

## Mark Scheme (Results) Summer 2009

**GCE** 

GCE Mathematics (6663/01)





## June 2009 6663 Core Mathematics C1 Mark Scheme

Ques	stion nber	Scheme	Mark	S
Q1	(a)	$(3\sqrt{7})^2 = 63$ $(8+\sqrt{5})(2-\sqrt{5}) = 16-5+2\sqrt{5}-8\sqrt{5}$	B1	(1)
	(b)	$(8 + \sqrt{5})(2 - \sqrt{5}) = 16 - 5 + 2\sqrt{5} - 8\sqrt{5}$	M1	
		$=11, -6\sqrt{5}$	A1, A1	
				(3) [4]
	(a)	B1 for 63 only		
	(b)	M1 for an attempt to expand their brackets with $\geq 3$ terms correct.		
		They may collect the $\sqrt{5}$ terms to get $16-5-6\sqrt{5}$		
		Allow $-\sqrt{5} \times \sqrt{5}$ or $-\left(\sqrt{5}\right)^2$ or $-\sqrt{25}$ instead of the -5		
		These 4 values may appear in a list or table but they should have minus signs		
		included		
		The next two marks should be awarded for the final answer but check that correct values follow from correct working. Do not use ISW rule		
		1 <sup>st</sup> A1 for 11 from $16 - 5$ or $-6\sqrt{5}$ from $-8\sqrt{5} + 2\sqrt{5}$		
		$2^{\text{nd}} \text{ A1}$ for both 11 and $-6\sqrt{5}$ .		
		S.C - Double sign error in expansion		
		For $16-5-2\sqrt{5}+8\sqrt{5}$ leading to $11 + \dots$ allow <u>one</u> mark		



Question Number	Scheme	Marks
Q2	$32 = 2^5$ or $2048 = 2^{11}$ , $\sqrt{2} = 2^{\frac{1}{2}}$ or $\sqrt{2048} = (2048)^{\frac{1}{2}}$	B1, B1
	$a = \frac{11}{2}$ (or $5\frac{1}{2}$ or $5.5$ )	B1
		[3]
	1st B1 for $32 = 2^5$ or $2048 = 2^{11}$ This should be explicitly seen: $32\sqrt{2} = 2^a$ followed by $2^5\sqrt{2} = 2^a$ is OK Even writing $32 \times 2 = 2^5 \times 2 \left(=2^6\right)$ is OK but simply writing $32 \times 2 = 2^6$ is NOT $2^{\text{nd}}$ B1 for $2^{\frac{1}{2}}$ or $(2048)^{\frac{1}{2}}$ seen. This mark may be implied $3^{\text{rd}}$ B1 for answer as written. Need $a = \dots$ so $2^{\frac{11}{2}}$ is B0 $a = \frac{11}{2} \left( \text{ or } 5\frac{1}{2} \text{ or } 5.5 \right)$ with no working scores full marks.  If $a = 5.5$ seen then award 3/3 unless it is clear that the value follows from totally incorrect work.  Part solutions: e.g. $2^5\sqrt{2}$ scores the first B1.  Special case:  If $\sqrt{2} = 2^{\frac{1}{2}}$ is not explicitly seen, but the final answer includes $\frac{1}{2}$ , e.g. $a = 2\frac{1}{2}$ , $a = 4\frac{1}{2}$ , the second B1 is given by implication.	



