ where a and b are rational numbers.

(2)

(b) Hence, or otherwise, solve the equation

$$4x = 2\sqrt{2}x + 20\sqrt{2}$$

giving your answer in the form $p + q\sqrt{2}$ where p and q are rational numbers.

(3)

Mark scheme

2.(a)	$\frac{1}{4-2\sqrt{2}}=\frac{1}{4-2\sqrt{2}}\times\frac{4+2\sqrt{2}}{4+2\sqrt{2}}$	M1
	$=\frac{4+2\sqrt{2}}{16-8}=\frac{1}{2}+\frac{1}{4}\sqrt{2}$ oe	A1
		(2)
(b)	$4x=2\sqrt{2}x+20\sqrt{2}\Rightarrow\left(4-2\sqrt{2}\right)x=20\sqrt{2}$	M1
	$\Rightarrow x=\frac{20\sqrt{2}}{\left(4-2\sqrt{2}\right)}=20\sqrt{2}\times(\text{a})$	dM1
	$\Rightarrow x=20\sqrt{2}\times\left(\frac{1}{2}+\frac{1}{4}\sqrt{2}\right)=10+10\sqrt{2}$	A1
		(3) **(5 marks)**

(a)

M1 For sight of $\frac{1}{4-2\sqrt{2}}\times\frac{4+2\sqrt{2}}{4+2\sqrt{2}}$ oe

A1 For achieving $\frac{1}{2}+\frac{1}{4}\sqrt{2}$ or exact equivalent such as $0.5+\frac{\sqrt{2}}{4}$, $\frac{2}{4}+\frac{2}{8}\sqrt{2}$ or correct *a* and *b*.
Remember it does not have to be simplified and isw following a correct answer

(b) Hence

M1 For attempting to collect the terms in x on one side of the equation and the constant term on the other side . Condone slips but there must be an attempt to collect terms with a bracket or implied bracket

dM1 For using part (a) and attempting to find $k\sqrt{2}\times(\text{a})$

A1 $10\sqrt{2}+10$ or $10+10\sqrt{2}$ but NOT $10\left(\sqrt{2}+1\right)$. It cannot be awarded without sight of $k\sqrt{2}\times(\text{a})$

....

Otherwise (1)- SQUARING APPROACH

M1 Squaring both sides $4x=2\sqrt{2}x+20\sqrt{2}\rightarrow16x^2=8x^2+160x+800$ Condone slips on coefficients
Cannot be scored by squaring each term. Look $ax^2=px^2+qx+r$

dM1 Re-arranging and attempting to solve their 3TQ usual rules
Eg $8x^2-160x-800=0\Rightarrow x^2-20x-100=0\Rightarrow x=\frac{20\pm\sqrt{400+400}}{2}$

A1 $10\sqrt{2}+10$ or $10+10\sqrt{2}$ following a correct solution of the quadratic equation seen above.

....

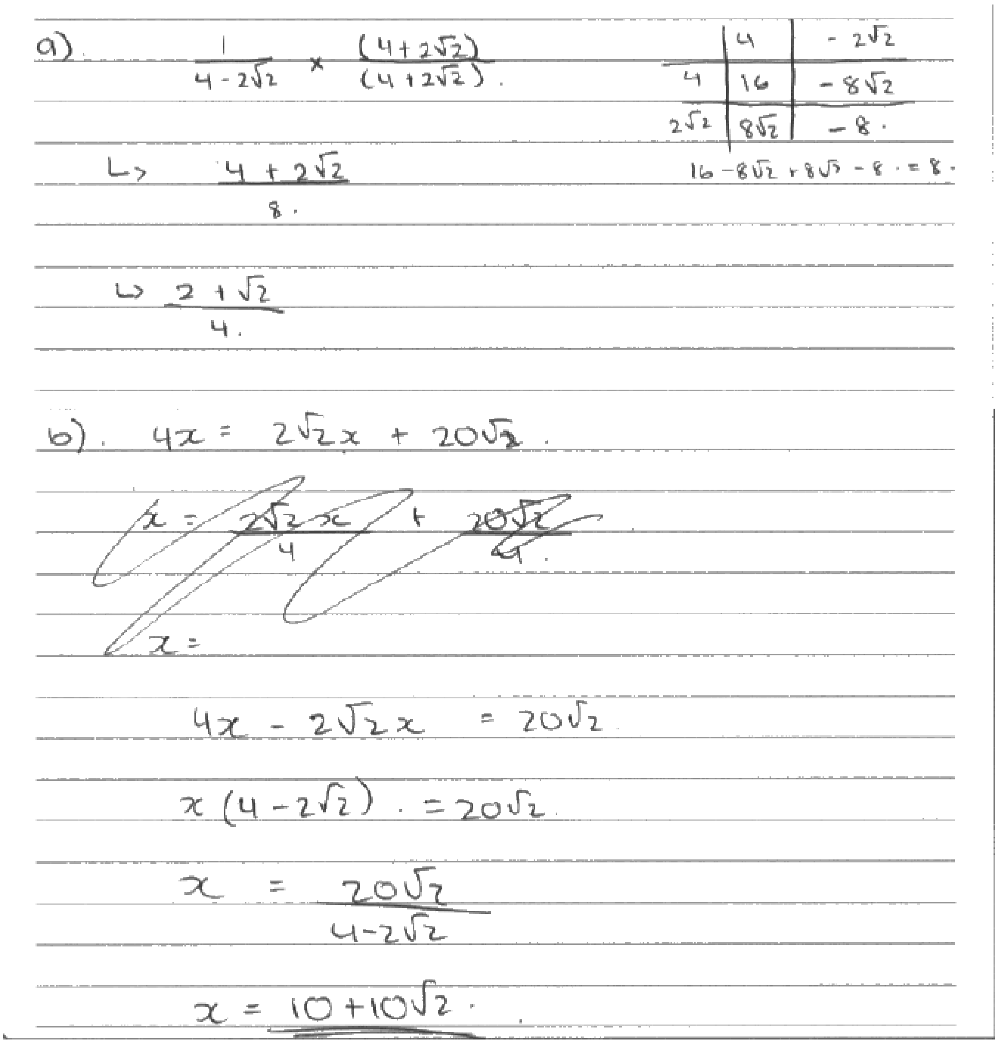
Otherwise (2)- REPEATING THE PROCESS OF PART (a)

