Mark Scheme (Results)

June 2011

GCE Core Mathematics C2 (6664) Paper 1

June 2011

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EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol ✓ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ☐ The second mark is dependent on gaining the first mark

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Core Mathematics C2 6664
Mark Scheme

Question Number	Scheme		Marks
1. (a)	$\text{f}(x) = 2x^3 - 7x^2 - 5x + 4$ Remainder $= \text{f}(1) = 2 - 7 - 5 + 4 = -6$ $= -6$	Attempts $\text{f}(1)$ or $\text{f}(-1)$. -6	M1 A1 **[2]**
(b)	$\text{f}(-1) = 2(-1)^3 - 7(-1)^2 - 5(-1) + 4$ and so $(x+1)$ is a factor.	Attempts $\text{f}(-1)$. $\text{f}(-1) = 0$ with no sign or substitution errors **and for conclusion.**	M1 A1 **[2]**
(c)	$\text{f}(x) = \{(x+1)\}(2x^2 - 9x + 4)$ $= (x+1)(2x-1)(x-4)$ (**Note**: *Ignore the ePEN notation of (b) (should be (c)) for the final three marks in this part*).		M1 A1 dM1 A1 **[4]** **8**

(a) M1 for ***attempting*** either $\text{f}(1)$ or $\text{f}(-1)$. Can be implied. Only one slip permitted.
M1 can also be given for an attempt (at least two "subtracting" processes) at long division to give a remainder which is independent of x. A1 can be given also for -6 seen at the bottom of long division working. Award A0 for a candidate who finds -6 but then states that the remainder is 6.
Award M1A1 for -6 without any working.

(b) M1: attempting only $\text{f}(-1)$. A1: must correctly show $\text{f}(-1) = 0$ and give a conclusion ***in part (b) only***.
Note: Stating "hence factor" or "it is a factor" or a "tick" or "QED" is fine for the conclusion.
Note also that a conclusion can be implied from a preamble, eg: "If $\text{f}(-1) = 0$, $(x+1)$ is a factor…."
Note: Long division scores no marks in part (b). The factor theorem is required.

(c) 1st M1: Attempts long division or other method, to obtain $(2x^2 \pm ax \pm b)$, $a \neq 0$, even with a remainder. Working need not be seen as this could be done "by inspection." $(2x^2 \pm ax \pm b)$ must be seen ***in part (c) only***. Award 1st M0 if the quadratic factor is clearly found from dividing $\text{f}(x)$ by $(x-1)$. Eg. Some candidates use their $(2x^2 - 5x - 10)$ in part (c) found from applying a long division method in part (a).
1st A1: For seeing $(2x^2 - 9x + 4)$.
2nd dM1: Factorises a 3 term quadratic. (see rule for factorising a quadratic). This is dependent on the previous method mark being awarded. This mark can also be awarded if the candidate applies the quadratic formula correctly.
2nd A1: is cao and needs all three factors on one line. Ignore following work (such as a solution to a quadratic equation.)
Note: Some candidates will go from $\{(x+1)\}(2x^2 - 9x + 4)$ to $\{x = -1\}$, $x = \frac{1}{2}$, 4, and not list all three factors. Award these responses M1A1M1A0.
Alternative: 1st M1: For finding either $\text{f}(4) = 0$ or $\text{f}\left(\frac{1}{2}\right) = 0$.
1st A1: A second correct factor of usually $(x-4)$ or $(2x-1)$ found. Note that any one of the other correct factors found would imply the 1st M1 mark.
2nd dM1: For using two known factors to find the third factor, usually $(2x \pm 1)$.
2nd A1 for correct answer of $(x+1)(2x-1)(x-4)$.
Alternative: (for the first two marks)
1st M1: Expands $(x+1)(2x^2 + ax + b)$ {giving $2x^3 + (a+2)x^2 + (b+a)x + b$ } then compare coefficients to find values for a and b. 1st A1: $a = -9$, $b = 4$
Not dealing with a factor of 2: $(x+1)(x - \frac{1}{2})(x-4)$ or $(x+1)(x - \frac{1}{2})(2x-8)$ scores M1A1M1A0.
Answer only, with one sign error: eg. $(x+1)(2x+1)(x-4)$ or $(x+1)(2x-1)(x+4)$ scores M1A1M1A0. **(c) Award M1A1M1A1 for Listing all three correct factors with no working**.

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Question Number	Scheme		Marks
2. (a)	$\{(3+bx)^5\} = (3)^5 + \underline{{}^5C_1}(3)^4(b\underline{x}) + \underline{{}^5C_2}(3)^3\underline{(bx)^2} + \ldots$ $= 243 + 405bx + 270b^2x^2 + \ldots$	243 as a constant term seen. $405bx$ $({}^5C_1 \times \ldots \times x)$ or $({}^5C_2 \times \ldots \times x^2)$ $270b^2x^2$ or $270(bx)^2$	B1 B1 M1 A1 **[4]**
(b)	$\{2(\text{coeff } x) = \text{coeff } x^2\} \Rightarrow 2(405b) = 270b^2$ So, $\left\{b = \frac{810}{270} \Rightarrow\right\} b = 3$	Establishes an equation from their coefficients. Condone 2 on the wrong side of the equation. $b = 3$ (Ignore $b = 0$, if seen.)	M1 A1 **[2]** **6**

(a) The terms can be "listed" rather than added. Ignore any extra terms.