Ques Num		Scheme	Marks
Q3	(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 6x^2 - 6x^{-3}$	M1 A1 A1 (3)
	(b)	$\frac{dy}{dx} = 6x^2 - 6x^{-3}$ $\frac{2x^4}{4} + \frac{3x^{-1}}{-1}(+C)$ $\frac{x^4}{2} - 3x^{-1} + C$	M1 A1
		$\frac{x^4}{2} - 3x^{-1} + C$	A1 (3) [6]
			[0]
	(a)	M1 for an attempt to differentiate $x^n \to x^{n-1}$ 1 <sup>st</sup> A1 for $6x^2$	
		$2^{\text{nd}} \text{ A1}$ for $-6x^{-3}$ or $-\frac{6}{x^3}$ Condone + $-6x^{-3}$ here. Inclusion of +c sco	res A0 here.
	(b)	M1 for some attempt to integrate an x term of the given y. $x^n \to x^{n+1}$	+1
		$1^{st}$ A1 for <b>both</b> $x$ terms correct but unsimplified- as printed or better. Ighere	gnore +c
		$2^{\text{nd}}$ A1 for both x terms correct and simplified and +c. Accept $-\frac{3}{x}$ but 1	NOT
		$+-3x^{-1}$	
		Condone the $+c$ appearing on the first (unsimplified) line but missinal (simplified) line	ssing on the
		Apply ISW if a correct answer is seen	
		If part (b) is attempted first and this is clearly labelled then apply the sche allow the marks. Otherwise assume the first solution is for part (a).	me and



	stion nber	Scheme	Mark	S
Q4	(a)	$5x > 10$ , $x > 2$ [Condone $x > \frac{10}{2} = 2$ for M1A1]	M1, A1	(2)
	(b)	$(2x+3)(x-4) = 0$ , 'Critical values' are $-\frac{3}{2}$ and 4	M1, A1	(2)
		$-\frac{3}{2} < x < 4$	M1 A1ft	
	(c)	2 < x < 4	B1ft	(4) (1) [7]
	(a)	M1 for attempt to collect like terms on each side leading to $ax > b$ , or $ax < b$ , or $ax = b$		
		Must have a or b correct so eg $3x > 4$ scores M0		
	(b)	1 <sup>st</sup> M1 for an attempt to factorize or solve to find critical values. Method must potentially give 2 critical values		
		1 <sup>st</sup> A1 for $-\frac{3}{2}$ and 4 seen. They may write $x < -\frac{3}{2}$ , $x < 4$ and still get this A1		
		2 <sup>nd</sup> M1 for choosing the "inside region" for their critical values 2 <sup>nd</sup> A1ft follow through their 2 distinct critical values		
		Allow $x > -\frac{3}{2}$ with "or" "," " $\cup$ " " " $x < 4$ to score M1A0 but "and" or " $\cap$ " score		
		M1A1 $x \in (-\frac{3}{2}, 4)$ is M1A1 but $x \in [-\frac{3}{2}, 4]$ is M1A0. Score M0A0 for a number line or graph only		
	(c)	B1ft Allow if <b>a correct answer is seen</b> or follow through their answer to (a) and their answer to (b) but their answers to (a) and (b) <u>must be regions</u> . Do not follow through single values.  If their follow through answer is the empty set accept ∅ or {} or equivalent in words  If (a) or (b) are not given then score this mark for cao		
		NB You may see x<4 (with anything or nothing in-between) x < -1.5 in (b) and empty set in (c) for B1ft  Do not award marks for part (b) if only seen in part (c)		
		Use of $\leq$ instead of $<$ (or $\geq$ instead of $>$ ) loses one accuracy mark only, at first occurrence.		



