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Principal Examiner Feedback

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Pearson Edexcel GCE
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Paper 02 Pure Mathematics

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General

This paper offered plenty of opportunity for candidates to show what they had learnt. The early questions were accessible to candidates of all abilities, and this was reflected by the modal mark for several of the early questions being full marks. It was often the case that prepared candidates scored the majority of the marks for the first 6 questions with 7 (b) usually offering the first significant challenge. The longer, later questions provided suitable challenge for stronger candidates but also gave opportunities for restarts for those who struggled with earlier parts of questions. There was also a range of independent marks that candidates could access. There were some instances where candidates embarked on lengthy and time-consuming algebraic solutions when it may have been beneficial to consider if there would be a more efficient approach. This was particularly true with question 14(b).

There were instances where marks were lost unnecessarily. The following observations should be helpful to students:

- specific answers not identified when a selection was required e.g. in 2(c) and 4(b)(i)
- values sometimes not given to the required accuracy as demanded in the question, e.g. in 13(a)
- answers not written as printed on the question paper e.g. the "= 0" missing from 3(b)
- answers not given as requested, e.g. the equation given as $y = \ldots$ not $H = \ldots$ in 9(a) and the partial fractions not written down in 12(a)
- take care with the use of brackets e.g. writing $\cos 3\theta = 1 - \frac{3\theta^2}{2}$ when $\cos 3\theta = 1 - \frac{(3\theta)^2}{2}$ was meant.

These points aside, there were many succinct and elegant solutions to the more demanding problems towards the end of the paper.

Question 1

This question was very accessible with the vast majority obtaining full marks on all three parts.

In part (a), a few candidates integrated the expression and hence obtained no marks as they did not have a linear equation to solve in part (b), but this was a rarity. One of the more common reasons to lose a mark was the inclusion of a spurious "+ c" with one or both derivatives. A few candidates attempted the differentiation but made mistakes with the coefficients (usually resulting from dividing by rather than multiplying by the index). Another error seen was to "simplify" the second derivative from
$24x - 14$ to $12x - 7$.
In part (b) candidates could access the first mark for setting their linear second derivative equal to 0 and solving for x. There were a few candidates who lost the accuracy mark by giving a value of $\frac{24}{14}$ instead of $\frac{14}{24}$ and there were a surprising number of candidates who went from $24x = 14$ to $x = \frac{1}{2}$.

Question 2

Part (a) was very accessible and very well answered. The vast majority of candidates deduced the value of the common difference and then substituted this into the standard term formula for an arithmetic sequence. Those candidates that were unfamiliar with this formula were able to produce a correct calculation, or even just list the payments for all 12 months, which was acceptable. Occasionally the formula was simplified to $410 - 10n$ before substituting in $n = 12$.

Part (b) was usually answered well, and candidates were able to substitute the given values into the sum formula correctly, expand brackets and obtain the required quadratic equation in N. The fact that the common difference was negative caused some candidates algebraic difficulties. Some candidates did not realise they needed to set their expression equal to 8100 but soon realised this and added it in later in their working but a few persisted with e.g. = 0 rather than = 8100. A few had bracketing and/or arithmetic errors and some missed out the '= 0' in the given answer therefore losing the final accuracy mark. A small number of candidates attempted to use the term formula in this part. Another rarer approach involved using a series summation approach. These attempts were mostly successful as it required further maths knowledge and were therefore performed by most proficient mathematicians.

In part (c), most candidates were able to solve the given quadratic equation and achieve the method mark. Overall, this was done with a calculator. Most chose the correct answer but quite a few left both answers or chose $N = 45$, appearing not to take into account the context of the question. They needed to realise that, when the loan was fully repaid after 36 months, the borrower would not continue to make loan repayments, so the second answer of 45 months needed to be rejected. This took significant effort for some - rechecking the sum for 36 and 45 terms and then finding the value of the 36th and 45th term and then rejecting 45 when the 45th term was negative - when thinking about the context would have been easier.

Question 3

This question was very effective at highlighting many candidates' misconceptions about the relationship between function notation and graph transformations. Whilst the majority of candidates coped well with single transformations, a significant proportion demonstrated a lack of secure understanding of multi-step transformations.

Part (i) presented few difficulties for most candidates. They either understood, or had memorised, that $y = \mathrm{f}(x - 2)$ represented a translation by two units in the positive x direction. The most common incorrect answer here was (1, –2) where candidates had confused the direction of the translation.

