Pearson
Edexcel

INTERNATIONAL ADVANCED LEVEL

PURE MATHEMATICS

Exemplars with examiner commentaries
Unit 2 - WMA12

Contents

Introduction

1.1 About this booklet

This booklet has been produced to support teachers delivering the Pearson Edexcel International A Level Pure Mathematics specification. The Paper WMA12 exemplar materials will enable teachers to guide their students in the application of knowledge and skills required to successfully complete this course. The booklet looks at questions 1, 3, 7, 8, 10 from the June 2019 examination series, showing real candidate responses to questions and how examiners have applied the mark schemes to demonstrate how student responses should be marked.

1.2 How to use this booklet

Each example covered in this booklet contains:

- Question
- Mark scheme
- Example responses for the selected question
- Example of the marker grading decision based on the mark scheme, accompanied by examiner commentary including the justification for the decision and guidance on how the answer can be improved to earn more marks.

The examples highlight the achievement of the assessment objectives at lower to higher levels of candidate responses.

Centres should use this content to support their internal assessment of students and incorporate examination skills into the delivery of the specification.

1.3 Further support

A range of materials are available from the Pearson qualifications website to support you in planning and delivering this specification.

Centres may find it beneficial to review this document in conjunction with the Examiner's Report and other assessment and support materials available on the Pearson Qualifications website.

Question 1

1. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = 4 - a_n$$
$$a_1 = 3$$

Find the value of

(a) (i) a_2

(ii) a_{107}

(2)

(b) $\sum_{n=1}^{200}(2a_n - 1)$

(2)

Mark scheme

Question Number	Scheme	Marks
1.(a)	(i) $a_2 = 1$	B1
	(ii) $a_{107} = 3$	B1
		(2)
(b)	$\sum_{n=1}^{200}(2a_n - 1) = 5+1+5+1+\ldots\ldots.5+1 = 100\times(5+1)$	M1
	$= 600$	A1
		(2)
		(4 marks)

Notes

(a) (i)
B1 $a_2 = 1$ Accept the sight of 1. Ignore incorrect working

(a)(ii)
B1 $a_{107} = 3$ Accept sight of just 3. Ignore incorrect working
If there are lots of 1's and 3's without reference to any suffices they need to choose 3.

(b)
M1 Establishes an attempt to find the sum of a series with two distinct terms.
Look for $100\times a + 100\times b$ or $200\times a + 200\times b$ where a and b are allowable terms.
Examples of allowable terms are

$a, b = 1, 5$ (which are correct)

$a, b = 1, 3$ (which are the values for (a))

$a, b = 3, 7$ (which is using $2a_n + 1$)

$a, b = 0, 5$ (which is a slip on the first value)

Methods using AP (and GP) formulae are common and score 0 marks.

A1 600.
600 should be awarded both marks as long as no incorrect working is seen

Exemplar response A

a) i. $a_2 = 4 - 3$

$a_2 = 1$

ii. $a_{107} = 3$

b) $\sum_{n=1}^{200} (2a_n - 1)$

$2 \times 1 - 1 \quad + \quad 2 \times 2 - 1 \quad + \quad 2 \times 3 - 1$

$1 \quad + \quad 3 \quad + \quad 5$

$d = +2$

$S_{200} = \frac{200}{2}\left(2 \times 1 + (199 \times 2)\right)$

$S_{200} = 40,000$

Examiner's comments:

This response was given 2 marks. (a)(i) B1 (a)(ii) B1 (b) M0 A0

Part (a) is completely correct.

In part (b), the candidate has an incorrect method and wrongly assumes that the series is arithmetic. In this case, the candidate found $\sum_{n=1}^{200} 2n - 1$. Hence, M0 A0 is awarded.

Exemplar response B

(2)

a) i) $a_2 = 4-3 = 1$ $a_3 = 3$ $a_4 = 1$

ii) 1

b) $n = 200$
$a = 3$

$100 \times (2(3) - 1) = 500$
$100 \times (2(1) - 1) = 100$

$500 + 100 = 600$

Examiner's comments:

This response was given 3 marks. (a)(i) B1 (a)(ii) B0 (b) M1 A1

This candidate knows that the sequence is made up of two distinct terms as witnessed by the 1, 3, 1 in part (a). Whilst a_2 is correct, in the candidate's calculation of odd and even terms there has been a slip made on finding the value of a_{107}.

Part (b) is a completely correct method and answer.

Question 3

3. (i) Use algebra to prove that for all real values of x

$$(x-4)^2 \geqslant 2x-9$$

(3)

(ii) Show that the following statement is untrue.