Ques		Scheme	Mark	S
Q5	(a)	$a + 9d = 2400 \qquad a + 39d = 600$	M1	
		$d = \frac{-1800}{30}$ $d = -60$ (accept $\pm 60$ for A1)	M1 A1	(3)
	(b)	$   \begin{array}{ll}     30 \\     a - 540 = 2400 & a = 2940   \end{array} $	M1 A1	(2)
	(c)	Total = $\frac{1}{2}n\{2a + (n-1)d\} = \frac{1}{2} \times 40 \times (5880 + 39 \times -60)$ (ft values of a and d)	M1 A1ft	
		$\frac{2}{2} = 70800$	A1cao	(3)
				[8]
		Note: If the sequence is considered 'backwards', an equivalent solution may be given using $d = 60$ with $a = 600$ and $l = 2940$ for part (b). This can still score full marks. <b>Ignore labelling of (a) and (b)</b>		
	(a)	$1^{\text{st}}$ M1 for an attempt to use 2400 and 600 in $a + (n-1)d$ formula. Must use both values		
		i.e. need $a + pd = 2400$ and $a + qd = 600$ where $p = 8$ or 9 and $q = 38$ or 39 (any combination)		
		$2^{\text{nd}}$ M1 for an attempt to solve <u>their</u> 2 linear equations in a and d as far as $d =$		
		A1 for $d = \pm 60$ . Condone correct equations leading to $d = 60$ or $a + 8d = 2400$ and $a + 38d = 600$ leading to $d = -60$ . They should get penalised in (b) and (c).		
		NB This is a "one off" ruling for A1. Usually an A mark must follow from their		
		<b>work.</b> ALT $1^{\text{st}}$ M1 for $(30d) = \pm (2400 - 600)$		
		$2^{\text{nd}} \text{ M1 for } (d =) \pm \frac{\overline{(2400 - 600)}}{30}$		
		A1 for $d = +60$		
		$a + 9d = 600$ , $a + 39d = 2400$ only scores M0 BUT if they solve to find $d = \pm 60$ then		
	(b)	use ALT scheme above. M1 for use of their $d$ in a <b>correct</b> linear equation to find $a$ leading to $a =$		
	( )	A1 their a must be compatible with their d so $d = 60$ must have $a = 600$ and $d = -60$ , $a = 2940$		
		So for example they can have $2400 = a + 9(60)$ leading to $a =$ for M1 but it scores A0		
		Any approach using a list scores M1A1 for a correct a but M0A0 otherwise		
	(c)	M1 for use of a correct $S_n$ formula with $n = 40$ and at least one of $a$ , $d$ or $l$ correct or correct ft.		
		$1^{\text{st}}$ A1ft for use of a correct $S_{40}$ formula and both $a$ , $d$ or $a$ , $l$ correct or correct follow		
		through		
		ALT Total = $\frac{1}{2}n\{a+l\} = \frac{1}{2} \times 40 \times (2940 + 600)$ (ft value of a) M1 A1ft		
		2 <sup>nd</sup> A1 for 70800 only		



Question Number	Scheme	Marks
Q6	$b^2 - 4ac$ attempted, in terms of $p$ . $(3p)^2 - 4p = 0$ o.e. Attempt to solve for $p$ e.g. $p(9p-4)=0$ Must potentially lead to $p=k,\ k \neq 0$ $p=\frac{4}{9}$ (Ignore $p=0$ , if seen)	M1 A1 M1 A1cso [4]
	1st M1 for an attempt to substitute into $b^2-4ac$ or $b^2=4ac$ with $b$ or $c$ correct Condone $x$ 's in one term only. This can be inside a square root as part of the quadratic formula for example. Use of inequalities can score the M marks only  1st A1 for any correct equation: $(3p)^2-4\times1\times p=0$ or better  2nd M1 for an attempt to factorize or solve their quadratic expression in $p$ . Method must be sufficient to lead to their $p=\frac{4}{9}$ .  Accept factors or use of quadratic formula or $(p\pm\frac{2}{9})^2=k^2$ (o.e. eg) $(3p\pm\frac{2}{3})^2=k^2$ or equivalent work on their eqn. $9p^2=4p\Rightarrow \frac{9p^{\frac{1}{3}}}{R}=4 \text{ which would lead to } 9p=4 \text{ is OK for this } 2^{\text{nd}} \text{ M1}$ ALT Comparing coefficients  M1 for $(x+\alpha)^2=x^2+\alpha^2+2\alpha x$ and A1 for a correct equation eg $3p=2\sqrt{p}$ M1 for forming solving leading to $\sqrt{p}=\frac{2}{3}$ or better  Use of quadratic/discriminant formula (or any formula) Rule for awarding M mark  If the formula is quoted accept some correct substitution leading to a partially correct expression.  If the formula is not quoted only award for a fully correct expression using their values.	