Similarly in part (ii), a large proportion of candidates, though fewer than in part (i), knew that $y = \mathrm{f}(2x)$ represented a stretch of scale factor $\frac{1}{2}$ in the x direction. Common incorrect answers were (6, –2), from multiplying the x coordinate by 2 instead of dividing, and (3, –4), which came from multiplying the y coordinate by 2.

Part (iii) highlighted that many candidates are potentially guessing the net effect of a multi-step transformation rather than separately considering the stepwise effect of the individual transformations. It appears that they are mainly unsure of the order in which the transformations take place and, probably because of this, what effect each step will have, and on which variable. The variety and range of incorrect answers was perhaps surprising. Whilst some candidates provided evidence to explain their logic, most simply wrote an answer. In many cases candidates either scored no marks or both marks here. When just one out of the two marks was earned, there was often little evidence to suggest where the correct and incorrect values had come from. Where piecewise transformations were seen, it was clear, perhaps unsurprisingly that translations were more successfully handled than reflection or stretch transformations and sign errors were fairly common. Some candidates drew small sketches to help visualise the transformations which often proved helpful in achieving the correct answer. Occasionally, other candidates came unstuck by apparently misunderstanding that the transformations for each part of the question should be applied to $P(3, -2)$ rather than sequentially applied to the result of the previous part of the question.

Question 4

Generally, candidates found this question accessible, with many scoring full marks or nearly full marks.

In part (a) most candidates used the given recurrence relation along with $u_1 = 6$ to find $u_2 = 6k - 5$ and then $u_3 = 6k^2 - 5k - 5$, and then set $u_3 = -1$ and rearranged to obtain the given answer $6k^2 - 5k - 4 = 0$. Algebraic errors were few, but some candidates forgot to include the '= 0' in their final line and lost the second mark in this part. There were occasional attempts at using arithmetic or geometric series which gained no credit.

In part (b)(i) most candidates were able to solve the quadratic equation, but not all identified that $k = \frac{4}{3}$ was the only correct answer, or subsequently used this value, so lost the mark in this part of the question. Some candidates spent more time than they needed using the quadratic formula, where a factorisation of the quadratic would have been more direct. Use of the calculator equation solver was also permitted here. Candidates must learn to check their answers against any limitations given in the question.

When errors did occur, they were most likely to appear in part (b)(ii), although most still answered it well. Most candidates identified the requirement of the question to add the first three terms of the sequence together, and correctly used $k = \frac{4}{3}$ to calculate $u_2 = 6\left(\frac{4}{3}\right) - 5$ before finding $6 + 3 + (-1) = 8$. Others first simplified the sum to obtain $6k$ or $6k^2 + k - 4$ before substituting the value of k. A common incorrect approach was to use $u_3 = 6k^2 - 5k - 4$ in the sum. Some also used $k = \frac{4}{3}$ and $k = -\frac{1}{2}$ to calculate the sum, without identifying which was the required answer. Alternatively, a good number of candidates attempted, correctly and incorrectly, to work out the values of u_1 and u_3 even though they were given in the question. It was also noticeable that a number of candidates used the quadratic expression from part (a) in order to work out values in (b)(ii), usually assuming $u_3 = k^2 - 5k - 4$. They, therefore, did not clearly understand what this equation represented. A few candidates incorrectly attempted to use the recurrence relation to find things like $\left(\frac{4}{3}\right)(1) - 5$, $\left(\frac{4}{3}\right)(2) - 5$ and $\left(\frac{4}{3}\right)(3) - 5$, using these expressions to find their sum, and there were again occasional attempts using arithmetic or geometric series which also gained no credit. There were also a few attempts at integration. Overall, this was a question that allowed candidates to gain a good number of marks with confidence.

Question 5

This question was very accessible with many candidates scoring full marks. The most common approach was to directly apply the small angle approximations to the given expression and on the whole, this was successfully simplified to give the correct value.

Using the small angle approximation $\theta \approx 2\theta$ was most successful for the numerator, however there were a few candidates who were unable to process the multiplication. The denominator proved to be trickier to process with errors seen in the bracketing, squaring the angle, using $3\cos\theta$ rather than $\cos 3\theta$ or omitting the "1–". It was very common to see errors such as $\cos 3\theta = 1 - \frac{3\theta^2}{2}$ and $1 - \cos 3\theta = 1 - 1 - \frac{3\theta^2}{2} = -\frac{3\theta^2}{2}$. Once the numerator and denominator had been simplified, there was a sizeable minority who struggled with cancelling the fraction. Incorrect answers of $\frac{4}{3}$ and $-\frac{4}{3}$ were seen frequently, resulting from the poor use of brackets.