$2^n + 1$ is a prime number for all values of n, $n \in \mathbb{N}$

(1)

Mark scheme

Question Number	Scheme	Marks
3. (i)	$(x-4)^2 \geqslant 2x-9 \Rightarrow x^2-10x+25 \ldots 0$	M1
	$\Rightarrow (x-5)^2 \ldots 0$	A1
	Explains that "square numbers are greater than or equal to zero" hence (as $x \in \mathbb{R}$), $\Rightarrow (x-4)^2 \geqslant 2x-9$ *	A1*
		(3)
(ii)	Shows that it is not true for a value of n Eg. When $n=3$, $2^n+1=8+1=9$ × Not prime	B1
		(1) **(4 marks)**

Notes

(i) A proof starting with the given statement

M1 Attempts to expand $(x-4)^2$ and work from form $(x-4)^2 \ldots 2x-9$ to form a 3TQ on one side of an equation or an inequality

A1 Achieves both $x^2-10x+25$ and $(x-5)^2$. Allow $(x-5)^2$ written as $(x-5)(x-5)$

A1* For a correct proof. Eg

"square numbers are **greater than or equal** to zero", hence (as $x \in \mathbb{R}$), $(x-5)^2 \geqslant 0$

$\Rightarrow (x-4)^2 \geqslant 2x-9$

This requires (1) Correct algebra throughout, (2) a correct explanation concerning square numbers and (3) a reference back to the original statement

Answers via b^2-4ac are unlikely to be correct. Whilst it is true that there is only one root and therefore it touches the x-axis, it does not show that it is always positive. The explanation could involve a sketch of $y=(x-5)^2$ but it must be accurate with a minimum on the +ve x axis with some statement alluding to why this shows $(x-5)^2 \geqslant 0$

Approaches via odd and even numbers will usually not score anything. They would need to proceed using the main scheme via $(2m-4)^2 \geqslant 4m-9$ and $(2m-1-4)^2 \geqslant 2(2m-1)-9$

Alt to (i) via contradiction

Proof by contradiction is acceptable and marks in a similar way

M1 For setting up the contradiction

'Assume that there is an x such that $(x-4)^2 < 2x-9 \Rightarrow x^2 - 10x + 25 ... 0$

A1 $\Rightarrow (x-5)^2 ... 0$ or $(x-5)(x-5)...0$

A1* This is not true as square numbers are always greater than or equal to 0, hence $(x-4)^2 \geqslant 2x-9$

Alt to part (i) States $(x-5)^2 \geqslant 0$

$\Rightarrow x^2 - 10x + 25 \geqslant 0$

$\Rightarrow x^2 - 8x - 16 \geqslant 2x - 9$

$\Rightarrow (x-4)^2 \geqslant 2x - 9$

M1 States $(x-5)^2 \geqslant 0$ and attempts to expand. There is no explanation required here

A1 Rearranges to reach $x^2 - 8x - 16 \geqslant 2x - 9$

A1* Reaches the given answer $(x-4)^2 \geqslant 2x-9$ with no errors

(ii)

B1 Shows that it is not true for a value of n

This requires a calculation (and value found) with a minimal statement that it is not true

Eg. '$2^6 + 1 = 65$ which is not prime' or '$2^5 + 1 = 33$ ×'

Condone sloppily expressed proofs. Eg. ' $2^7 + 1 = \frac{129}{3} = 43$ which is not prime'

Condone implied proofs where candidates write $2^5 + 1 = 33$ which has a factor of 11

If there are lots of calculations mark positively.

Only one value is required to be found (with the relevant statement) to score the B1

The calculation cannot be incorrect. Eg. $2^3 + 1 = 10$ which is not prime

Exemplar response A

i) $(1-4)^2 \geq 2(1)-9$

$25 \geq -6$

$(-1-4)^2 \geq 2(-1)-9$

$25 \geq -11$

$(1.5-4)^2 \geq 2(1.5)-9$

$6.25 \geq -6$

$(0-4)^2 \geq 2(0)-9$

$16 \geq -9$

All positive, negative, decimals and zero (all real values) obey this.

ii) $2^1+1=3$

$2^2+1=5$

$2^3+1=9$ → divisible by 3 as well, not a prime number

$2^4+1=$

Examiner's comments:

This response was given 1 mark. (i) M0 A0 A0 (ii) B1

This sort of approach where a candidate merely substitutes various values into the given inequality were common and resulted in no marks. The question demands that algebra is used.

Part (ii) scores the B1 mark as there is a correct calculation and a correct statement. In this case, 9 is not prime as it is divisible by 3.

Exemplar response B

(i) Assume a number $= (n-5)$

$(n-5)^2$ must be equal to or greater than 0

$(n-5)^2 \geqslant 0$

$n^2 - 10n + 25 \geqslant 0$ (adding ~~(2n-9)~~ $2x-9$ on both sides)

$n^2 - 8n + 16 \geqslant 2x - 9$

$(n-4)^2 \geqslant 2n - 9$

* Shown

(ii) when $n = 3$

$2^n + 1 = 8 + 1 = 9$

9 is not a prime number as it is divisible by 3. *(Shown)

Examiner's comments:

This response was given 4 marks. (i) M1 A1 A1 (ii) B1

Part (i) is a perfect response by the candidate.