1st B1: A constant term of 243 seen. Just writing $(3)^5$ is B0.

2nd B1: Term must be simplified to $405bx$ for B1. The x is required for this mark. Note $405 + bx$ is B0.

M1: For <u>either</u> the x term <u>or</u> the x^2 term. Requires <u>correct</u> binomial coefficient in any form <u>with the correct power of x</u>, but the other part of the coefficient (perhaps including powers of 3 and/or b) may be wrong or missing.

<u>Allow</u> binomial coefficients such as $\binom{5}{2}, \left(\frac{5}{2}\right), \binom{5}{1}, \left(\frac{5}{1}\right), {}^5C_2, {}^5C_1$.

A1: For either $270b^2x^2$ or $270(bx)^2$. (If $270bx^2$ follows $270(bx)^2$, isw and allow A1.)

<u>Alternative:</u>

Note that a factor of 3^5 can be taken out first: $3^5\left(1+\frac{bx}{3}\right)^5$, but the mark scheme still applies.

<u>Ignore subsequent working (isw)</u>: Isw if necessary after correct working:

e.g. $243 + 405bx + 270b^2x^2 + \ldots$ leading to $9 + 15bx + 10b^2x^2 + \ldots$ scores B1B1M1A1 isw.

Also note that full marks could also be available in part (b), here.

<u>Special Case</u>: Candidate writing down the first three terms in ***descending*** powers of x usually get $(bx)^5 + {}^5C_4(3)^1(bx)^4 + {}^5C_3(3)^2(bx)^3 + \ldots = b^5x^5 + 15b^4x^4 + 90b^3x^3 + \ldots$

So award SC: B0B0M1A0 for either $({}^5C_4 \times \ldots \times x^4)$ or $({}^5C_3 \times \ldots \times x^3)$

(b) M1 for equating 2 times their coefficient of x to the coefficient of x^2 to get an equation in b, <u>or</u> equating their coefficient of x to 2 times that of x^2, to get an equation in b.

Allow this M mark even if the equation is trivial, providing their coefficients from part (a) have been used, eg: $2(405b) = 270b$, but beware $b = 3$ from this, which is A0.

<u>An equation in b alone</u> is required:

e.g. $2(405b)x = 270b^2x^2 \Rightarrow b = 3$ or similar will be **Special Case** SC: M1A0 (as equation in coefficients only is not seen here).

e.g. $2(405b)x = 270b^2x^2 \Rightarrow 2(405b) = 270b^2 \Rightarrow b = 3$ will get M1A1 (as coefficients rather than terms have now been considered).

Note: Answer of 3 from no working scores M1A0.

Note: The mistake $k\left(1+\frac{bx}{3}\right)^5$, $k \neq 243$ would give a maximum of 3 marks: B0B0M1A0, M1A1

Note: For $270bx^2$ in part (a), followed by $2(405b) = 270b^2 \Rightarrow b = 3$, in part (b), allow recovery M1A1.

Question Number	Scheme	Marks
3.	(a) $5^x = 10$ and (b) $\log_3(x-2) = -1$	
(a)	$x = \dfrac{\log 10}{\log 5}$ or $x = \log_5 10$	M1
	$x\{= 1.430676558...\} = 1.43$ (3 sf) 1.43	A1 **cao**
		[2]
(b)	$(x-2) = 3^{-1}$ $(x-2) = 3^{-1}$ or $\frac{1}{3}$	M1 oe
	$x\left\{= \frac{1}{3} + 2\right\} = 2\frac{1}{3}$ $2\frac{1}{3}$ or $\frac{7}{3}$ or $2.\dot{3}$ or awrt 2.33	A1
		[2]
		4
(a)	M1: for $x = \dfrac{\log 10}{\log 5}$ or $x = \log_5 10$. Also allow M1 for $x = \dfrac{1}{\log 5}$ 1.43 with no working (or any working) scores M1A1 (even if left as $5^{1.43}$). Other answers which round to 1.4 with no working score M1A0. **Trial & Improvement Method:** M1: For a method of trial and improvement by trialing f(value between 1.4 and 1.43) = Value below 10 and f(value between 1.431 and 1.5) = Value over 10. A1 for 1.43 cao. **Note**: $x = \log_{10} 5$ by itself is M0; but $x = \log_{10} 5$ followed by $x = 1.430676558...$ is M1.	
(b)	M1: Is for correctly eliminating log out of the equation. **Eg 1:** $\log_3(x-2) = \log_3(\frac{1}{3}) \Rightarrow x - 2 = \frac{1}{3}$ only gets M1 when the logs are correctly removed. **Eg 2:** $\log_3(x-2) = -\log_3(3) \Rightarrow \log_3(x-2) + \log_3(3) = 0 \Rightarrow \log_3(3(x-2)) = 0$ $\Rightarrow 3(x-2) = 3^0$ only gets M1 when the logs are correctly removed, but $3(x-2) = 0$ would score M0. **Note**: $\log_3(x-2) = -1 \Rightarrow \log_3\left(\dfrac{x}{2}\right) = -1 \Rightarrow \dfrac{x}{2} = 3^{-1}$ would score M0 for incorrect use of logs. **Alternative: changing base** $\dfrac{\log_{10}(x-2)}{\log_{10} 3} = -1 \Rightarrow \log_{10}(x-2) = -\log_{10} 3 \Rightarrow \log_{10}(x-2) + \log_{10} 3 = 0$ $\Rightarrow \log_{10} 3(x-2) = 0 \Rightarrow 3(x-2) = 10^0$. At this point M1 is scored. A correct answer in (b) without any working scores M1A1.	