A significant number of candidates complicated the process by introducing trigonometric identities before applying small angle approximations, presumably not realising they could substitute the appropriate approximations directly. There were many variations in the processing such as double angle formulae and compound angle formulae which all involved a lot more work and inevitably led to complicated expressions with higher order indices in the denominator. Unfortunately, only a few candidates were successful with such approaches as there were often errors in the algebra needed for the simplification or a lack of recognition that they needed to make a limiting argument.

Question 6

In part (a) candidates struggled with the differentiation of the exponential function and were much more confident in dealing with the natural logarithm. For the exponential function, the most successful candidates often wrote an intermediate step such as $u = 4x^2 - 1$ and then used the chain rule.

Common incorrect responses seen in (a)(i) were $A\mathrm{e}^{4x^2-1}$ (A was often 4 or 8 when this was seen), $\left(4x^2-1\right)\mathrm{e}^{4x^2-1}$ (with or without brackets), $4x^2\mathrm{e}^{4x^2-1}$ and expressions involving e^{4x^2}.

Common incorrect responses seen in (a)(ii) were $\frac{1}{8x}$ and $\frac{1}{x}$.

In (a)(ii) a few candidates wrote $8\ln x = \ln x^8$ first and achieved the correct alternative $\frac{8x^7}{x^8}$.

It was disappointing to see a sizeable number of candidates confused between the "dash" notation for differentiation and inverses. So, it was fairly common to see attempts at the inverse of each function, resulting in no marks for parts (a) or (b).

Success in part (b) was dependent on having the correct forms of the derivatives in part (a).

There were often mistakes in using log rules (e.g. $\log(ab) = \log(a) \times \log(b)$) and some did not know what to do with $\ln\frac{1}{x^2}$. As with question 4, of those who proceeded correctly, some forgot to include the "= 0". Generally, candidates did whatever they could to manipulate their working, and many wrote down the given answer following completely incorrect work.

In part (c) many candidates scored all three marks often with no working, just by writing down the two numerical answers required. Candidates should however be advised to show their method as they risk losing both marks. In contrast, some candidates wrote down all the iterations they thought necessary, including the working for each one. Amongst those that didn't get the final mark, the common mistakes were, stopping at too early an iterate such as x_2 or x_3, stating an answer of 0.6707 instead of 0.6706 and not rounding to 4dp. There were a small number of candidates who didn't seem to realise they needed to keep applying the iteration to find the value for α and instead tried to solve the equation given in part (b). Others thought 0.6 or 0.7109 should be substituted into the equation for part (b), not realising a sequence needed to be generated.

Question 7

This question proved to be a differentiator between candidates. Some were unable to make any meaningful progress while others demonstrated a very intuitive grasp of the topic and were able to produce answers with very little working.

Most candidates were successful in part (a) although there were a small number of incorrect responses which had arithmetic errors, used addition of vectors rather than subtraction or had poor notation.

In part (b), many candidates did not appreciate the scaffolding in the question and did not use their answer to part (a) to help them with a strategy for part (b). Relatively few candidates used the most straightforward method to find both two possible positions of *P*, e.g., $\overrightarrow{OP} = \overrightarrow{OA} + 2\overrightarrow{AB}$ and $\overrightarrow{OP} = \overrightarrow{OA} + \frac{2}{3}\overrightarrow{AB}$. Some candidates were able to find one of the positions (usually 8**i**+15**j**+11**k**) but not the other, with a common misconception being that the second point was on the opposite side of *A*, attempting $\overrightarrow{OP} = \overrightarrow{OA} - 2\overrightarrow{AB}$. There was also some confusion with the ratio with some candidates using $\overrightarrow{OP} = \overrightarrow{OA} + \frac{1}{3}\overrightarrow{AB}$. Many candidates failed to appreciate that there were two points satisfying $|\overrightarrow{AP}| = 2|\overrightarrow{BP}|$. Relatively few candidates drew a diagram or used the given diagram on the question paper to give them some idea of where *P* could be.

It was much more common for candidates to take an unnecessarily complicated algebraic approach. Many responses calculated $|\overrightarrow{AB}| (= 3\sqrt{11})$ and progressed no further. Many responses attempted to express $|\overrightarrow{AP}| = 2|\overrightarrow{BP}|$ in terms of variables representing components of an unknown vector, showing a good understanding of how to calculate the magnitude, but struggled to formulate suitable equations. Some candidates used a fully correct method of considering the point *P* as the general point on the line $\overrightarrow{OP} = \overrightarrow{OA} + \lambda\overrightarrow{AB}$ and then setting up the equation $|\overrightarrow{AP}| = 2|\overrightarrow{BP}|$. These candidates tended to arrive at two final answers for *P*, having found two values for λ, but many had made processing errors and could only gain the method marks as a result. Other candidates set up a general point *P* in the form $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, then used $|\overrightarrow{AP}| = 2|\overrightarrow{BP}|$, extracted and solved $(x-2)^2 = 4(x-5)^2$, $(y+3)^2 = 4(y-6)^2$ and $(z-5)^2 = 4(z-8)^2$ to generate 3 pairs of possible values for the *x*, *y* and *z* components. These candidates then tended to struggle to determine the possible positions of *P* from their 6 values. Vector notation was very variable with a wide variety of non-standard notations being encountered.