Part (ii) scores the B1 mark as there is a correct calculation and a correct statement. In this case, 9 is not a prime number as it is divisible by 3.

Question 7

7. Kim starts working for a company.

- In year 1 her annual salary will be £16 200
- In year 10 her annual salary is predicted to be £31 500

Model *A* assumes that her annual salary will increase by the same amount each year.

(a) According to model *A*, determine Kim's annual salary in year 2.

(3)

Model *B* assumes that her annual salary will increase by the same percentage each year.

(b) According to model *B*, determine Kim's annual salary in year 2. Give your answer to the nearest £10

(3)

(c) Calculate, according to the two models, the difference between the total amounts that Kim is predicted to earn from year 1 to year 10 inclusive. Give your answer to the nearest £10

(3)

Mark scheme

Question Number	Scheme	Marks
7.(a)	Attempts to use $31\,500 = 16\,200 + 9d$ to find 'd'	M1
	For 16 200 + their d = (1700) where d has been found by an allowable method	M1
	Year 2 salary is (£)17 900	A1
		(3)
(b)	Attempts to use $31\,500 = 16\,200r^9$ to find 'r'	M1
	For 16 2000 × their $r = (1.077)$ where r has been found by an allowable method	M1
	Year 2 salary in the range $17440 \leqslant S \leqslant 17450$	A1
		(3)
(c)	Attempts $\frac{10}{2}\{16200+31500\}$ or $\frac{16200\left(1.077^{10}-1\right)}{1.077-1}$	M1
	Finds $\pm\left(\frac{10}{2}\{16200+31500\}-\frac{16200\left('1.077'^{10}-1\right)}{'1.077'-1}\right)$	dM1
	Difference = £7480 cao	A1
		(3)
		(9 marks)

Notes
(a) M1 Attempts to use the AP formula in an attempt to find 'd' Accept an attempt at $31\,500 = 16\,200 + 9d$ resulting in a value for d. Accept the calculation $\frac{31500-16200}{9}$ condoning slips on the 31500 and 16200 M1 A correct attempt to find the second term by adding 16 200 to their 'd' which must have been found via an allowable method. Allow d to be found from an "incorrect" AP formula with $10d$ being used instead of $9d$. Eg $31\,500 = 16\,200 + 10d$ or more likely $\frac{31500-16200}{10} = 1530$ usually leading to an answer of 17730 A1 Year 2 salary is (£) 17 900 (b) M1 Attempts to use the GP formula in an attempt to find 'r' Accept an attempt at $31\,500 = 16\,200r^9 \Rightarrow r^9 = \frac{31500}{16200} \Rightarrow r = ...$ condoning numerical slips. Accept the calculation $\sqrt[9]{\frac{31500}{16200}}$ or $\sqrt[9]{\frac{35}{18}}$ condoning slips on the 31500 and 16200. It will most likely be implied by a value of r rounding to 1.08 Accept an attempt at $31\,500 = 16\,200r^9$ via logs condoning slips but correct log work must be seen
M1 A correct attempt to find the second term by multiplying 16 200 by their 'r' which must have been found via an allowable method. Allow r to be found from an "incorrect" GP formula with 10 being used instead of 9. Eg following $31\,500 = 16\,200r^{10}$ or $\sqrt[10]{\frac{31500}{16200}}$. You may also award, condoning slips, for an attempt at $16200 \times r$ where r is their solution of $31\,500 = 16\,200r^n$ where $n = 9$ or 10 A1 For an answer in the range $£17440 \leqslant S \leqslant 17450$ Note that $r = 1.077 \Rightarrow 17447.40$ (c) M1 A correct method to find the sum of either the AP or the GP For the AP accept an attempt at either $\frac{10}{2}\{16200+31500\}$ or $\frac{10}{2}\{2\times16200+9\times'd'\}$ For the GP accept an attempt at either $\frac{16200('r'^{10}-1)}{'r'-1}$ or $\frac{16200(1-'r'^{10})}{1-'r'}$ dM1 Both formulae must be attempted "correctly" (see above) and the difference taken (either way around) FYI if d and r are correct, the sums are £238 500 and £231 019.(24) A1 Difference = £7480 **CAO. Note that this answer is found using the unrounded value for *r*.** Note that using the rounded value will give £7130 which is A0 If the solutions for (a) and (b) are reversed, eg GP in (a) and AP in (b) then please send to review. **(i) General approach to marking part (i) This is now marked M1 A1 M1 A1 on epen** M1 Takes log of both sides and uses the power law. Accept any base. Condone missing brackets A1 For a correct linear equation in x which only involve logs of base 2 usually $\log_2 6$, $\log_2 2$ or $\log_2 8$ but sometimes $\log_2 \frac{3}{4}$ and others so read each solution carefully M1 Attempts to use a log law to create a linear equation in $\log_2 3$ Eg. $\log_2 6 = \log_2 2 + \log_2 3$ which is implied by $\log_2 6 = 1 + \log_2 3$ Eg. $\log_2 \frac{3}{4} = \log_2 3 - \log_2 4$ which may be implied by $\log_2 3 - 2$ A1 For $x = -\frac{1}{3} + \frac{\log_2 3}{6}$ oe in the form required by the question. Note that $x = \frac{\log_2 3 - 2}{6}$ is A0