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Question Number	Scheme	Marks
4.	$x^2 + y^2 + 4x - 2y - 11 = 0$	
(a)	$\{(x+2)^2 - 4 + (y-1)^2 - 1 - 11 = 0\}$ $(\pm 2, \pm 1)$, see notes.	M1
	Centre is $(-2, 1)$. $(-2, 1)$.	A1 **cao** **[2]**
(b)	$(x+2)^2 + (y-1)^2 = 11 + 1 + 4$ $r = \sqrt{11 \pm "1" \pm "4"}$	M1
	So $r = \sqrt{11+1+4} \Rightarrow r = 4$ 4 or $\sqrt{16}$ (Award A0 for ± 4).	A1 **[2]**
(c)	When $x = 0$, $y^2 - 2y - 11 = 0$ Putting $x = 0$ in C or their C.	M1
	$y^2 - 2y - 11 = 0$ or $(y-1)^2 = 12$, etc	A1 aef
	$y = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2(1)}$ $\left\{= \frac{2 \pm \sqrt{48}}{2}\right\}$ Attempt to use formula or a method of completing the square in order to find $y = ...$	M1
	So, $y = 1 \pm 2\sqrt{3}$ $1 \pm 2\sqrt{3}$	A1 **cao cso** **[4]**
		8

Note: Please mark parts (a) and (b) together. Answers only in (a) and/or (b) get full marks.
Note in part (a) the marks are now M1A1 and not B1B1 as on ePEN.

(a) M1: for $(\pm 2, \pm 1)$. Otherwise, M1 for an attempt to complete the square eg. $(x \pm 2)^2 \pm \alpha$, $\alpha \neq 0$ or $(y \pm 1)^2 \pm \beta$, $\beta \neq 0$. M1A1: Correct answer of $(-2, 1)$ stated from any working gets M1A1.

(b) M1: to find the radius using 11, "1" and "4", ie. $r = \sqrt{11 \pm "1" \pm "4"}$. By applying this method candidates will usually achieve $\sqrt{16}$, $\sqrt{6}$, $\sqrt{8}$ or $\sqrt{14}$ and not 16, 6, 8 or 14.

Note: $(x+2)^2 + (y-1)^2 = -11 - 5 = -16 \Rightarrow r = \sqrt{16} = 4$ should be awarded M0A0.

Alternative: M1 in part (a): For comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$ to write down centre $(-g, -f)$ directly. Condone sign errors for this M mark. M1 in part (b): For using $r = \sqrt{g^2 + f^2 - c}$. Condone sign errors for this method mark.

$(x+2)^2 + (y-1)^2 = 16 \Rightarrow r = 8$ scores M0A0, but $r = \sqrt{16} = 8$ scores M1A1 isw.

(c) 1st M1: Putting $x = 0$ in either $x^2 + y^2 + 4x - 2y - 11 = 0$ or their circle equation usually given in part (a) or part (b). 1st A1 for a correct equation in y **in any form** which can be implied by later working.

2nd M1: See rules for using the formula. Or completing the square on a 3TQ to give $y = a \pm \sqrt{b}$, where $\sqrt{b}$ is a surd, $b \neq$ their 11 and $b > 0$. This mark should not be given for an attempt to factorise.

2nd A1: Need exact pair in simplified surd form of $\{y =\}\ 1 \pm 2\sqrt{3}$. This mark is also cso.

Do not need to see $(0, 1 + 2\sqrt{3})$ and $(0, 1 - 2\sqrt{3})$. Allow 2nd A1 for bod $(1 + 2\sqrt{3}, 0)$ and $(1 - 2\sqrt{3}, 0)$.

Any incorrect working in (c) gets penalised the final accuracy mark. So, **beware:** incorrect $(x-2)^2 + (y-1)^2 = 16$ leading to $y^2 - 2y - 11 = 0$ and then $y = 1 \pm 2\sqrt{3}$ scores M1A1M1A0.