It would be helpful for candidates to bear in mind that a four-mark question should not involve pages of work and that there are more efficient strategies. Whatever method was adopted, work was sometimes difficult to follow as there was little explanation and unclear notation used. In part (b), because of the volume of work they were producing, candidates often needed more space to complete their solution. Candidates are strongly advised that if they run out of space, they should use supplementary paper rather than complete their answer within the space of another question.

Question 8

A significant number of candidates found this question challenging.

In part (a) the first challenge candidates faced was combining the two terms to produce a single fraction. This was completed successfully by many candidates, either as a fraction in $\operatorname{cosec}\theta$ or a fraction in $\sin\theta$. The second challenge was using trigonometric identities correctly. The majority used $\sin^2\theta + \cos^2\theta = 1$ but the more successful candidates typically used $1 + \cot^2\theta = \operatorname{cosec}^2\theta$. It was only a minority who managed to complete the proof successfully. It was possible to complete the proof starting either from either side of the identity, however, generally those candidates who started with the left-hand side were more successful. Few, if any, candidates gave a completely correct solution starting from the right-hand side and the majority only obtained the mark for using a correct Pythagorean identity. A small number of candidates used a "meet in the middle" approach with varying degrees of success. Notational errors were not widespread, but variables were sometimes missing, or indices misplaced resulting in the loss of the final mark. Many also lost the accuracy mark because they did not show all the necessary steps by proceeding to the given answer using sine and cosine.

In part (b), candidates that spotted the connection with part (a) tended to achieve at least 3 out of the 4 marks available. Those who failed to get the first line right with the variable of $2x$ on both sides but just substituted $2\tan x\sec x$ or even $2\tan\theta\sec\theta$ on the left-hand side gained no marks in this part. Candidates took a number of different approaches to solving the correct equation. The most frequent was to obtain $\tan^2 2x = \frac{1}{2}$ but approaches involving $3\sin^2 2x = 1$ or $3\cos^2 2x = 2$ were relatively common. There were a few candidates who used the identity $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$ to reach a quadratic in $\tan^2 x$. These attempts were usually successful. Some candidates stated that the square root of $\frac{1}{2}$ is $\frac{1}{4}$ which lost them the final two marks in this part. However, those candidates that did get to one of the correct equations could normally proceed to one correct answer (which was typically $x = 17.6°$). It was only a minority of candidates who gave both correct answers. The final mark was regularly lost because only one solution was identified, or candidates had incorrectly found 45 degrees by incorrectly solving $\sec 2x = 0$ and interpreting this as $\cos 2x = 0$ rather than being undefined. Most candidates that achieved marks in this part of the questions could correctly identify the order of operations to solve their equations although there were some candidates who struggled with this. For the final mark, a few wrote down 17.7 instead of 17.6 and some had extra answers in the given range and therefore lost the accuracy mark. A small minority of candidates did not show appropriate working in part (b), sometimes merely writing down the two correct answers to the given equation. The question explicitly asked for all stages of a candidate's working as the answers can be found using a calculator. Hence candidates that gave the correct answers without appropriate working were not awarded any marks in this part of the question.

Question 9

There was a very mixed response to this question. There was a fairly even split between the two alternative methods of using a model of the form $H = ax^2 + bx + c$ and the completed square form of $H = A(x-9)^2 + B$. Most candidates recognised the need for a coefficient of x^2 (other than 1) and were therefore able to access some, if not all, the marks for this part.

Using $H = ax^2 + bx + c$, most candidates accessed the first mark by identifying that $c = 2$ and correctly using the point (20, 0.8) to form an equation in a and b. The majority of candidates then used $x = 9$ and $\frac{\mathrm{d}H}{\mathrm{d}x} = 0$ to form a second equation in a and b. A very small number of candidates formed a second equation by using another point which the curve passed through, either (–2, 0.8) or (18, 2), identified by using the symmetry of the curve. Unfortunately, not all candidates realised that the value of c was the H intercept of 2 and so lost all marks in this part.