Exemplar response A

(3)

a) $31500 - 16200 = £15300$

$\frac{15300}{10} = 1530$

~~salary in year 2 = 31500 + 1530 = £33030~~

$= 16200 + 1530$

$= £17730$

b) $\frac{31500}{16200} = r^{10-1}$

$r^{9} = \frac{35}{18}$

$r = 1.08$

$U_2 = 16200 \times 1.08^{2-1}$

$= £17490$

c) ~~$S_n = \frac{1}{2}n(a+l)$~~

$$S_n = \frac{1}{2}n\left(2a + (n-1)d\right)$$

$$= \frac{10}{2}\left(2 \times 16200 + 1530(10-1)\right)$$

$$= £230\,850$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$= \frac{16200(1-1.08^{10})}{1-1.08}$$

$$= £231019.2444$$

difference $= 231019.2444 - 230850$

$= £169.2$

$= £170$

Examiner's comments:

This response was given 6 marks. (a) M0 M1 A0 (b) M1 M1 A1 (c) M1 M1 A0

In part (a), the candidate uses an incorrect method to find d, the common difference. With '10' being used instead of '9 ' in the formula, it was possible, as seen here, to score the next mark for a correct method of finding the salary in year 2.

Part (b) is a completely correct method with suitable accuracy shown.

In part (c), both method marks are scored for using the appropriate formulae correctly with their values of n, a, d and r. The accuracy mark is lost due to their value of d being 1530.

Exemplar response B

7) a) $a = 16200$
$n = 2$
$d = 1700$

$31500 - 16200 = 15300$

$\frac{15300}{9} = 1700 = d$

$u_2 = 16200 + (2-1) \times 1700$
$= £17900$

b) $31500 = 16200 \times r^9$

$\frac{35}{18} = r^9$

$\sqrt[9]{\frac{35}{18}} = r = 1.077$

$u_2 = 16200 \times 1.077^{(2-1)} = 17447.4 \Rightarrow £17447$

c) $S_n = \frac{16200(1 - 1.077^{10})}{1 - 1.077} = 231365.24 \Rightarrow £231365$

$S_n = \frac{1}{2}5(2 \times 16200 + (9) \times 1700) = £238500$

$238500 - 231365$ $£7135$

Examiner's comments:

This response was given 8 marks. (a) M1 M1 A1 (b) M1 M1 A1 (c) M1 M1 A0

Parts (a) and (b) are completely correct. Note that the answer for part (b) lies in the allowable range of £17440 to £17450.

In part (c), both method marks are scored for using the appropriate formulae correctly with their values of n, a, d and r. The accuracy mark is lost due to a failure of not using the unrounded value for r. An answer of £7480 was the only acceptable response here.

Question 8

8. (i) Find the exact solution of the equation

$$8^{2x+1} = 6$$

giving your answer in the form $a + b\log_2 3$, where a and b are constants to be found.

(4)

(ii) Using the laws of logarithms, solve

$$\log_5(7 - 2y) = 2\log_5(y + 1) - 1$$

(5)

Mark scheme

Question Number	Please read notes for 8(i) before looking at scheme				Marks
8.(i)	$8^{2x+1} = 6 \Rightarrow 2x+1 = \log_8 6$	M1	$2^{6x+3} = 6$		
	$\Rightarrow 2x+1 = \dfrac{\log_2 6}{\log_2 8}$	A1	$\Rightarrow (6x+3)\log_2 2 = \log_2 6$	M1 A1	
	$\Rightarrow 2x+1 = \dfrac{\log_2 2 + \log_2 3}{3}$	M1	$\Rightarrow (6x+3) = \log_2 2 + \log_2 3$	M1	
	$\Rightarrow x = \dfrac{\log_2 3}{6} - \dfrac{1}{3}$	A1	$\Rightarrow x = \dfrac{\log_2 3}{6} - \dfrac{1}{3}$	A1	
					(4)
(ii)	$\log_5(7-2y) = 2\log_5(y+1) - 1$		$2\log_5(y+1) - \log_5(7-2y) = 1$		
	$\log_5(7-2y) = \log_5(y+1)^2 - 1$		$\log_5(y+1)^2 - \log_5(7-2y) = 1$		M1
	$\log_5(7-2y) = \log_5(y+1)^2 - \log_5 5$		$\log_5 \dfrac{(y+1)^2}{(7-2y)} = 1$		dM1
	$(7-2y) = \dfrac{(y+1)^2}{5}$		$\dfrac{(y+1)^2}{(7-2y)} = 5$		A1
	$y^2 + 12y - 34 = 0 \Rightarrow y =$		$y^2 + 12y - 34 = 0 \Rightarrow y =$		ddM1
	$y = -6 + \sqrt{70}$ oe only				A1
					(5)
					(9 marks)

Notes

There are many different ways to attempt this but essentially can be marked in a similar way.