M1 Rearranges $4x=2\sqrt{2}x+20\sqrt{2}\Rightarrow\left(4\pm2\sqrt{2}\right)x=20\sqrt{2}$ condoning slips. May even divide by 2 first

dM1 Then divide, rationalise and attempt to simplify. Eg $x=\frac{20\sqrt{2}}{\left(4-2\sqrt{2}\right)}\times\frac{\left(4+2\sqrt{2}\right)}{\left(4+2\sqrt{2}\right)}=\frac{80\sqrt{2}+40\sqrt{2}\times\sqrt{2}}{16-8}$ oe

A1 $10\sqrt{2}+10$ or $10+10\sqrt{2}$ only. It cannot be awarded without sight of the correct intermediate line seen above

Exemplar response A



a) $\frac{1}{4-2\sqrt{2}} \times \frac{(4+2\sqrt{2})}{(4+2\sqrt{2})}$.

	4	$-2\sqrt{2}$
4	16	$-8\sqrt{2}$
$2\sqrt{2}$	$8\sqrt{2}$	-8.

$16-8\sqrt{2}+8\sqrt{2}-8 = 8$.

↳ $\frac{4+2\sqrt{2}}{8}$.

↳ $\frac{2+\sqrt{2}}{4}$.

b). $4x = 2\sqrt{2}x + 20\sqrt{2}$.

~~$x = \frac{2\sqrt{2}x}{4} + \frac{20\sqrt{2}}{4}$.~~

~~$x =$~~

$4x - 2\sqrt{2}x = 20\sqrt{2}$

$x(4-2\sqrt{2}) = 20\sqrt{2}$.

$x = \frac{20\sqrt{2}}{4-2\sqrt{2}}$

$x = 10+10\sqrt{2}$.

Examiner's comments:

This response was given 2 marks. (a) M1 A0 (B) M1 dM0 A0

Part (a): The candidate scores the method mark for multiplying both the numerator and denominator by $4+2\sqrt{2}$ but fails to write their answer in the form $a+b\sqrt{2}$ as required by the question.

Part (b): Attempted by "Otherwise 2" as seen in the notes to the mark scheme. The candidate collects the terms in *x* and divides to make *x* the subject. The demand of the question, "Answer this question showing each stage in your working" meant that candidates needed to show all steps in proceeding to the answer $10+10\sqrt{2}$. To score the dM1 mark the candidate would need to show the steps $\frac{20}{4-2\sqrt{2}} \times \frac{4+2\sqrt{2}}{4+2\sqrt{2}} = \frac{20\left(4+2\sqrt{2}\right)}{16-8}$ **before** writing down the correct answer. This candidate has merely written down the answer, possibly using a calculator.

Exemplar response B

a) $\frac{1}{4-2\sqrt{2}}$

$= \frac{1}{4-2\sqrt{2}} \times \frac{4+2\sqrt{2}}{4+2\sqrt{2}}$

$= \frac{4+2\sqrt{2}}{16-8} = \frac{4+2\sqrt{2}}{8}$

$= \frac{1}{2} + \frac{1}{4}\sqrt{2}$.

b). $4x = 2\sqrt{2}(x+10)$

~~x~~ $x+10 = \frac{4x}{2\sqrt{2}}$

$x+10 = \sqrt{2}\,x$.

~~$\frac{x+10}{x} = \sqrt{2}$~~

$(x+10)^2 = (\sqrt{2}x)^2$

$x^2+20x+100 = 2x^2$

$2x^2-x^2-20x-100=0$

$x^2-20x-100=0$.

$x^2-20x-100=0$.

~~x=~~

$x = \frac{20 \pm \sqrt{20^2-4\times-100}}{2\times1}$

$x = \frac{20 \pm \sqrt{800}}{2} = 10+10\sqrt{2},\ 10-10\sqrt{2}$.

Examiner's comments:

This response was given 4 marks. (a) M1 A1 (b) M1 dM1 A0

Part (a) is completely correct.

Part (b) is attempted by ''Otherwise 1'' as seen on the notes to the mark scheme. The M1 is awarded for squaring as witnessed by the work in moving from line 4 to line 5. The dM1 is awarded for a correct method to solve the quadratic equation. The candidate then writes down the answers to the quadratic, both $10+10\sqrt{2}$ and $10-10\sqrt{2}$. To score the A1 under this method, the solution $10+10\sqrt{2}$ must be chosen as their only answer.

Note that it is good practice in some questions involving squares and square roots to check that all of the answers are solutions of the original equation. In this case $10-10\sqrt{2}$ is not a solution of $4x = 2\sqrt{2}x + 20\sqrt{2}$. This is due to the fact that the left hand side of the equation is $4\left(10-10\sqrt{2}\right) = 40-40\sqrt{2}$ where as the right hand side is $2\sqrt{2}\left(10-10\sqrt{2}\right)+20\sqrt{2} = 40\sqrt{2}-40$. As these are different, the solution of $10-10\sqrt{2}$ should be rejected.

Question 3

3.