Using $H = A(x-9)^2 + B$, the most common error was for candidates to incorrectly assume that $B = 2$. Otherwise, candidates had frequent success using this format for the model. The vast majority of responses correctly used (0, 2) and (20, 0.8) to form 2 equations in A and B. A small number of candidates used an equation of the form $H = A\left((x-9)^2 - 81\right) + B$ and some attempted a linear transformation of the curve. These were less successful. Some candidates switched their model from $H = ax^2 + bx + c$ to $H = b + a(x-9)^2$ and then confused their a's and b's assuming they were the same value.

Very few errors were made once correct equations in a and b (or A and B) had been found so via both approaches, candidates were successful in correctly solving their simultaneous equations and achieving a fully correct equation for the model. Whilst the vast majority of candidates left their final answer as an equation, some lost the final mark for failing to have $H = \mathrm{f}(x)$ and instead giving their answer as $y = \mathrm{f}(x)$. When no marks were scored in this part, the most common error was to assume $B = 2$ for the form $H = A(x-9)^2 + B$. A surprising number of candidates also attempted a linear model and also scored no marks in this part.

In part (b), most candidates were able to identify a limitation of the model, so this part was well answered, often independently to the rest of the question. Most successful responses referred to the fact that air resistance hadn't been considered. Other common responses achieving the mark included "the spin of the ball has not been considered" or "the ground is unlikely to be completely horizontal/flat". A small number of responses did correctly refer to the fact that H is unlikely to be a perfect quadratic function in x or that the ball is unlikely to travel in a vertical plane therefore adding a third dimension to the motion. The most common incorrect responses involved candidates referring to the ball outside its flight path e.g. "H will become negative" or "the ball will go below the ground". Some candidates did give vague single word answers like "air resistance" or "spin" which did not gain them any credit as there was no link to the model.

In part (c), the vast majority of candidates recognised the need to substitute $x = 16$ into their model and candidates who achieved an incorrect quadratic model in part (a) were still able to achieve the method mark. When candidates achieved full marks in part (a) they usually went on to achieve full marks in part (c) too. It was extremely rare for $H = 2.96$ to be achieved and an incorrect conclusion to follow as most candidates knew to conclude correctly, fully linking back to the question.

Very few candidates attempted the alternative approach of substituting in $H = 2.5$ into the model to create and solve a quadratic equation in x. This approach did then require candidates to understand that the (larger) solution of $x = 17.0$ meant that Chandra could not catch the ball. This seemed a more challenging approach than just substituting $x = 16$ in the model.

Question 10

Most candidates were successful in this question, showing that parametric differentiation was overall well understood by candidates.

In part (a), many candidates successfully found the given equation of the tangent using parametric differentiation as specified in the question. By far, the most common approach seen involved finding the value of t and then substituting it into an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ obtained by using $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \times \frac{\mathrm{d}t}{\mathrm{d}x}$ and then using standard coordinate geometry to find the equation of the tangent.
There were a number of careless slips and algebraic errors, with $2(t + 3) = 2t + 3$ being disappointingly common. Once the numerical gradient had been obtained, there were usually no issues in finding the equation of the tangent in the required form, showing at least one intermediate step as this was a given answer. Most candidates achieved the equation of the tangent via the Point-Slope form although a large number went via the Slope-Intercept form. A few candidates made coefficient and/or sign errors when manipulating their equation to obtain the given answer and unfortunately lost the final accuracy mark.
Those candidates who gained few, if any, marks in this question often failed to see the need to find the value of the parameter 't' for the given point P (4, 2) and were unsuccessful in using the alternative method to continue to find a value for $\frac{\mathrm{d}y}{\mathrm{d}x}$. Some candidates used an incorrect value of 't', usually $t = -5$ rather than $t = -1$, as they either did not note the restrictions for t given in the question, or did not find the t value for the y coordinate of P. A few gave two equations for each value of t found.
The attempted use of the Chain Rule was not always completed accurately. Some candidates converted their gradient from the use of the Chain Rule into the gradient of the normal which resulted in an inappropriate equation. When finding the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$, some candidates substituted 4 for their value of t, confusing it with the x coordinate of point P. Some candidates did not follow the instructions stating that the question had to be answered 'using parametric differentiation' and, consequently, scored no marks even though they achieved the required result by obtaining the Cartesian equation of the curve. Some candidates tried to complete the question by unnecessarily

converting their $\frac{dy}{dt} \times \frac{dt}{dx}$ in terms of t to be in terms of x and/or y which often led to incorrect solutions due to algebraic and/or differentiation errors. There were many candidates who initially went wrong and restarted the question. A number of these ran out of space and incorrectly used the back pages of their answer booklet to continue this question rather than using supplementary paper.