If index work is used marks are not scored until the log work is seen

Eg 1: $8^{2x+1} = 6 \Rightarrow 8^{2x} \times 8 = 6 \Rightarrow 8^{2x} = \frac{3}{4}$.

1^{ST} M1 is scored for $2x = \log_8 \frac{3}{4}$ and then 1^{ST} A1 for $2x = \frac{\log_2 \frac{3}{4}}{\log_2 8}$

but BOTH of these marks would be scored for $2x \log_2 8 = \log_2 \frac{3}{4}$

2^{nd} M1 would then be awarded for $\log_2 \frac{3}{4} = \log_2 3 - \log_2 4$ which may be implied by $\log_2 3 - 2$

Two more examples where the candidate initially uses index work.

$8^{2x+1} = 6 \Rightarrow 2^{3(2x+1)} = 6$ $3(2x+1) = \log_2 6$ is M1 A1 as it is a correct linear equation in x involving a $\log_2$ term	$8^{2x+1} = 6 \Rightarrow 64^x = \frac{3}{4}$ $\Rightarrow x = \log_{64} \frac{3}{4}$ is M1 But $\Rightarrow x \log_2 64 = \log_2 \frac{3}{4}$ is M1 A1

(ii)

M1 Attempts a correct log law. This may include

$2\log_5(y+1) \to \log_5(y+1)^2 \quad 1 \to \log_5 5$

You may award this following incorrect work. Eg

$1 = 2\log_5(y+1) - \log_5(7-2y) \Rightarrow 1 = \log_5 2(y+1) - \log_5(7-2y) \Rightarrow 1 = \log_5 \frac{2(y+1)}{(7-2y)}$

dM1 Uses two correct log laws. It may not be awarded following errors (see above)

It is awarded for $2\log_5(y+1) - 1 = \log_5 \frac{(y+1)^2}{5}$, $2\log_5(y+1) - \log_5(7-2y) = \log_5 \frac{(y+1)^2}{(7-2y)}$

$1 + \log_5(7-2y) = \log_5 5(7-2y)$ or $2\log_5(y+1) - 1 = \log_5(y+1)^2 - \log_5 5$

A1 A correct equation in 'y' not involving logs

ddM1 A correct attempt at finding at least one value of y from a 3TQ in y

All previous M's must have been awarded. It can be awarded for decimal answer(s), 2.4 and − 14.4

A1 $y = -6 + \sqrt{70}$ or exact equivalent only.

It cannot be the decimal equivalent but award if the candidate chooses 2.4 following the exact answer. If $y = -6 \pm \sqrt{70}$ then the final A mark is withheld

---

Special case:

Candidates who write

$\log_5(y+1)^2 - \log_5(7-2y) = 1 \Rightarrow \frac{\log_5(y+1)^2}{\log_5(7-2y)} = 1 \Rightarrow \frac{(y+1)^2}{(7-2y)} = 5$

can score M1 dM0 A0 ddM1 A1 if they find the correct answer.

Exemplar response A

i) $8^{2x+1} = 6$

$\log 8^{2x+1} = \log 6$

$2x+1 \log 8 = \log 6$

$2x+1 = \frac{\log 6}{\log 8}$

$2x+1 = 0.8617$; $2x = 1.8616$

$x = 0.931$

~~[illegible]~~ $2 + \log_2 3$

ii) $\log_5 (7-2y) = 2\log_5 (y+1) - 1$

$\log_5 (7-2y) = 2\log_5 (y+1)$ ~~[illegible]~~ $- \log_5 5$

$\log_5 (7-2y) - 2\log_5 (y+1) = -\log_5 5$

$\log_5 (7-2y) - \log_5 (y+1)^2 = -\log_5 5$ $\quad (y+1)(y+1)$

$y^2+y+y+1$

y^2+2y+1

$\log_5 \frac{7-2y}{(y+1)^2} = -\log_5 5$

$\frac{7-2y}{(y+1)^2} = 5$; $7-2y = 5(y+1)^2$

$7-2y = 5(y^2+2y+1)$

$7-2y = 5y^2+10y+5$

$7 = 5y^2+12y+5$

$5y^2+12y-2=0$

$\frac{-12 \pm \sqrt{12^2-4\times5\times(-2)}}{10}$

$\frac{-12 \pm \sqrt{184}}{10} = \frac{-12+\sqrt{184}}{10} = \frac{-6+\sqrt{46}}{5}$ or $\frac{\pm 6+\sqrt{46}}{5}$

Examiner's comments:

This response was given 4 marks. (i) M1 A0 M0 A0 (ii) M1 M1 A0 M1 A0

In part (i), the candidate takes logs of both sides and uses the power law. No more marks can be awarded as there is no attempt to write the answer in the form $\log_2$, as demanded by the question.