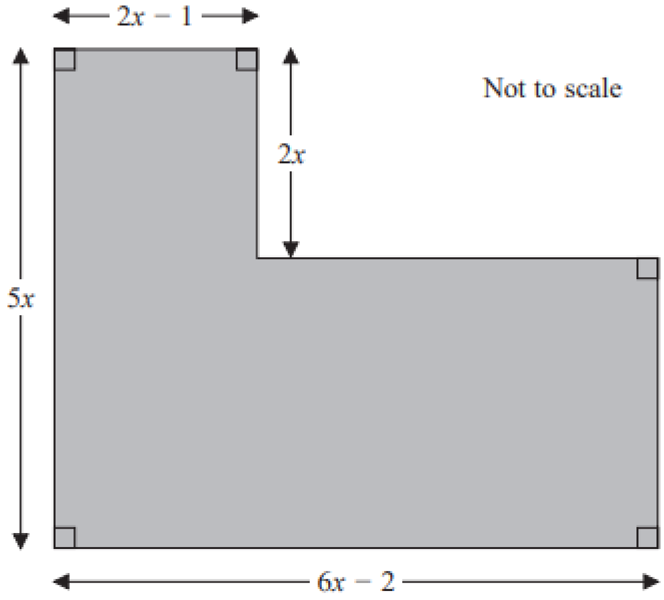


Figure 1

Figure 1 shows the plan of a garden. The marked angles are right angles.

The six edges are straight lines.

The lengths shown in the diagram are given in metres.

Given that the perimeter of the garden is greater than 29 m,

(a) show that $x > 1.5$ m

(3)

Given also that the area of the garden is less than 72 m^2,

(b) form and solve a quadratic inequality in x.

(5)

(c) Hence state the range of possible values of x.

(1)

Mark scheme

Question Number	Scheme	Marks
3.(a)	Attempts perimeter of garden = $2\times 5x+2\times(6x-2)$	M1
	Sets $2\times 5x+2\times(6x-2)>29 \Rightarrow 22x>33$	dM1
	$\Rightarrow x>\frac{33}{22} \Rightarrow x>1.5$ *	A1*
		(3)
(b)	Attempts area of garden = $2x(2x-1)+3x(6x-2)$	M1
	Sets $A<72 \Rightarrow 22x^2-8x-72<0$	A1
	Finds critical values $11x^2-4x-36 \Rightarrow x=-\frac{18}{11}, 2$	M1
	Chooses inside region	ddM1
	$-\frac{18}{11}<x<2$	A1
		(5)
(c)	$1.5<x<2$	B1
		(1)
		(9 marks)

(a)

M1 An attempt at finding the perimeter of the garden.
Scored for sight of $5x+2x-1+2x+6x-2+$ additional term(s) involving x
Individual lengths may not be seen so imply for sight of a total of $ax+b$, where $a>15$

dM1 Sets their $P>29$ and attempts to solve by proceeding to $ax>c$
You may condone an attempt in which $P=29 \Rightarrow ax=c$

A1* cso with at least one correct intermediate (simplified) line $22x>33$ or $x>\frac{33}{22}$ before $x>1.5$ seen.
Condone an attempt in which you see $P=29 \Rightarrow x=1.5$ before $x>1.5$ seen
Note that it is possible to start with $x>1.5$ and prove $P>29$ but for the A1* to be scored there must be a final statement of the type "hence $x>1.5$". There is no requirement for any units

(b) **Mark part (b) and (c) together**

M1 For an attempt at finding the area of the garden. For this to be scored look for
The sum of two areas $2x(2x-1)+...x(6x-2)$ condoning slips
The sum of two areas $5x(2x-1)+...x(..x\pm...)$ condoning slips
The difference between two areas $5x(6x-2)-2x(..x\pm...)$ condoning slips.

A1 A "correct and simplified" equality or inequality, condoning $< \leftrightarrow \leqslant \leftrightarrow =$ Eg. $22x^2-8x-72<0$ oe

M1 A valid attempt to find the critical values of their 3TQ. Allow factorisation, formula, completion of square or use of calculator. If a calculator is used then the answer(s) must be correct for their 3TQ.
Condone candidates who fail to state the negative root of their quadratic.

ddM1 Dependent upon both M's. For choosing the inside region for their critical values. Condone $< \leftrightarrow \leqslant$
Condone for this mark replacing a negative root with 0, 0.5 or 1.5. So accept for example one of either $1.5<x<"2"$, $0<x<"2"$ or $0.5<x<"2"$

A1 $-\frac{18}{11}<x<2$ Allow $0<x<2$ or $0.5<x<2$ due to context. Allow alternative notation. See below

(c)

B1 $1.5<x<2$. Accept versions such as $(1.5,2)$, $x>1.5$ **and** $x<2$, $x>1.5 \cap x<2$
Do not allow $x>1.5$ or $x<2$ $\quad$ $x>1.5,\ x<2$

Exemplar response A

(1)