It was noted by examiners that part (b) was not attempted by a significant number of candidates. When it was attempted, it was generally not done well, with very many candidates not scoring the mark. Many achieved the correct value only to lose the mark for not stating the units. A common wrong answer was to use $x = 0$ in the tangent equation to give 5m. Another was to use $t = -3$ and then substitute this into $y = 1 - t^3$ to give 28m, ignoring the set of values for which t was valid. Others set $\frac{dy}{dx} = 0$ thinking this would give a maximum and then substituted their t value into y. Some wrongly assumed that the largest value of t for the given domain would lead to the maximum height. There was little evidence that candidates had used the sketch to help them find the maximum height.

Question 11

Almost all candidates recognised that this question required integration and were able to correctly identify that integration by parts was needed to access any of the marks in this question. The most common method was for candidates to apply by parts twice using the formula given in the formula book, although this did frequently result in candidates getting signs wrong for their terms. A number of candidates used the "DI method" which tended to be more successful and less complicated for those candidates who tried this approach. The word LATE or ILATE written on scripts which serves to assess the priority of the first function was also seen. This seemed to be helpful for candidates to identify which function should be differentiated. A small number of candidates used a substitution, usually $u = -3x$, with varying degrees of success. Although this resulted in fewer minus signs to manipulate, it did not simplify the processing.

For most candidates the first step of by parts was generally correct, although coefficient and sign errors were common. A few integrated the $8x^2$ which led to incorrect forms. Of those candidates who got the correct form for the first step most got the second stage into the correct form though many made errors with signs. It was disappointing to see how few candidates simplified double negatives. Candidates who completed the integration to a correct form generally understood the substitution of the limits although again errors in signs occasionally cost candidates the final method mark as their working did not imply use of $e^0 = 1$. Candidates should be encouraged to fully show their working when applying limits so that it is clear that the lower limit is being subtracted. Most responses culminated with an attempt at an answer in its exact form but candidates that obtained the correct values for A and B were in the minority. Although not penalised if the intention was clear, some notation was poor with missing dx's and spurious or missing integral signs.

Question 12

This question tested the candidates' skill in various areas, i.e. finding partial fractions, integration, rules of logarithms, manipulating equations to eliminate logarithms using exponentials, indices, and commentating on the expected behaviour of a variable as another tends to infinity. This was a challenging question which in a number of instances, candidates struggled with after part (a). Those however, who were well prepared, were able to score full marks often with relatively concise solutions.

Part (a) was well answered. Most candidates were familiar with the process of rewriting expressions using partial fractions and obtained a correct answer such as $\frac{1}{25V}+\frac{1}{25(25-V)}$. There were occasional sign slips in the coefficients, and some candidates failed to write down their final answer, or use it in part (b), losing the second mark here.

In part (b) a number of correct forms of integration were possible. Many candidates appreciated that an attempt at separation of variables and integration was required, and many reached correct equations such as $\frac{1}{25}\ln V-\frac{1}{25}\ln(25-V)=\frac{1}{10}t+c$ or $\frac{1}{25}\ln 25V-\frac{1}{25}\ln 25(25-V)=\frac{1}{10}t+c$. There were however, many slips with coefficients, and the omission of the minus sign in front of the ln(25 – V) or ln25(25 – V) term was common. Additionally, some omitted a constant of integration entirely, which meant that no further progress could be made. A surprising number of candidates did not see any link with part (a) and instead, integrated a quadratic in V, or did not integrate at all. Most candidates who attempted integration on both sides and included a constant of integration were able to score 2 method marks by using $t = 0$ and $V = 20$ to find the value of their constant and then substituting $V = 24$ to find the time taken. Most who got this far converted their final answer into minutes as required by the question although some candidates left the answer as 0.7167 which lost the final accuracy mark in this part. A few candidates used the alternative method of applying limits of 20 and 24 to their integral with respect to V and 0 and T for their integral with respect to t.

Many candidates did not reach part (c), and some who did were not able to gain marks there because they either had not established a numerical constant of integration, or because their integrated expression was not of a correct form to earn credit. Successful attempts started with expressions such as $\frac{1}{25}\ln V-\frac{1}{25}\ln(25-V)=\frac{1}{10}t+\frac{1}{25}\ln 4$ to obtain expressions such as $\frac{1}{25}\ln\frac{V}{4(25-V)}=\frac{1}{10}t$ and then eliminated the log to obtain $\frac{V}{4(25-V)}=e^{2.5t}$ and made V the subject. Some candidates obtained a correct expression for V but failed to reach the form required by the question. Incorrect log work, however, was common and expressions such as $\frac{V}{4(25-V)}=e^{2.5t}+4$ were seen frequently. Candidates often made sign or coefficient errors but were awarded the method marks in (c) for a correct algebraic approach for the required rearrangement. A significant number didn't reach the required form, leaving their answer with two $e^{2.5t}$ terms rather than the required single $e^{-2.5t}$ term.