In part (ii), two log laws are used correctly in writing $\log_5(7-2y)-2\log_5(y+1)$ in the form $\log_5\frac{(7-2y)}{(y+1)^2}$. Although -1 is written correctly as $-\log_5 5$, an incorrect equation in y is formed.

This is solved correctly; however, scoring the method mark.

Exemplar response B

i) $8^{2x+1} = 6$

$\Rightarrow (2^3)^{2x+1} = 2 \times 3$

$\Rightarrow 2^{6x+3} = 2 \times 3.$

$\Rightarrow 6x+3 = \log_2(2\times 3)$

$\Rightarrow 6x+3 = \log_2 2 + \log_2 3$

$\Rightarrow 6x+3 = 1 + \log_2 3$

$\Rightarrow 6x = 1-3 + \log_2 3$

$\Rightarrow x = \dfrac{-2+\log_2 3}{6}$

$= -\dfrac{1}{3} + \dfrac{\log_2 3}{6}$

$\therefore a = -\dfrac{1}{3}$

$b = \dfrac{1}{6}$

ii) $\log_5(7-2y) = 2\log_5(y+1) - 1$

$\Rightarrow \log_5(7-2y) = \log_5(y+1)^2 - 1$

$\Rightarrow \log_5(7-2y) - \log_5(y+1)^2 = -1$

$\Rightarrow \log_5\left[\dfrac{7-2y}{(y+1)^2}\right] = -1$

$\Rightarrow \dfrac{7-2y}{(y+1)^2} = \dfrac{1}{5}$

$(y+1)^2$

$y^2 + 2(y)(1) + (1)^2$

$y^2 + 2y + 1$

$\Rightarrow 5(7-2y) = y^2 + 2y + 1$

$\Rightarrow 35 - 10y = y^2 + 2y + 1$

$\Rightarrow y^2 + 2y + 10y + 1 - 35 = 0$

$y^2 + 12y - 34 = 0$

$\dfrac{-(12) \pm \sqrt{(12)^2 - 4(1)(-34)}}{2(1)}$

$y = -6+\sqrt{70}\ ,\ -6-\sqrt{70}$

Examiner's comments:

This response was given 8 marks. (i) M1 A1 M1 A1 (ii) M1 M1 A1 M1 A0

In part (i), the candidate scores M1 A1 on their 4th line before proceeding to the correct answer.

In part (ii), an almost perfect solution is marred by the fact that the candidate gave both answers to their quadratic. In this question, the solution $-6-\sqrt{70}$, if found, had to be rejected as it could not be considered a solution of $\log_5(7-2y) = 2\log_5(y+1) - 1$.

Question 10

10.

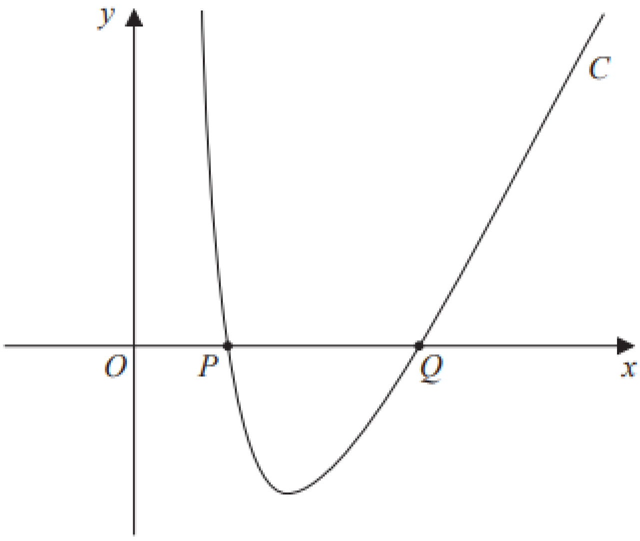


Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = f(x)$ where

$$f(x) = \frac{36}{x^2} + 2x - 13 \qquad x > 0$$

Using calculus,

(a) find the range of values of x for which f(x) is increasing,

(4)

(b) show that $\int_2^9 \left(\frac{36}{x^2} + 2x - 13\right)dx = 0$

(4)

The point $P(2, 0)$ and the point $Q(6, 0)$ lie on C.