(a) Perimeter > 29

$5x + 2x - 1 + 2x + 3x + (6x - 2) + 4x - 1 > 29$

$6x - 2 - (2x - 1)$
$= 4x - 1$

$= 22x - 4 > 29$

$22x > 33$

$x > 1 \cdot 5$

(b) Area of small rectangle $= (2x - 1)(2x)$
$= 4x^2 - 2x$

Area of larger rectangle $= (6x - 2)(3x)$
$= 18x^2 - 6x$

Total area

$= (4x^2 - 2x) + (18x^2 - 6x) < 72$

$22x^2 - 8x < 72$

$22x^2 - 8x - 72 > 0$

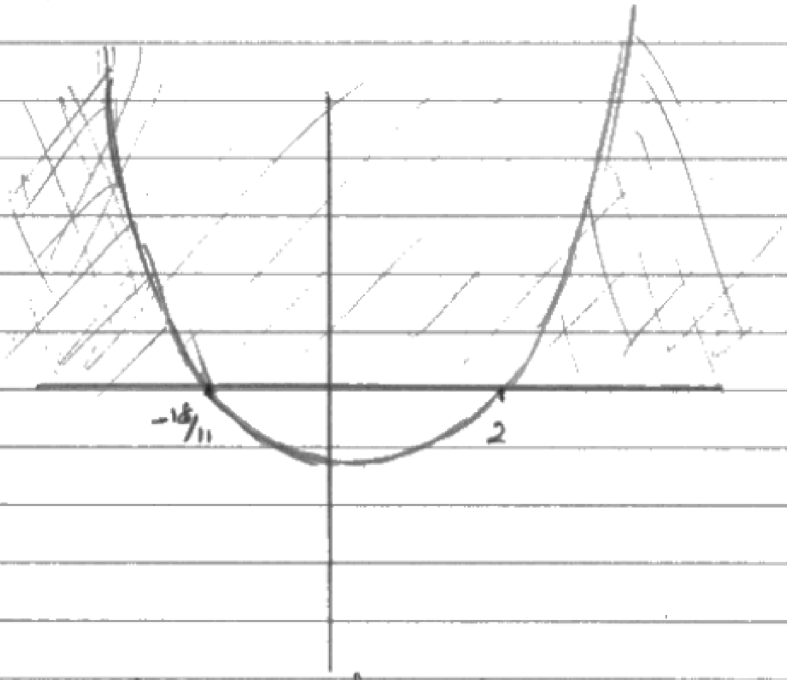
~~$22x^2 +$~~ $22x^2 - 44x + 36x - 72 > 0$

$22x(x - 2) + 36(x - 2) > 0$

$(x - 2)(22x + 36) > 0$

$x > 2 \quad x < -18/11$

(c)



range of possible values of $x \Rightarrow$ $x > 2$

$x < -18/11$

Examiner's comments:

This response was given 5 marks. (a) M1 dM1 A1 (b) M1 A0 M1 ddM0 A0 (c) B0

Part (a) is completely correct.

In part (b), the candidate makes a correct attempt at finding the area of the garden but proceeds to an incorrect inequality during the rearrangement. The method mark is scored for solving their quadratic equation but gives the outside region rather than the inside region.

Part (c) is incorrect.

Exemplar response B

(a) $(2x-1)+2x+(6x-2-(2x-1))+3x+(6x-2)+5x>29$

$(2x-1)+10x+(6x-2)+(4x-1)>29$

$2x-1+10x+6x-2+4x-1>29$

$22x>33$

$x>\frac{33}{22}$

$x>1.5\text{ m}$

(b) $3x(6x-2)+2x(2x-1)<72$

$18x^2-6x+4x^2-2x<72$

$22x^2-8x-72<0$

$22x^2-8x-72=0$

$11x^2-4x-36=0$

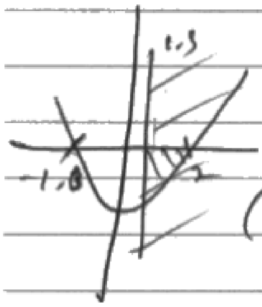
99×4

33×12

11×36

$x=2$ $\quad x=-1.636$

↑ Length cannot be negative



~~x $1.5>x$~~

(c) $1.5<x<2$

Examiner's comments:

This response was given 8 marks. (a) M1 dM1 A1 (b) M1 A1 M1 ddM1 A0 (c) B1

Parts (a) and (c) are completely correct.

In part (b), the candidate makes a correct attempt at finding the area of the garden and proceeds to a correct inequality on line 3. The resulting quadratic equation is solved correctly as witnessed by roots of 2 and - 1.636 and they give an inside region when they write $1.5<x<2$. They cannot score the A1 in this part as they have not answered the question, which was to solve the quadratic inequality.

Question 5

5. (a) Find, using algebra, all real solutions of

$$2x^3 + 3x^2 - 35x = 0$$

(3)

(b) Hence find all real solutions of

$$2(y-5)^6 + 3(y-5)^4 - 35(y-5)^2 = 0$$

(4)

Mark scheme

Question Number	Scheme	Marks
5.(a)	$2x^3+3x^2-35x=0 \Rightarrow x\left(2x^2+3x-35\right)=0$	M1
	$(2x-7)(x+5)=0 \Rightarrow x=...$	dM1
	$x=-5, 0, \frac{7}{2}$	A1
		(3)
(b)	$2(y-5)^6+3(y-5)^4-35(y-5)^2=0$	
	States that $y=5$ is a solution	B1
	$(y-5)^2 = "\frac{7}{2}" \Rightarrow y=...$	M1
	$y=5+\sqrt{\frac{7}{2}}$ or $y=5-\sqrt{\frac{7}{2}}$ or exact equiavlent	A1ft
	Both $y=5+\sqrt{\frac{7}{2}}$ and $y=5-\sqrt{\frac{7}{2}}$ or exact equivalent.	A1
		(4)
		(7 marks)

(a)

M1 Takes out a common factor of x. Score if each term is divided by x.

dM1 Attempts to solve the resulting quadratic **via algebra** (usual rules). Allow factorisation, formula or completion of square. They cannot just write down answers from their calculator for this mark.

A1 $x=-5, 0, \frac{7}{2}$

Note 1: Some candidates will just write down their answers from a calculator. This scores 0,0,0

Note 2: Some students will attempt to solve the cubic by the quadratic formula Eg.

$2x^3+3x^2-35x=0 \Rightarrow a=2, b=3, c=-35$ and use $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \Rightarrow x=-5, \frac{7}{2}$

This scores 0,0,0 as the method used is incorrect

(b)

B1 States that 5 is a solution of the given equation in (b)

M1 Realises that $x=(y-5)^2$ and proceeds to find a value for y using $(y-5)^2="\frac{7}{2}" \Rightarrow y=...$ Follow through on any positive value from (a). Allow decimal answers here. Don't be overly concerned by the mechanics of their solution.