In part (d), candidates sometimes identified that L = 25, even when little else of credit was presented throughout, but usually those who had not attempted part (c) also made no attempt at (d). The reasons given were often not rigorous enough to gain credit, even when 25 had been established as the required value of L. When correct explanations were seen they usually stated that $L = 25$ because $e^{-2.5t} \to 0$ as $t \to \infty$ or that $L = 25$ because $\frac{dV}{dt} = 0$.

Question 13

Part (a) was generally well answered with candidates commonly using the main method in the mark scheme, having made the link with $y = mx + c$ and taking logarithms. Most candidates recognised log $a = 0.81$ and log $b = 0.0054$ with minimal preamble. When this was recognised it almost always led to correct answers. There were occasional errors with logs such as using base e instead of base 10. The misread of the 0.0054 as 0.054 was seen quite often. A common issue was candidates not reading the 3 decimal place requirement and instead writing things such as $b = 1.0125$ (awrt was applied so no marks were lost) or giving 3 significant figures. The method where values for t were used to obtain the values for a and b was sometimes used, but often candidates made limited progress towards obtaining an accurate answer.

For part (b)(i), most candidates understood that this was the initial population but the word billion was often omitted, showing a lack of full comprehension of the model.

Part (b)(ii) was less well answered than (b)(i). The proportionality or rate was often recognised but "per year" was sometimes omitted. The wording of answers also cost candidates the mark as they often talked about the "amount" or "how much" the population went up by, or they wrote correct responses followed by it increases by "1.013 billion".

Part (c) was generally well answered with the majority of candidates substituting $t = 26$ into their model and remembering to write billions (despite often having forgotten this in part b(i)). Occasionally candidates used $t = 27$ and more rarely 25. When candidates attempted to write their answer as the full form of the decimal, they often made errors in the number of digits.

Part (d) was not well answered on the whole, and credit seemed to be rarely given. Many candidates focussed on an assumption that death/disease was not incorporated into the model. This showed poor understanding of modelling as the growth rate can factor things like this in when it is derived. Candidates who did make a link to extrapolation, were often unable to recognise that this was a prediction far into the future so the model may not be appropriate. Responses to this part often seemed to be rote answers that indicated a lack of understanding of the model.

Question 14

A common marking profile in this question was to score all three marks in (a) followed by zero marks in (b).

In part (a) the most common approach was to complete the square and very often it was successful although there were a variety of incorrect answers for the centre – usually variations of $(\pm 3, \pm 7)$.

Quite a few knew the strategy for finding the radius although not all of them knew how to use it correctly and $r = \sqrt{91}$ was a frequently seen incorrect answer from attempting $(x-3)^2 + 9 + (y+7)^2 + 49 + 33 = 0$. A surprising number of candidates evaluated 7^2 as something other than 49.

In part (b), it was common for no marks to be scored and there were a large number of non-attempts. The most successful candidates were those who drew diagrams and looked at the problem geometrically. However, the most common approach was an algebraic one. This typically involved solving the 2 circle equations simultaneously, but the majority of such attempts were aborted before any marks were gained. There was also some poor use of algebra with things such as $(x-3)^2 + (y+7)^2 = 25 \Rightarrow x - 3 + y + 7 = 5$ seen a surprising number of times and such attempts gained no credit. Many candidates did recognise that there would be a discriminant involved so they often arrived at a point when they thought that it would be appropriate to calculate one. However, this was usually prematurely. Despite this, it was noted that some examiners did report the occasional successful algebraic attempt. Some candidates calculated the equation of the line passing through the centres but then did not realise what they had to do with it.

Most who were able to find the correct distance between the centres recognised they had to add and subtract the radius found earlier to give the limits of the solution set. Candidates who made it this far sometimes lost the final accuracy mark due to the use of incorrect notation. For candidates who got as far as the correct interval for *k*, the most successful were those who gave the final answer as a single set. Candidates who broke the answer into two sets, often used the incorrect symbol between them.