Given $\int_2^6 \left(\frac{36}{x^2} + 2x - 13\right)dx = -8$

(c) (i) state the value of $\int_6^9 \left(\frac{36}{x^2} + 2x - 13\right)dx$

(ii) find the value of the constant k such that $\int_2^6 \left(\frac{36}{x^2} + 2x + k\right)dx = 0$

(3)

Mark scheme

Question Number	Scheme	Marks
10 (a)	$(\mathrm{f}'(x)) = -\frac{72}{x^3} + 2$	M1 A1
	Attempts to solve $\mathrm{f}'(x) = 0 \Rightarrow x = \ldots$ via $x^{\pm n} = k, \quad k > 0$	dM1
	$x > \sqrt[3]{36}$ oe	A1
		(4)
(b)	$\int \frac{36}{x^2} + 2x - 13\,\mathrm{d}x = -\frac{36}{x} + x^2 - 13x\,(+c)$	M1 A1
	Uses limits 9 and 2 $= \left(-\frac{36}{9} + 9^2 - 13\times 9\right) - \left(-\frac{36}{2} + 2^2 - 13\times 2\right) = 0$ *	dM1 A1*
		(4)
(c)(i)	8	B1
(ii)	$\int_2^6 \left(\frac{36}{x^2} + 2x + k\right)\mathrm{d}x = 0 \Rightarrow \left[-\frac{36}{x} + x^2 + kx\right]_2^6 = 0 \Rightarrow (30+6k) - (-14+2k) = 0$	
	$44 + 4k = 0 \Rightarrow k = -11$	M1 A1
		(3)
		(11 marks)

Notes

(a)

M1 Attempts $\mathrm{f}'(x)$ with one index correct. Allow for $x^{-2} \to x^{-3}$ or $2x \to 2$

A1 $\mathrm{f}'(x) = -\frac{72}{x^3} + 2$ correct but may be unsimplified $\mathrm{f}'(x) = 36 \times -2x^{-3} + 2$

dM1 Attempts to find where $\mathrm{f}'(x) = 0$. Score for $x^n = k$ where $k > 0$ and $n \neq \pm 1$ leading to $x = \ldots$
Do not allow this to be scored from an equation that is adapted incorrectly to get a positive k.
Allow this to be scored from an attempt at solving $\mathrm{f}'(x)\ldots 0$ where **...** can be any inequality

A1 Achieves $x > \sqrt[3]{36}$ or $x > 6^{\frac{2}{3}}$ Allow $x \geqslant \sqrt[3]{36}$ or $x \geqslant 6^{\frac{2}{3}}$ but not $x > \left(\frac{1}{36}\right)^{-\frac{1}{3}}$

We require an exact value but remember to isw. An answer of 3.302 usually implies the first 3 marks.

(b)

M1 For $x^n \to x^{n+1}$ seen on either $\frac{36}{x^2}$ or $2x$. Indices must be processed. eg $x^{1+1} \to x^2$

A1 $\int \frac{36}{x^2} + 2x - 13\,\mathrm{d}x = -\frac{36}{x} + x^2 - 13x$ which may be unsimplified. Eg $x^2 \leftrightarrow \frac{2x^2}{2}$ Allow with $+ c$

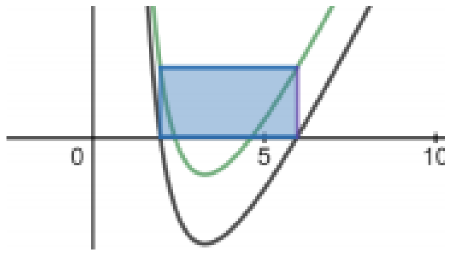
dM1 Substitutes 9 and 2 into their integral and subtracts either way around. Condone missing brackets
Dependent upon the previous M

A1* Completely correct integration with either embedded values seen or calculated values $(-40) - (-40)$
Note that this is a given answer and so the bracketing must be correct.

(c)(i)
B1 For sight of 8. Allow this to be scored from a restart, from a calculator or … = 8

(c)(ii)
M1 This may be awarded in a variety of ways

- A restart (See scheme). For this to be awarded all terms must be integrated with $k \to kx$, the limits 6 and 2 applied, the linear expression in k must be set equal to 0 and a solution attempted.
- An attempt at solving $\int_2^6 k + 13 \, dx = 8$ or equivalent. Look for the linear equation $-8 + 4(13 + k) = 0$ or $4(13 + k) = 8$ and a solution attempted.
- Recognising that the curve needs to be moved up 2 units.
- Sight of $\frac{8}{6-2}$ or $-13 + 2$



A1 $k = -11$. This alone can be awarded both marks as long as no incorrect working is seen.