A1ft A solution of $5+\sqrt{\frac{7}{2}}$ or $5-\sqrt{\frac{7}{2}}$ You should follow through on their positive root. Allow decimals for this mark only. So accept awrt 6.87 or awrt 3.13

A1 Both $5+\sqrt{\frac{7}{2}}$ and $5-\sqrt{\frac{7}{2}}$ with no other solutions for part (b) apart from 5. Do not allow decimal equivalents. Don't allow complex solutions.

Exemplar response A

a) $2x^3 + 3x^2 - 35x = 0$

$x(2x^2 + 3x - 35) = 0$

$x = 0$, $2x^2 + 3x - 35 = 0$

$x = 7/2$, $x = -5$.

Real solutions of $2x^3 + 3x^2 - 35x = 0$

$x = 7/2$, $x = -5$

b) $2(y-5)^6 + 3(y-5)^4 - 35(y-5)^2$

Let $(y-5)^2$ be x

$2x^3 + 3x^2 - 35x = 0$

$x(2x^2 + 3x - 35) = 0$

$x = 0$, $2x^3 + 3x - 35 = 0$

$x = 7/2$; $x = -5$

$(y-5)^2 = 7/2$ $\quad$ $(y-5)^2 = -5$

$y - 5 = \pm\sqrt{5}$

$y - 5 = \pm\sqrt{7/2}$ $\quad$ $y = 5 \pm \sqrt{5}$

$y = 5 \pm \sqrt{7/2}$

Examiner's comments:

This response was given 3 marks. (a) M1 dM0 A0 (b) B0 M1 A1 A0

Part (a): a common factor of x is taken out **but** the roots of the resulting quadratic are just written down. The demand of the question "Find, using algebra, all real solutions of " meant that factorisation or formula was required to solve the quadratic equation.

In part (b), the candidate does use their positive root from part (a) to find at least one value for y. In fact, $y = 5 \pm \sqrt{\frac{7}{2}}$ are two of the three correct values. The B1 mark is not awarded as $y = 5$ is not found and the A1 mark at the end of the question is withheld due to the extra incorrect solutions of $5 \pm \sqrt{5}$.

Exemplar response B

a

$2x^3+3x^2-35x=0$

$x(2x^2+3x-35)=0$

$x(x+5)(2x-7)=0$

$x=0$ or $x=-5$ or $x=\frac{7}{2}$

(b) $2(y-5)^6+3(y-5)^4-3(y-5)^2=0$

Let $x=(y-5)^2$

$2x^3+3x^2-35x=0$

$(y-5)^2=0 \longrightarrow y=5$

$(y-5)^2=-5 \longrightarrow$ impossible

$(y-5)^2=\frac{7}{2}$

$\Downarrow$

$y-5=\pm\sqrt{\frac{7}{2}}$ $\quad y=\pm\sqrt{\frac{7}{2}}+5$ $\quad \therefore y=\pm\frac{\sqrt{14}}{2}+5$

Examiner's comments:

This response was given 7 marks. (a) M1 dM1 A1 (b) B1 M1 A1 A1

Part (a) is completely correct. Note that in this response the candidate uses algebra to find all three solutions (via factorisation).

In part (b), the candidate rejects the solution from the negative root and gives the 3 correct solutions. Hence, all three marks can be awarded.

Question 7

7.

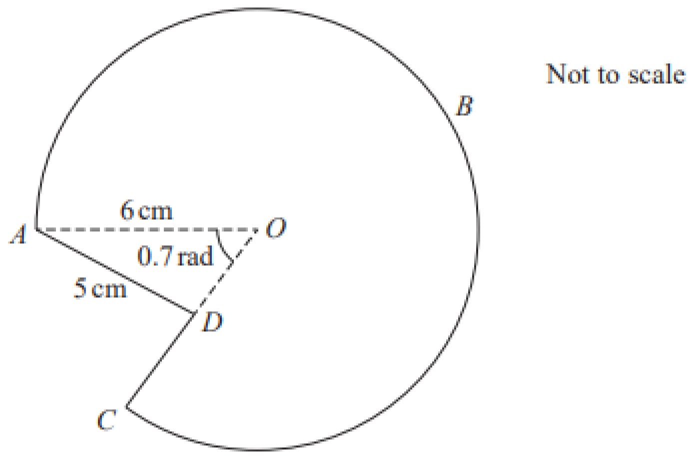


Figure 2

The shape *ABCDA* consists of a sector *ABCOA* of a circle, centre *O*, joined to a triangle *AOD*, as shown in Figure 2.

The point *D* lies on *OC*.

The radius of the circle is 6 cm, length *AD* is 5 cm and angle *AOD* is 0.7 radians.

(a) Find the area of the sector *ABCOA*, giving your answer to one decimal place.
(3)

Given angle *ADO* is obtuse,

(b) find the size of angle *ADO*, giving your answer to 3 decimal places.
(3)

(c) Hence find the perimeter of shape *ABCDA*, giving your answer to one decimal place.
(4)