Ques		Scheme	Mari	KS
Num	inel			
Q7	(a)	$(a_2 =)2k - 7$	B1	(1)
	(b)	$(a_3 =)2(2k-7)-7 \text{ or } 4k-14-7, = 4k-21$ (*)	M1, A1	cso
				(2)
	(c)	$(a_4 =) 2(4k-21)-7  (=8k-49)$	M1	
		$(a_4 =) 2(4k-21)-7  (=8k-49)$ $\sum_{r=1}^{4} a_r = k + "(2k-7)" + (4k-21) + "(8k-49)"$	M1	
		k + (2k - 7) + (4k - 21) + (8k - 49) = 15k - 77 = 43 $k = 8$	M1 A1	(4) [7]
	(b)	M1 must see 2(their $a_2$ ) - 7 or $2(2k-7)-7$ or $4k-14-7$ . Their $a_2$ must be a		
		function of $k$ .		
		A1cso must see the $2(2k-7)-7$ or $4k-14-7$ expression and the $4k-21$ with no incorrect working		
	(c)	incorrect working $1^{st}$ M1 for an attempt to find $a_4$ using the given rule. Can be awarded for $8k$ - 49		
		seen.		
		Use of formulae for the sum of an arithmetic series scores M0M0A0 for the		
		next 3 marks. 2 <sup>nd</sup> M1 for attempting the sum of the 1 <sup>st</sup> 4 terms. Must have "+" not just, or clear		
		attempt to sum.		
		Follow through their $a_2$ and $a_4$ provided they are linear functions of $k$ .		
		Must lead to linear expression in k. Condone use of their linear $a_3 \neq 4k - 21$		
		here too. $3^{rd}$ M1 for forming a linear equation in $k$ using their sum and the 43 and attempt to		
		solve for k as far as $pk = q$		
		A1 for $k = 8$ only so $k = \frac{120}{15}$ is A0		
		Answer Only (e.g. trial improvement)		
		Accept $k = 8$ only if $8 + 9 + 11 + 15 = 43$ is seen as well		
		<u>Sum</u> $a_2 + a_3 + a_4 + a_5$ or $a_2 + a_3 + a_4$		
		Allow: M1 if $8k - 49$ is seen, M0 for the sum (since they are not adding the $1^{st}$ 4 terms) then M1		
		if they use their sum along with the 43 to form a linear equation and attempt to solve		
		but A0		



Question Number	Scheme	Marks	
Q8 (a)	AB: $m = \frac{2-7}{8-6}$ , $\left(=-\frac{5}{2}\right)$	B1	
	Using $m_1 m_2 = -1$ : $m_2 = \frac{2}{5}$	M1	
	$y-7=\frac{2}{5}(x-6)$ , $2x-5y+23=0$ (o.e. with integer coefficients)	M1, A1 (4)	)
(b)	Using $x = 0$ in the answer to (a), $y = \frac{23}{5}$ or 4.6	M1, A1ft (2	)
(c)	Area of triangle = $\frac{1}{2} \times 8 \times \frac{23}{5} = \frac{92}{5}$ (o.e) e.g. $\left(18\frac{2}{5}, 18.4, \frac{184}{10}\right)$	M1 A1 (2	
(a) (b)	B1 for an expression for the gradient of $AB$ . Does not need the $=-2.5$ $1^{\text{st}}$ M1 for use of the perpendicular gradient rule. Follow through their $m$ $2^{\text{nd}}$ M1 for the use of $(6, 7)$ and their changed gradient to form an equation for $l$ . Can be awarded for $\frac{y-7}{x-6} = \frac{2}{5}$ o.e.  Alternative is to use $(6, 7)$ in $y = mx + c$ to find a value for $c$ . Score when $c = \dots$ is reached.  A1 for a correct equation in the required form and must have "= 0" and integer coefficients  M1 for using $x = 0$ in their answer to part (a) e.g. $-5y + 23 = 0$ A1ft for $y = \frac{23}{5}$ provided that $x = 0$ clearly seen or $C(0, 4.6)$ . Follow through their equation in (a)  If $x = 0$ , $y = 4.6$ are clearly seen but $C$ is given as $(4.6,0)$ apply ISW and award the mark.  This A mark requires a simplified fraction or an exact decimal Accept their $4.6$ marked on diagram next to $C$ for M1A1ft  M1 for $\frac{1}{2} \times 8 \times y_C$ so can follow through their $y$ coordinate of $C$ .  A1 for $18.4$ (o.e.) but their $y$ coordinate of $C$ must be positive  Use of 2 triangles or trapezium and triangle  Award M1 when an expression for area of $OCB$ only is seen  Determinant approach  Award M1 when an expression containing $\frac{1}{2} \times 8 \times y_C$ is seen		