Special Case for setting $y = 0$: Award SC: M0A0M1A0 for an attempt at applying the formula

$$x = \frac{-4 \pm \sqrt{(-4)^2 - 4(1)(-11)}}{2(1)} \quad \left\{= \frac{-4 \pm \sqrt{60}}{2} = -2 \pm \sqrt{15}\right\}$$

Award SC: M0A0M1A0 for completing the square to their equation in x which will usually be $x^2 + 4x - 11 = 0$ to give $a \pm \sqrt{b}$, where $\sqrt{b}$ is a surd, $b \neq$ their 11 and $b > 0$.

Special Case: For a candidate not using $\pm$ but achieving one of the correct answers then award SC: M1A1 M1A0 for one of either $y = 1 + 2\sqrt{3}$ or $y = 1 - 2\sqrt{3}$ or $y = 1 + \sqrt{12}$ or $y = 1 - \sqrt{12}$.

Question Number	Scheme		Marks
5. (a)	$\frac{1}{2}r^2\theta = \frac{1}{2}(6)^2\left(\frac{\pi}{3}\right) = 6\pi$ or 18.85 or awrt 18.8 $(\text{cm})^2$	Using $\frac{1}{2}r^2\theta$ (See notes) 6π or 18.85 or awrt 18.8	M1 A1 **[2]**
(b)	$\sin\left(\frac{\pi}{6}\right) = \frac{r}{6-r}$ $\frac{1}{2} = \frac{r}{6-r}$ $6 - r = 2r \Rightarrow r = 2$	$\sin\left(\frac{\pi}{6}\right)$ or $\sin 30^\circ = \frac{r}{6-r}$ Replaces sin by numeric value $r = 2$	M1 dM1 A1 **cso** **[3]**
(c)	Area $= 6\pi - \pi(2)^2 = 2\pi$ or awrt $6.3\ (\text{cm})^2$	their area of sector $-\ \pi r^2$ 2π or awrt 6.3	M1 A1 **cao** **[2]** **7**

(a) M1: Needs θ in radians for this formula.
Candidate could convert to degrees and use the degrees formula.
A1: Does not need units. Answer should be either 6π or 18.85 or awrt 18.8
Correct answer with no working is M1A1.
This M1A1 can only be awarded in part (a).

(b) M1: Also allow $\cos\left(\frac{\pi}{3}\right)$ or $\cos 60^\circ = \frac{r}{6-r}$.

1st M1: Needs correct trigonometry method. Candidates could state $\sin\left(\frac{\pi}{6}\right) = \frac{r}{x}$ and $x + r = 6$ or equivalent in their working to gain this method mark.

dM1: Replaces sin by numerical value. $0.009... = \frac{r}{6-r}$ from working "incorrectly" in degrees is fine here for dM1.
A1: For $r = 2$ from correct solution only.

Alternative: 1st M1 for $\frac{r}{OC} = \sin 30$ or $\frac{r}{OC} = \cos 60$. 2nd M1 for $OC = 2r$ and then A1 for $r = 2$.

Note seeing $OC = 2r$ is M1M1.

Special Case: If a candidate states an answer of $r = 2$ (must be in part (b)) as a guess or from an incorrect method then award SC: M0M0B1. Such a candidate could then go on to score M1A1 in part (c).

(c) M1: For "their area of sector – their area of circle", where $r > 0$ is ft from their answer to part (b).
Allow the method mark if "their area of sector" < "their area of circle". The candidate must show somewhere in their working that they are subtracting the correct way round, even if their answer is negative.
Some candidates in part (c) will either use their value of r from part (b) or even introduce a value of r in part (c). You can apply the scheme to award either M0A0 or M1A0 or M1A1 to these candidates.

Note: Candidates can get M1 by writing "their part (a) answer $-\ \pi r^2$", where the radius of the circle is not substituted.

A1: cao – accept exact answer or awrt 6.3
Correct answer only with no working in (c) gets M1A1