Mark scheme

Question Number	Scheme	Marks
7.(a)	Attempts to use $\frac{1}{2}r^2\theta$ with $r=6$ and any allowable angle θ	M1
	Full method to find area $\frac{1}{2}\times6^2\times(2\pi-0.7)$ or $\pi\times6^2-\frac{1}{2}\times6^2\times0.7$	M1
	$=100.5\,\text{cm}^2$ (awrt)	A1
		(3)
(b)	Attempts $\frac{\sin\angle ADO}{6}=\frac{\sin0.7}{5}\Rightarrow\sin\angle ADO=0.77...$	M1 A1
	$\angle ADO=2.258$ (awrt)	A1
		(3)
(c)	Attempts arc length $ABC=6\times(2\pi-0.7)$ 33.50	M1
	Attempts length OD $\frac{\sin(\pi-0.7-\text{"}2.258\text{"})}{OD}=\frac{\sin0.7}{5}\Rightarrow OD=...$ 1.42	M1
	Full method to find perimeter = "33.50"+5+ 6−"1.42"	ddM1
	= 43.1 cm	A1
		(4)
		(10 marks)
Alt (c)	Alternative for arc length $ABC=12\pi-6\times0.7$	M1
	Alternative for finding OD using the cosine rule $OD^2=6^2+5^2-2\times6\times5\cos(\pi-0.7-\text{"}2.258\text{"})\Rightarrow OD$	M1
	Solutions where candidate changes to degrees Look for angle AOD = awrt 40° to score M marks	
7.(a)	Attempts to use $\frac{\theta}{360}\pi r^2$ with $r=6$ and angle $\theta=$ awrt 40 or 320	M1
	Full method to find area $\frac{(360-\text{awrt }40)}{360}\times\pi6^2$ or $\pi\times6^2-\frac{\text{awrt }40}{360}\times\pi6^2$	M1
	$=100.5\,\text{cm}^2$ (awrt)	A1
		(3)
(b)	Attempts $\frac{\sin\angle ADO}{6}=\frac{\sin40°}{5}\Rightarrow\sin\angle ADO=0.77...$	M1 A1
	$\angle ADO=129.4°$ (awrt)	A1
		(3)
(c)	Attempts arc length $ABC=\frac{(360-40)}{360}\times2\pi6$ 33.50	M1
	Attempts length OD $\frac{\sin(180-40-\text{"}129.4\text{"})}{OD}=\frac{\sin40}{5}\Rightarrow OD=...$ 1.42	M1
	Full method to find perimeter = "33.50"+5+ 6−"1.42"	ddM1
	= 43.1 cm	A1
		(4)
		(10 marks)

Notes

(a)

M1 Attempts to use $\frac{1}{2}r^2\theta$ with $r = 6$ and any angle allowable angle θ

Allowable angles are; 0.7 $\pi - 0.7 =$ allow awrt 2.4 $2\pi - 0.7 =$ allow awrt 5.6

M1 A correct attempt to find the area of sector *ABCOA*. See scheme Accept awrt 100 or 101 for this mark

A1 awrt $100.5(\text{cm}^2)$ The units are not required

(b)

M1 Attempts the sine rule with the lengths and angles in the correct positions $\frac{\sin \angle ADO}{6} = \frac{\sin 0.7}{5}$

A1 Correct value for $\sin \angle ADO = 0.77....$ Be careful here! $\angle ADO = 0.77...$ is A0
May be implied by either a correct answer or awrt 0.88

A1 awrt $\angle ADO = 2.258$ or $129.4°$

(c)

M1 A correct method to find arc length *ABC* May be implied by sight of $6 \times$ awrt 5.6 or awrt 33.5 or 33.6

M1 A correct method to find length *OD* using either the sine rule or cosine rule. The angle *OAD* must be attempted using a correct method $(\pi - 0.7 - "2.258")$.

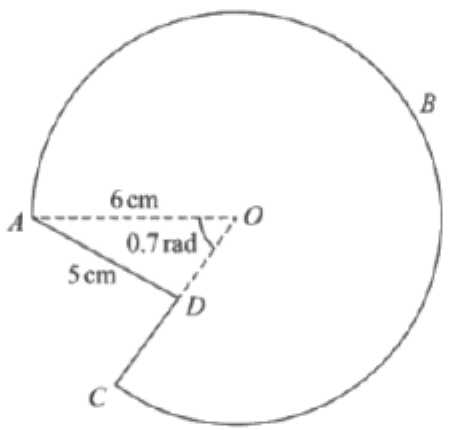
Eg. For the sine rule $\frac{OD}{\sin(\pi - 0.7 - "2.258")} = \frac{5}{\sin 0.7} = \frac{6}{\sin "2.258"} \Rightarrow OD = ...$

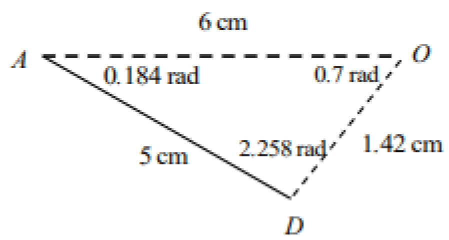
For cosine rule it could be $5^2 = 6^2 + x^2 - 2 \times 6 \times x \times \cos(0.7) \Rightarrow$ 3TQ in x which must be solved by correct methods

ddM1 Both previous M's must have been scored. It is for a correct method to find the perimeter of the shape.
Look for " 33.5" +5 + 6 − "1.42". It is implied by awrt 43

A1 cso and cao 43.1 cm. Units are not required

Handy Diagrams





Exemplar response A

a. $A = \frac{1}{2} r^2 \theta$.

$A = \frac{1}{2} \times 6^2 \times 0.7$.

$A = 12.6 cm^2$.

b. $\frac{\sin 0.7}{5} = \frac{\sin x}{6}$.

$\frac{\sin 0.7 \times 6}{5} = \sin x$.

$\sin x = 0.884$ rad (3 dp.).

c. $2\pi - 0.7 = 5.583$ rad

$L = r\theta$.

$L = 6 \times 5.583$

$L = 33.4991184$ cm.

$AC = 33.5$ cm (3 s.f.)

$CD = 3$ cm.

$AD = 5$ cm.

Perimeter $= 2r + r\theta$.

$= 3 + 5 + 33.4991184$

$= 41.4991$ cm

$= 41.5$ cm (3 s.f.)