Exemplar response A

(3)

(a) $f(x) = 36x^{-2} + 2x - 13$

$f'(x) = \frac{36x^{-1}}{-1} + \frac{2x^2}{2} - 13x$

$f'(x) = -36x^{-1} + x^2 - 13x$

$-\frac{36}{x} + x^2 - 13x > 0 \quad \times x$

Question 10 continued

(a) $-36 + x^3 - 13x^2 > 0$

$x^3 - 13x^2 - 36 > 0$

(b) $\int_2^9 \left(\frac{36}{x^2} + 2x - 13\right) dx$

$\int = 36x^{-2} + 2x - 13$

$\int = \frac{36x^{-1}}{-1} + \frac{2x^2}{2} - 13x$

$= -36x^{-1} + x^2 - 13x$

$\int_2^9 \left(\frac{36}{x^2} + 2x - 13\right) = \left[\frac{-36}{(9)} + (9)^2 - 13(9)\right] - \left[\frac{-36}{(2)} + (2)^2 - 13(2)\right]$

$= -40 - (-40)$

$= \underline{\underline{0}}$

(c)(i) $\int_6^9 \left(\frac{36}{x^2} + 2x - 13\right) dx = -6$

(ii) $\int \left(36x^{-2} + 2x + k\right) dx = \frac{36x^{-1}}{-1} + \frac{2x^2}{2} + kx$

$= -36x^{-1} + x^2 + kx$

$\int_2^6 \left(36x^{-2} + 2x + k\right) dx = \left[-36(6)^{-1} + (6)^2 + k(6)\right] - \left[-36(2)^{-1} + (2)^2 + k(2)\right] = (30 + 6k) - (2k - 14) = 0$

$30 - 6k - (2k - 14) = 0$

$44 - 8k = 0$

$-8k = -44$

$k = \frac{-44}{-8} = \frac{11}{2}$

Examiner's comments:

This response was given 5 marks. (a) M0 A0 M0 A0 (b) M1 A1 M1 A1 (c) B0 M1 A0

In part (a), the candidate integrates and multiplies by *x*. This type of response was common.

Part (b) is fully correct and shows all the necessary steps required leading to an answer of 0.

In part (c)(ii), the method mark is scored for setting the integrated function with limits of 2 and 6 equal to zero. A slip in the 4th last line where +6k is written -6k means that the accuracy mark is lost. Part (c)(i) is incorrect.

Exemplar response B

(a) $f(x) = 36x^{-2} + 2x - 13$

$\frac{dy}{dx} = -72x^{-3} + 2 \leq 0$

$\frac{-72}{x^3} \leq \frac{-2}{1}$

$-2x^3 \leq -72$

$x^3 \leq 36 \qquad x \leq 3.3$

(b) $\int_2^9 36x^{-2} + 2x - 13$

$= \frac{36x^{-1}}{-1} + \frac{2x^2}{2} - 13x$

$\left[\frac{-36}{9} + 9^2 - 13(9)\right] - \left[\frac{-36}{2} + 2^2 - 13(2)\right]$

$-40 + 40 = 0$

c (i) -8

(ii) $\frac{36x^{-1}}{-1} + x^2 - kx = \frac{-36}{x} + x^2 - kx$

$\left[\frac{-36}{6} + 6^2 - 6k\right] - \left[\frac{-36}{2} + 2^2 - 2k\right] = 0$

$(-6 + 36 - 6k) - (-18 + 4 - 2k) = 0$

$-6 + 36 - 6k + 18 - 4 + 2k = 0$

$44 = 4k$

~~$k = -11$~~ $\qquad k = -11$

Examiner's comments:

This response was given 9 marks. (a) M1 A1 M1 A0 (b) M1 A1 M1 A1 (c) B0 M1 A1

In part (a), the candidate differentiates correctly and finds the value where $f'(x) = 0$. This is implied by $x^3 \ldots 36$ followed by $x \ldots 3.3$. The accuracy mark is not scored as the inequality is incorrect and this answer had to be exact. Note that either $x > \sqrt[3]{36}$ or $x \geqslant \sqrt[3]{36}$ scored the A1 mark.

Part (b) is fully correct and shows all the necessary steps required leading to an answer of 0.

The candidate's answer of -8 in (c)(i) was common and scored B0.

In part (c)(ii), the method mark is scored for setting the integrated function with limits of 2 and 6 equal to zero. The candidate then proceeds to the correct answer *k* = -11.

FOR INFORMATION ABOUT EDEXCEL, BTEC OR LCCI QUALIFICATIONS
VISIT QUALIFICATIONS.PEARSON.COM

EDEXCEL IS A REGISTERED TRADEMARK OF PEARSON EDUCATION LIMITED

PEARSON EDUCATION LIMITED. REGISTERED IN ENGLAND AND WALES NO. 872828
REGISTERED OFFICE: 80 STRAND, LONDON WC2R 0RL
VAT REG NO GB 278 537121

Pearson
Edexcel