Beware: The answer in (c) is the same as the arc length of the pendant

Question Number	Scheme	Marks
6. (a)	$\{ ar = 192$ and $ar^2 = 144 \}$	
	$r = \frac{144}{192}$ Attempt to eliminate a. (See notes.)	M1
	$r = \frac{3}{4}$ or 0.75 $\frac{3}{4}$ or 0.75	A1
		[2]
(b)	$a(0.75) = 192$	M1
	$a \left\{= \frac{192}{0.75}\right\} = 256$ 256	A1
		[2]
(c)	$S_\infty = \frac{256}{1-0.75}$ Applies $\frac{a}{1-r}$ correctly using both their a **and** their $\lvert r\rvert < 1$.	M1
	So, $\{S_\infty =\}$ 1024 1024	A1 cao
		[2]
(d)	$\frac{256(1-(0.75)^n)}{1-0.75} > 1000$ Applies S_n with their a and r and "uses" 1000 at any point in their working. (Allow with = or <).	M1
	$(0.75)^n < 1 - \frac{1000(0.25)}{256} \left\{= \frac{6}{256}\right\}$ Attempt to isolate $+(r)^n$ from S_n formula. (Allow with = or >).	M1
	$n\log(0.75) < \log\left(\frac{6}{256}\right)$ Uses the power law of logarithms correctly. (Allow with = or >). (See notes.)	M1
	$n > \frac{\log\left(\frac{6}{256}\right)}{\log(0.75)} = 13.0471042... \Rightarrow n = 14$ See notes **and** $n = 14$	A1 **cso**
		[4] **10**
(a) (b)	M1: **for eliminating *a*** by eg. $192r = 144$ or by either dividing $ar^2 = 144$ by $ar = 192$ or dividing $ar = 192$ by $ar^2 = 144$, to achieve an equation in r or $\frac{1}{r}$ Note that $r^2 - r = \frac{144}{192}$ is M0. Note also that any of $r = \frac{144}{192}$ or $r = \frac{192}{144}\left\{= \frac{4}{3}\right\}$ or $\frac{1}{r} = \frac{192}{144}$ or $\frac{1}{r} = \frac{144}{192}$ are fine for the award of M1. **Note:** A candidate just writing $r = \frac{144}{192}$ with no reference to a can also get the method mark. **Note**: $ar^2 = 192$ and $ar^3 = 144$ leading to $r = \frac{3}{4}$ scores M1A1. This is because r is the ratio between any two consecutive terms. These candidates, however, will usually be penalised in part (b). M1 for inserting their r into either of the correct equations of either $ar = 192$ or $\{a =\}\ \frac{192}{r}$ or $ar^2 = 144$ or $\{a =\}\ \frac{144}{r^2}$. No slips allowed here for M1. M1: can also be awarded for writing down $144 = a\left(\frac{192}{a}\right)^2$ A1 for $a = 256$ only. Note 256 from any working scores M1A1. **Note**: Some candidates incorrectly confuse notation to give $r = \frac{4}{3}$ or 1.33 in part (a) (getting M1A0). In part (b), they recover to write $a = 192 \times \frac{4}{3}$ for M1 and then 256 for A1.	

Question Number	Scheme	Marks
(c)	M1: for applying $\frac{a}{1-r}$ correctly (no slips allowed!) using both their *a* **and** their *r*, where $\lvert r \rvert < 1$. A1: for 1024, cao. In parts (a) or (b) or (c), the correct answer with no working scores full marks.	
(d)	1st M1: For applying S_n with their *a* and either "the letter *r*" or their *r* and "uses" 1000. 2nd M1: For isolating $+(r)^n$ and not $(ar)^n$, (eg. $(192)^n$) as the subject of an equation or inequality. $+(r)^n$ must be derived from the S_n formula. 3rd M1: For applying the power law to $\lambda^k = \mu$ to give $k\log\lambda = \log\mu$ oe. where $\lambda, \mu > 0$. or 3rd M1: For solving $\lambda^k = \mu$ to give $k = \log_\lambda \mu$, where $\lambda, \mu > 0$. A1: cso If a candidate uses inequalities, a fully correct method with inequalities is required here. So, an <u>incorrect</u> inequality statement at any stage in a candidate's working for this part loses this mark. **Note:** Some candidates do not realise that the direction of the inequality is reversed in the final line of their solution. Or A1: cso Note a candidate can achieve full marks here if they do not use inequalities. So, if a candidate uses equations rather than inequalities in their working then they need to state in the final line of their working that $n = 13.04$ (truncated) or $n =$ awrt $13.05 \Rightarrow n = 14$ for A1. $n = 14$ from no working gets SC: M0M0M1A1. A method of $T_n > 1000 \Rightarrow 256(0.75)^{n-1} > 1000$ can score M0M0M1A0 for a correct application of the power law of logarithms. **<u>Trial & Improvement Method:</u>** For $a = 256$ and $r = 0.75$, apply the following scheme:	

$S_{13} = \frac{256(1-(0.75)^{13})}{1-0.75} = 999.6725616...$	Attempt to find either S_{13} or S_{14}.	M1
	EITHER (1) $S_{13} =$ awrt 999.7 or truncated 999 OR (2) $S_{14} =$ awrt 1005.8 or truncated 1005.	M1
$S_{14} = \frac{256(1-(0.75)^{14})}{1-0.75} = 1005.754421...$	Attempt to find both S_{13} and S_{14}.	M1
So, $n = 14$.	BOTH (1) $S_{13} =$ awrt 999.7 or truncated 999 AND (2) $S_{14} =$ awrt 1005.8 or truncated 1005 AND $n = 14$.	A1