Question Number	Scheme	Mark	(S
Q9 (a)	$\left[ (3 - 4\sqrt{x})^2 = \right] 9 - 12\sqrt{x} - 12\sqrt{x} + (-4)^2 x$	M1	(2)
(b)	$9x^{-\frac{1}{2}} + 16x^{\frac{1}{2}} - 24$ $f'(x) = -\frac{9}{2}x^{-\frac{3}{2}}, +\frac{16}{2}x^{-\frac{1}{2}}$	M1 A1, A	(3) A1ft
(c)	$f'(9) = -\frac{9}{2} \times \frac{1}{27} + \frac{16}{2} \times \frac{1}{3} = -\frac{1}{6} + \frac{16}{6} = \frac{5}{2}$	M1 A1	<ul><li>(3)</li><li>(2)</li><li>[8]</li></ul>
(a)	M1 for an attempt to expand $(3-4\sqrt{x})^2$ with at least 3 terms correct- as printed or better		
(b)	M1 for an attempt to differentiate an $x$ term $x^n \to x^{n-1}$ $1^{\text{st}} \text{ A1}$ for $-\frac{9}{2}x^{-\frac{3}{2}}$ and their constant $B$ differentiated to zero. NB $-\frac{1}{2} \times 9x^{-\frac{3}{2}}$ is A0 $2^{\text{nd}} \text{ A1ft}$ follow through their $Ax^{\frac{1}{2}}$ but can be scored without a value for $A$ , i.e. for $\frac{A}{2}x^{-\frac{1}{2}}$		
(c)	M1 for some correct substitution of $x = 9$ in their expression for $f'(x)$ including an attempt at $(9)^{\pm \frac{k}{2}}$ ( $k$ odd) somewhere that leads to some appropriate multiples of $\frac{1}{3}$ or $3$ A1 accept $\frac{15}{6}$ or any exact equivalent of 2.5 e.g. $\frac{45}{18}, \frac{135}{54}$ or even $\frac{67.5}{27}$ Misread (MR) Only allow MR of the form $\frac{\left(3-k\sqrt{x}\right)^2}{\sqrt{x}}$ N.B. Leads to answer in (c) of $\frac{k^2-1}{6}$ Score as M1A0A0, M1A1A1ft, M1A1ft		



Question Number	Scheme	Marl	<b>KS</b>
Q10 (a) (b)	$x(x^{2}-6x+9)$ $= x(x-3)(x-3)$ Shape $\frac{\text{Through origin (not touching)}}{\text{Touching } x\text{-axis only once}}$ $\text{Touching at (3, 0), or 3 on } x\text{-axis}$ $\text{[Must be on graph not in a table]}$	B1 M1 A1 B1 B1 B1 B1ft	(3)
(c)	Moved horizontally (either way) (2, 0) and (5, 0), or 2 and 5 on x-axis	M1 A1 (2)	[9]
(a)	B1 for correctly taking out a factor of <i>x</i>		
S.C.	M1 for an attempt to factorize their 3TQ e.g. $(x+p)(x+q)$ where $ pq =9$ . So $(x-3)(x+3)$ will score M1 but A0  A1 for a fully correct factorized expression - accept $x(x-3)^2$ If they "solve" use ISW  If the only correct linear factor is $(x-3)$ , perhaps from factor theorem, award B0M1A0  Do not award marks for factorising in part (b)		
(b)	For the graphs  "Sharp points" will lose the 1 <sup>st</sup> B1 in (b) but otherwise be generous on shape Condone (0, 3) in (b) and (0, 2), (0,5) in (c) if the points are marked in the correct places.  2 <sup>nd</sup> B1 for a curve that starts or terminates at (0, 0) score B0		
	$4^{th}$ B1ft for a curve that touches (not crossing or terminating) at $(a, 0)$ where their $y = x(x-a)^2$		
(c)	M1 for their graph moved horizontally (only) or a fully correct graph Condone a partial stretch if ignoring their values looks like a simple translation A1 for their graph translated 2 to the right and crossing or touching the axis at 2 and 5 only Allow a fully correct graph (as shown above) to score M1A1 whatever they have in (b)		