Examiner's comments:

This response was given 4 marks. (a) M1 M0 A0 (b) M1 A1 A0 (c) M1 M0 dM0 A0

In part (a), the candidate uses a correct formula with an allowable angle. The second M cannot be awarded as they would need to either subtract the value found from 12π or use the same formula with an angle of $(2\pi - 0.7)$.

In part (b), a correct sine formula is used and an angle of 0.884 is found. (We condoned the fact that it is preceded by sin*x* =). This angle is the acute angle so would need to be subtracted from π to obtain the obtuse angle of 2.258 radians.

In part (c), a correct method is used to find the arc length ABC. This is the only mark that can be awarded here as there is no attempt to find the length OD, using either the sine or cosine rules.

Exemplar response B

(a) ABCOA $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 6^2 \times 5.58$

$= 100.44 \text{ cm}^2$

(b) $\frac{\sin ADO}{6} = \frac{\sin 0.7}{5}$

$\sin ADO = \frac{6 \sin 0.7}{5}$

$\sin ADO = 0.773$

$ADO = 0.8837$

$ADO = \pi - 0.8837$

$= 2.258$ radians

(c) $OD^2 = 5^2 + 6^2 - 2 \times 5 \times 6 \cos 0.184$

$= 2.0128$

$OD = 1.42$

$CD = 6 - 1.42 = 4.58$ cm

$ABC = r\theta$

$= 6 \times 5.583$

$ABC = 33.498$

Perimeter $= 33.498 + 4.58 + 5$

$= 43.078$ cm

Examiner's comments:

This response was given 8 marks. (a) M1 M1 A0 (b) M1 A 1 A1 (c) M1 M1 dM1 A0

In part (a), the candidate uses a correct formula with the 'correct' angle. Unfortunately, 5.58 lacks sufficient accuracy to produce the correct answer of 100.5 cm^2.

Part (b) is completely correct.

In part (c), a correct method is used to find the perimeter of shape ABCDA. This is the one mark on this paper that was used to test accuracy of calculations. Hence, the only acceptable answer here was 43.1 cm.

Question 9

9.

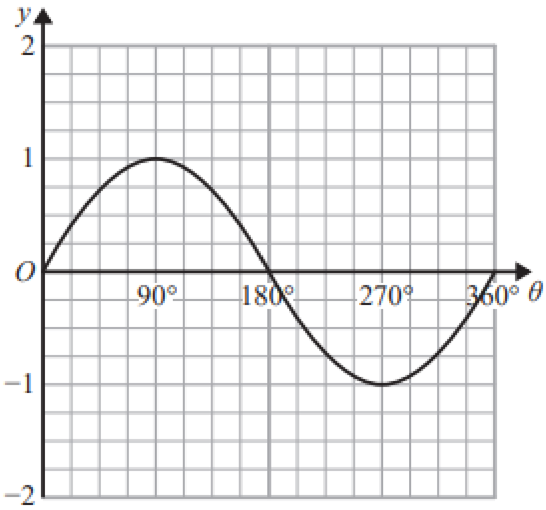


Figure 3

Figure 3 shows a plot of the curve with equation $y = \sin\theta, \quad 0 \leqslant \theta \leqslant 360°$

(a) State the coordinates of the minimum point on the curve with equation

$$y = 4\sin\theta, \quad 0 \leqslant \theta \leqslant 360°$$

(2)

A copy of Figure 3, called Diagram 1, is shown on the next page.

(b) On Diagram 1, sketch and label the curves

(i) $y = 1 + \sin\theta, \quad 0 \leqslant \theta \leqslant 360°$

(ii) $y = \tan\theta, \quad 0 \leqslant \theta \leqslant 360°$

(2)

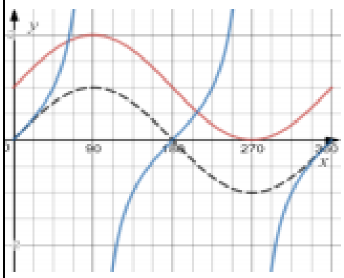
(c) Hence find the number of solutions of the equation

(i) $\tan\theta = 1 + \sin\theta$ that lie in the region $0 \leqslant \theta \leqslant 2160°$

(ii) $\tan\theta = 1 + \sin\theta$ that lie in the region $0 \leqslant \theta \leqslant 1980°$

(3)

Mark scheme

Question Number	Scheme	Marks
9. (a)	$(270°, -4)$	B1 B1 **(2)**
(b)	 For $y = 1 + \sin\theta$	B1
	$y = \tan\theta$	B1 **(2)**
(c)	(i) $6 \times 2 = 12$ (ii) 11	M1 A1 B1 ft **(3)** **(7 marks)**

(a)

B1 Either coordinate correct. Look for either $270^{(°)}$ or -4 in the correct position within (,) .

Alternatively look for either $x = 270$ or $y = -4$ Condone $\frac{3\pi}{2} = 270°$

Do not accept multiple answers unless one point is chosen or it is clearly part of their thought process. There is no need for the degrees symbol. Condone swapped coordinates, ie $(-4, 270)$ for this mark

B1 For correct coordinates.

$(270°, -4)$ with or without degrees symbol. Condone $x = 270(°), y = -4$

(b) These may appear on Figure 3 rather than Diagram 1

B1 For $y = 1 + \sin\theta$ Score for a curve passing through $(0,1), (90°,2), (180°,1), (270°,0), (360°,1)$ with acceptable curvature. Do not accept straight lines

B1 For $y = \tan\theta$ with acceptable curvature. Must go beyond $y = 1$ and -1

Score for the general shape of the curve rather than specific coordinates. See practice and qualification items for clarification.