Question Number	Scheme	Marks
	Note: A similar scheme would apply for T&I for candidates using their a and their r. So,... 1st M1: For attempting to find one of the correct S_n 's either side (but next to) 1000. 2nd M1: For one of these S_n 's correct for their a and their r. (You may need to get your calculators out!) 3rd M1: For attempting to find both of the correct S_n 's either side (but next to) 1000. A1: Cannot be gained for wrong a and/or r. **Trial & Improvement Cumulative Approach:** A similar scheme to T&I will be applied here: 1st M1: For getting as far as the cumulative sum of 13 terms. 2nd M1: (1) $S_{13} =$ awrt 999.7 or truncated 999. 3rd M1: For getting as far as the cumulative sum to 14 terms. Also at this stage $S_{13} < 1000$ and $S_{14} > 1000$. A1: BOTH (1) $S_{13} =$ awrt 999.7 or truncated 999 AND (2) $S_{14} =$ awrt 1005.8 or truncated 1005 AND $n = 14$. **Trial & Improvement Method:** for $(0.75)^n < \frac{6}{256} = 0.0234375$ 3rd M1: For evidence of examining both $n = 13$ and $n = 14$. Eg: $(0.75)^{13}\{= 0.023757...\}$ and $(0.75)^{14}\{= 0.0178179...\}$ A1: $n = 14$ **Any misreads,** $S_n > 10000$ etc, please escalate up to your Team Leader.	
7.	(a) $3\sin(x + 45^\circ) = 2$; $0 \le x < 360^\circ$ (b) $2\sin^2 x + 2 = 7\cos x$; $0 \le x < 2\pi$	
(a)	$\sin(x + 45^\circ) = \frac{2}{3}$, so $(x + 45^\circ) = 41.8103...$ $(\alpha = 41.8103...)$ — $\sin^{-1}\left(\frac{2}{3}\right)$ or awrt 41.8 or awrt 0.73^c	M1
	So, $x + 45^\circ = \{138.1897...,\ 401.8103...\}$ — $x + 45^\circ =$ either "180 − their α" or "360° + their α" (α could be in radians).	M1
	and $x = \{93.1897...,\ 356.8103...\}$ — Either awrt 93.2° or awrt 356.8°	A1
	Both awrt 93.2° and awrt 356.8°	A1
		[4]
(b)	$2(1 - \cos^2 x) + 2 = 7\cos x$ — Applies $\sin^2 x = 1 - \cos^2 x$	M1
	$2\cos^2 x + 7\cos x - 4 = 0$ — Correct 3 term, $2\cos^2 x + 7\cos x - 4\ \{= 0\}$	A1 **oe**
	$(2\cos x - 1)(\cos x + 4)\ \{= 0\}$, $\cos x = ...$ — Valid attempt at solving and $\cos x = ...$	M1
	$\cos x = \frac{1}{2}$, $\{\cos x = -4\}$ — $\cos x = \frac{1}{2}$ (See notes.)	A1 cso
	$\left(\beta = \frac{\pi}{3}\right)$	
	$x = \frac{\pi}{3}$ or $1.04719...^c$ — Either $\frac{\pi}{3}$ or awrt 1.05^c	B1
	$x = \frac{5\pi}{3}$ or $5.23598...^c$ — Either $\frac{5\pi}{3}$ or awrt 5.24^c or 2π − their β (See notes.)	B1 ft
		[6] **10**

Question Number	Scheme	Marks
(a)	1st M1: can also be implied for $x = \text{awrt } -3.2$ 2nd M1: for $x + 45^\circ$ = either "$180 - \text{their } \alpha$" or "$360^\circ + \text{their } \alpha$". This can be implied by later working. The candidate's α could also be in radians. **Note that this mark is not** for x = either "$180 - \text{their } \alpha$" or "$360^\circ + \text{their } \alpha$". **<u>Note:</u>** Imply the first two method marks or award M1M1A1 for either awrt 93.2° or awrt 356.8°. **<u>Note:</u>** Candidates who apply the following incorrect working of $3\sin(x + 45^\circ) = 2$ $\Rightarrow 3(\sin x + \sin 45) = 2$, etc will usually score M0M0A0A0. If there are any EXTRA solutions inside the range $0 \le x < 360$ and the candidate would otherwise score FULL MARKS then withhold the final aA2 mark (the final mark in this part of the question). Also ignore EXTRA solutions outside the range $0 \le x < 360$. **Working in Radians:** Note the answers in radians are x = awrt 1.6, awrt 6.2 If a candidate works in radians then mark part (a) as above awarding the A marks in the same way. If the candidate would then score FULL MARKS then withhold the final aA2 mark (the final mark in this part of the question.) **No working:** Award M1M1A1A0 for one of awrt 93.2° or awrt 356.8° seen without any working. Award M1M1A1A1 for both awrt 93.2° and awrt 356.8° seen without any working. Allow benefit of the doubt (FULL MARKS) for final answer of $\sin x \{\text{and not } x\} = \{\text{awrt } 93.2, \text{ awrt } 356.8\}$	