Question 15

This question provided a challenge to most candidates and was in general a good discriminator. In part (a), it was perhaps disappointing, although not unexpected, to see that so many candidates' instinctive first step was to multiply out the brackets rather than realising that a simpler, quicker and more accurate method was available. In part (a), the use of the chain rule and factorisation could greatly reduce the required effort and the potential for making mistakes. Those candidates with a good understanding of the chain rule and implicit differentiation were able to use this efficient method quickly to accumulate the 5 marks available. However, a common error of omitting the 1 in $(1 + \frac{\mathrm{d}y}{\mathrm{d}x})$ was seen many times. Furthermore, a significant proportion of candidates who differentiated the left-hand side using the chain rule, often chose to expand the derivative, nonetheless, thus complicating the remainder of the question.

Unfortunately, the majority of candidates chose to first expand $(x+y)^3$ before attempting to differentiate. Whilst this was, in many cases, completed correctly there was of course an inherent time penalty. A significant proportion of candidates made errors when expanding and displayed a lack of algebraic fluency and poor application of the binomial expansion. Common errors that were seen included missing out terms in the expansion and poor understanding of indices. Once the brackets had been expanded, there were clearly some gaps in understanding of how to successfully carry out the implicit differentiation, particularly for terms that also required application of the product rule. It was quite common to see for example $3x^2y$ going to $6x\frac{\mathrm{d}y}{\mathrm{d}x}$ and $3xy^2$ going to $6\frac{\mathrm{d}y}{\mathrm{d}x}$.

Overall, the right-hand-side was dealt with correctly although the constant term -2 was sometimes retained following differentiation and sometimes $6y\frac{\mathrm{d}y}{\mathrm{d}x}$ was seen. When candidates attempted to move all terms over to one side of the equation, there were sometimes sign errors or slips in copying terms. Upon completion of the process of differentiation, almost all candidates seemed to understand that the task was to gather the $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms together, factorise and divide in order to isolate the $\frac{\mathrm{d}y}{\mathrm{d}x}$, and the majority who reached this stage were able to earn credit here provided their differentiation had been sufficiently structurally correct to produce sufficient terms in $\frac{\mathrm{d}y}{\mathrm{d}x}$. It was very rare to see a spurious $\frac{\mathrm{d}y}{\mathrm{d}x}$ term retained which has been an issue in previous sessions.

In general, those candidates who were able to successfully complete part (a) found part (b) straightforward. However, the accuracy mark in this part was not available to those candidates who had not managed to achieve the correct derivative in part (a). Due to the nature of the function, setting $x = 1$ and $y = 0$, gave a 'correct' value of $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}$ in many cases even when $\frac{\mathrm{d}y}{\mathrm{d}x}$ was incorrect. As a result, candidates were often not alerted to the fact they had an incorrect $\frac{\mathrm{d}y}{\mathrm{d}x}$. Disappointingly, candidates who did not obtain $\frac{1}{2}$ due to errors in their differentiation, often attempted to manipulate their work to obtain the value that they could see was required from the printed answer and this often meant that the method mark was out of reach, which was a shame.

Part (c) provided a challenge for many candidates and was a good discriminator. It is possible that some candidates were running out of time by this part of the paper, and this seemed to lead to simple algebraic and sign errors. However, many candidates were able to make at least some progress and most recognised the need to substitute the equation from part (b), which had been provided, into the equation from part (a). Most were able to expand to obtain a cubic in x albeit with frequent errors and slips, in particular a common error of cancelling the constant terms leading to a cubic with only three terms was often seen. The alternative elimination of x to lead to a cubic in y was rarely seen.

Having obtained a cubic equation, many candidates made little creditworthy further progress and the majority seemed not to have made the link to earlier parts of the question and so did not attempt to take $(x - 1)$ out as a factor of their cubic. This was perhaps understandable when the cubic was incorrect although it seemed that even candidates with the correct cubic did not consider using the fact that $x = 1$ was a root. Many candidates used their calculator to obtain the three roots of their cubic and others employed dubious techniques either attempting to find a 'discriminant' by ignoring the x^3 term or attempting to take out a factor of x whilst ignoring the constant term. Those candidates who had a three-term cubic rather than a four-term cubic were unable to progress here.

The final two marks of the paper were only accessible to those who were very secure in their algebraic accuracy and knowledge as these marks were contingent on both a correct cubic and a correct quadratic factor. Most candidates who met these criteria successfully used the discriminant to demonstrate that the quadratic had no roots although some used more elaborate reasoning involving the gradient of the cubic or the minimum point of the quadratic. Unfortunately, a significant proportion of candidates who got this far, did not manage to earn both marks because their justification or conclusion was insufficient, and it was sometimes disappointing to see otherwise well-answered solutions lose the final mark because they concluded with 'no roots' rather than also relating this back to the question to conclude that there could be 'no further points of intersection'.

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