First quadrant from $(0,0) \rightarrow (90°, \infty)$

Second and third quadrants from $(90°, -\infty) \rightarrow (270°, \infty)$ passing through $(180°, 0)$

Fourth quadrant from $(270°, -\infty) \rightarrow (0,0)$

(c)(i) The question states hence so it is expected the results come from graphs.
If neither or only one graph is drawn then score for 12 in (i) for M1 A1 and 11 in (ii) B1

M1 For the calculation $\frac{2160}{360} = 6$ or $\frac{2160}{180} = 12$ or multiplying the number of intersections in their (b) by 6

Sight of 6 or 12 will imply this mark.

A1 12. 12 will score both marks.

(c) (ii)

B1 ft For either 11 (correct answer)
or follow through on n less than their answer to (c) (i) where n is their number of solutions in the range $180° < \theta \leqslant 360°$

Exemplar response A

(a) Cxn (270°, 3).

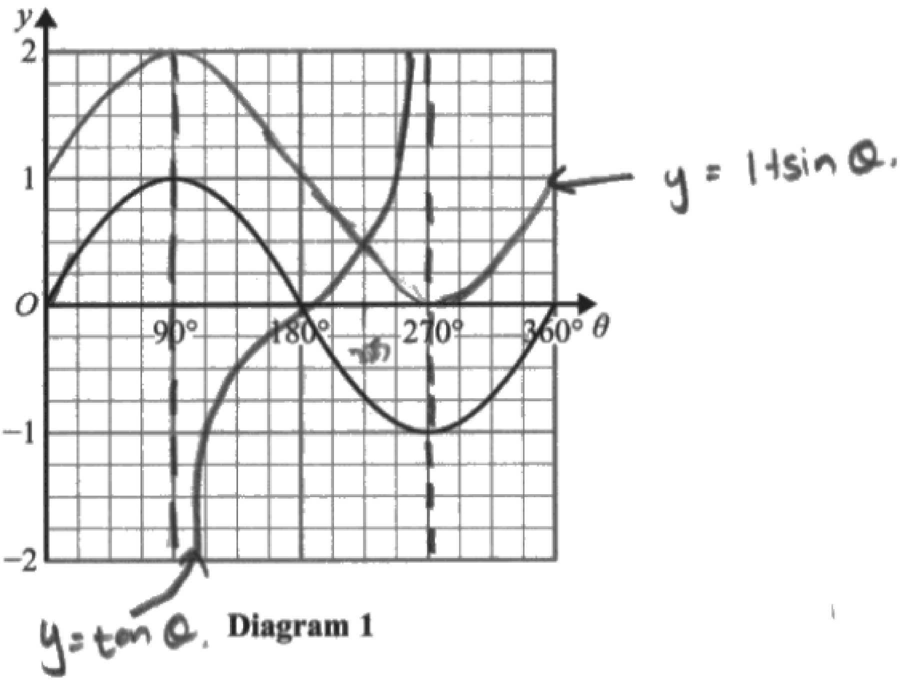


Diagram 1

(c) i) 6.
ii) 5

Examiner's comments:

This response was given 4 marks. (a) B1 B0 (b) B1 B0 (c) M1 A0 B1ft

In part (a), the $270°$ is correct but the "3" is not.

The graph for $y = 1 + \sin\theta$ in part (b) is completely correct, but the one for $y = \tan\theta$ is lacking the branches in first and fourth quadrants. This was a common error.

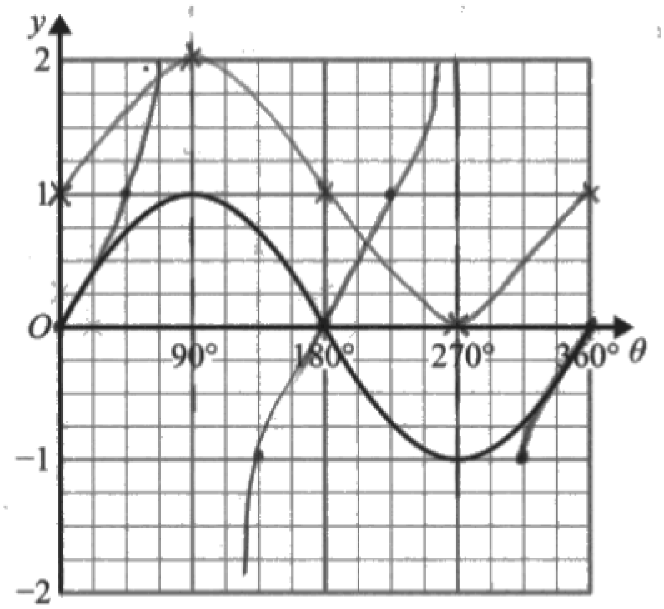
Note that in part (b), the candidates' graphs meet only once, at around $225°$. In part (c)(i), the "6" scores M1 as it is the number of intersections for the candidates' graphs multiplied by 6. The answer of 5 scores the B1 follow through mark as it is one less than the number of solutions for (c)(i).

Exemplar response B

(a). y = 4sinθ

(90, 1) → (90, 4)

(270, -1) → (270, -4)



(c).

(i): tanθ = 1+sinθ

360° → 2 solutions

2160° → x

$\frac{360°x}{360} = \frac{4320}{360}$

x = 12 solutions.

(ii). 360° → 2

1980 → x

$\frac{360x}{360} = \frac{1980 \times 2}{360}$

x = 11 solutions

Examiner's comments:

This response was given 6 marks. (a) B1 B1 (b) B1 B0 (c) M1 A1 B1

Parts (a) and (c) are fully correct.

In part (b), the graph for $y = 1 + \sin\theta$ is acceptable. The graph for $y = \tan\theta$ is acceptable in quadrants one, two and three but the branch in quadrant four is incomplete.

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