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Question Number	Scheme	Marks
(b)	1st M1: for a correct method to use $\sin^2 x = 1 - \cos^2 x$ on the given equation. Give bod if the candidate omits the bracket when substituting for $\sin^2 x$, but $2 - \cos^2 x + 2 = 7\cos x$, without supporting working, (eg. seeing "$\sin^2 x = 1 - \cos^2 x$") would score 1st M0. Note that applying $\sin^2 x = \cos^2 x - 1$, scores M0. 1st A1: for obtaining either $2\cos^2 x + 7\cos x - 4$ or $-2\cos^2 x - 7\cos x + 4$. 1st A1: can also awarded for a correct three term equation eg. $2\cos^2 x + 7\cos x = 4$ or $2\cos^2 x = 4 - 7\cos x$ etc. 2nd M1: for a valid attempt at factorisation of a quadratic (either 2TQ or 3TQ) in cos, can use any variable here, c, y, x or $\cos x$, and an attempt to find at least one of the solutions. See introduction to the Mark Scheme. *Alternatively*, using a correct formula for solving the quadratic. Either the formula must be stated correctly or the correct form must be implied by the substitution. 2nd A1: for $\cos x = \frac{1}{2}$, BY A CORRECT SOLUTION ONLY UP TO THIS POINT. Ignore extra answer of $\cos x = -4$, but penalise if candidate states an incorrect result e.g. $\cos x = 4$. If they have used a substitution, a correct value of their c or their y or their x. **Note:** 2nd A1 for $\cos x = \frac{1}{2}$ can be implied by later working. 1st B1: for either $\frac{\pi}{3}$ or awrt 1.05^c 2nd B1: for either $\frac{5\pi}{3}$ or awrt 5.24^c or can be ft from 2π − their β or 360° − their β where $\beta = \cos^{-1}(k)$, such that $0 < k < 1$ or $-1 < k < 0$, but $k \neq 0$, $k \neq 1$ or $k \neq -1$. If there are any EXTRA solutions inside the range $0 \le x < 2\pi$ and the candidate would otherwise score FULL MARKS then withhold the final bB2 mark (the final mark in this part of the question). Also ignore EXTRA solutions outside the range $0 \le x < 2\pi$. **Working in Degrees:** Note the answers in degrees are $x = 60,\ 300$ If a candidate works in degrees then mark part (b) as above awarding the B marks in the same way. If the candidate would then score FULL MARKS then withhold the final bB2 mark (the final mark in this part of the question.) **Answers from no working:** $x = \frac{\pi}{3}$ and $x = \frac{5\pi}{3}$ scores M0A0M0A0B1B1, $x = 60$ and $x = 300$ scores M0A0M0A0B1B0, $x = \frac{\pi}{3}$ ONLY or $x = 60$ ONLY scores M0A0M0A0B1B0, $x = \frac{5\pi}{3}$ ONLY or $x = 120$ ONLY scores M0A0M0A0B0B1. **No working:** You cannot apply the ft in the B1ft if the answers are given with NO working. Eg: $x = \frac{\pi}{5}$ and $x = \frac{9\pi}{3}$ FROM NO WORKING scores M0A0M0A0B0B0. **For candidates using trial & improvement, please forward these to your Team Leader.**	

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<table>
<tr><th>Question Number</th><th>Scheme</th><th></th><th>Marks</th></tr>
<tr><td>8. (a)</td><td>$\{V =\}\ 2x^2y = 81$
$\{L = 2(2x + x + 2x + x) + 4y \Rightarrow L = 12x + 4y\}$
$y = \frac{81}{2x^2} \Rightarrow L = 12x + 4\left(\frac{81}{2x^2}\right)$
So, $L = 12x + \frac{162}{x^2}$ AG</td><td>$2x^2y = 81$

Making y the subject of their expression and substitute this into the correct L formula.
Correct solution only. AG.</td><td>B1 oe

M1

A1 cso
[3]</td></tr>
<tr><td>(b)</td><td>$\frac{dL}{dx} = 12 - \frac{324}{x^3}\ \{= 12 - 324x^{-3}\}$

$\left\{\frac{dL}{dx} =\right\} 12 - \frac{324}{x^3} = 0 \Rightarrow x^3 = \frac{324}{12}; = 27 \Rightarrow x = 3$

$\{x = 3,\}\ L = 12(3) + \frac{162}{3^2} = 54$ (cm)</td><td>Either $12x \to 12$ or $\frac{162}{x^2} \to \frac{\pm\lambda}{x^3}$
Correct differentiation (need not be simplified).
$L' = 0$ and "their $x^3 = \pm$ value" or "their $x^{-3} = \pm$ value"
$x = \sqrt[3]{27}$ or $x = 3$
Substitute candidate's value of $x (\neq 0)$ into a formula for L.
54</td><td>M1
A1 aef
M1;

A1 cso
ddM1

A1 cao
[6]</td></tr>
<tr><td>(c)</td><td>$\{\text{For } x = 3\},\ \frac{d^2L}{dx^2} = \frac{972}{x^4} > 0 \Rightarrow$ Minimum</td><td>Correct ft L'' and considering sign.
$\frac{972}{x^4}$ and > 0 and conclusion.</td><td>M1
A1 [2]
11</td></tr>
<tr><td>(a)