Quest	tion	Schomo	Mar	ke
Numl	ber	Scheme	Mar	V.2
Q11	(a)	x = 2: $y = 8 - 8 - 2 + 9 = 7$ (*)	B1	(1)
	(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 4x - 1$	M1 A1	
		$x = 2$ : $\frac{dy}{dx} = 12 - 8 - 1 (= 3)$	A1ft	
	(a)	y-7=3(x-2), $y=3x+1$	M1, <u>A1</u>	(5)
	(C)	$m = -\frac{1}{3}$ (for $-\frac{1}{m}$ with their $m$ )	B1ft	
		$3x^2 - 4x - 1 = -\frac{1}{3}$ , $9x^2 - 12x - 2 = 0$ or $x^2 - \frac{4}{3}x - \frac{2}{9} = 0$ (o.e.)	M1, A1	
		$\left(x = \frac{12 + \sqrt{144 + 72}}{18}\right) \left(\sqrt{216} = \sqrt{36}\sqrt{6} = 6\sqrt{6}\right) \text{ or } (3x - 2)^2 = 6 \to 3x = 2 \pm \sqrt{6}$	M1	
		$x = \frac{1}{3} \left( 2 + \sqrt{6} \right) \tag{*}$	A1cso	(5)
				[11]
	(a)	B1 there must be a clear attempt to substitute $x = 2$ leading to 7		
	(b)	e.g. $2^3 - 2 \times 2^2 - 2 + 9 = 7$ $1^{st}$ M1 for an attempt to differentiate with at least one of the given terms fully		
	(0)	1 <sup>st</sup> M1 for an attempt to differentiate with at least one of the given terms fully correct.		
		1 <sup>st</sup> A1 for a fully correct expression		
		$2^{\text{nd}}$ A1ft for sub. $x=2$ in their $\frac{dy}{dx} \neq y$ accept for a correct expression e.g.		
		$3 \times (2)^2 - 4 \times 2 - 1$		
		2 <sup>nd</sup> M1 for use of their "3" (provided it comes from their $\frac{dy}{dx}$ ( $\neq y$ ) and $x=2$ ) to find		
		equation of tangent. Alternative is to use (2, 7) in $y = mx + c$ to <u>find a value</u> for c.		
		Award when $c = \dots$ is seen.		
		No attempted use of $\frac{dy}{dx}$ in (b) scores 0/5		
	(c)	1 <sup>st</sup> M1 for forming an equation from their $\frac{dy}{dx} (\neq y)$ and their $-\frac{1}{m}$ (must be		
		changed from $m$ )		
		$1^{\text{st}}$ A1 for a correct 3TQ all terms on LHS (condone missing =0) $2^{\text{nd}}$ M1 for proceeding to $x = \dots$ or $3x = \dots$ by formula or completing the square for a 3TQ.		
		Not factorising. Condone $\pm$ $2^{\text{nd}}$ A1 for proceeding to given answer with no incorrect working seen. Can still		
	ALT	have $\pm$ .		
	<b>\</b> ∟ I	Verify (for M1A1M1A1)  1 <sup>st</sup> M1 for attempting to square need $\geq 3$ correct values in $\frac{4+6+4\sqrt{6}}{9}$ , 1 <sup>st</sup> A1 for $\frac{10+4\sqrt{6}}{9}$		
		$2^{\text{nd}}$ M1 Dependent on $1^{\text{st}}$ M1 in this case for substituting in all terms of their $\frac{dy}{dx}$		
		$2^{\text{nd}}$ A1cso for cso with a full comment e.g. "the x co-ord of Q is"		