(b)

(c)</td><td colspan="3">B1: For any correct form of $2x^2y = 81$. (may be unsimplified). Note that $2x^3 = 81$ is B0. Otherwise, candidates can use any symbol or letter in place of y.
M1: Making y the subject of their formula and substituting this into a correct expression for L.
A1: Correct solution only. Note that the answer is given.
Note you can mark parts (b) and (c) together.
2nd M1: Setting their $\frac{dL}{dx} = 0$ and "candidate's ft correct power of x = a value". The power of x must be consistent with their differentiation. If inequalities are used this mark cannot be gained until candidate states value of x or L from their x without inequalities.
$L' = 0$ can be implied by $12 = \frac{324}{x^3}$.
2nd A1: $x^3 = 27 \Rightarrow x = \pm 3$ scores A0.
2nd A1: can be given for no value of x given but followed through by correct working leading to $L = 54$.
3rd M1: Note that this method mark is dependent upon the two previous method marks being awarded.
M1: for attempting correct ft second derivative and <u>considering its sign</u>.
A1: Correct second derivative of $\frac{972}{x^4}$ (need not be simplified) <u>and</u> a valid reason (e.g. > 0), <u>and</u> conclusion. Need to conclude minimum (allow x and not L is a minimum) or indicate by a tick that it is a minimum. The actual value of the second derivative, if found, can be ignored, although substituting their L and not x into L'' is A0. Note: 2 marks can be scored from a wrong value of x, no value of x found or from not substituting in the value of their x into L''.
Gradient test or testing values either side of their x scores M0A0 in part (c).
Throughout this question allow confused notation such as $\frac{dy}{dx}$ for $\frac{dL}{dx}$.</td></tr>
</table>

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Question Number	Scheme		Marks
9.	Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$		
(a)	$\{\text{Curve} = \text{Line}\} \Rightarrow -x^2 + 2x + 24 = x + 4$	Eliminating y correctly.	B1
	$x^2 - x - 20 \{= 0\} \Rightarrow (x-5)(x+4) \{= 0\} \Rightarrow x = ...$	Attempt to solve a ***resulting*** quadratic to give x = their values.	M1
	So, $x = 5, -4$	Both $x = 5$ and $x = -4$.	A1
	So corresponding y-values are $y = 9$ and $y = 0$.	See notes below.	B1ft **[4]**
(b)	$\left\{\int (-x^2 + 2x + 24)\,dx\right\} = \underline{-\frac{x^3}{3} + \frac{2x^2}{2} + 24x} \{+ c$	M1: $x^n \to x^{n+1}$ for any one term. 1st A1 at least two out of three terms. 2nd A1 for <u>correct answer</u>.	M1A1A1
	$\left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x\right]_{-4}^{5} = (......) - (......)$	Substitutes 5 and -4 (or their limits from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{\left(-\frac{125}{3} + 25 + 120\right) - \left(\frac{64}{3} + 16 - 96\right) = \left(103\frac{1}{3}\right) - \left(-58\frac{2}{3}\right) = 162\right\}$		
	Area of $\Delta = \frac{1}{2}(9)(9) = 40.5$	Uses correct method for finding area of triangle.	M1
	So area of R is $162 - 40.5 = 121.5$	Area under curve – Area of triangle.	M1
		121.5	A1 oe **cao**
			[7]
			11

Question Number	Scheme	Marks
(a)	1st B1: For correctly eliminating either x or y. Candidates will usually write $-x^2+2x+24=x+4$. This mark can be implied by the resulting quadratic. M1: For solving their quadratic (which must be different to $-x^2+2x+24$) to give $x=...$ See introduction for Method mark for solving a 3TQ. It must result from some attempt to eliminate one of the variables. A1: For both $x=5$ and $x=-4$. 2nd B1ft: For correctly substituting their values of x in equation of line or parabola to give ***both correct ft*** y-values. (You may have to get your calculators out if they substitute their x into $y=-x^2+2x+24$). **Note:** For $x=5,-4 \Rightarrow y=9$ and $y=0 \Rightarrow$ eg. $(-4,9)$ and $(5,0)$, award B1 isw. If the candidate gives additional answers to $(-4,0)$ and $(5,9)$, then withhold the final B1 mark. **Special Case**: Award SC: B0M0A0B1 for $\{A\}(-4,0)$. You may see this point marked on the diagram. **Note:** SC: B0M0A0B1 for solving $0=-x^2+2x+24$ to give $\{A\}(-4,0)$ and/or $(6,10)$. Note: Do not give marks for working in part (b) which would be creditable in part (a).	
(b)	1st M1 for an attempt to integrate meaning that $x^n \to x^{n+1}$ for at least one of the terms. Note that $24 \to 24x$ is sufficient for M1. 1st A1 at least two out of three terms correctly integrated. 2nd A1 for correct integration only and no follow through. Ignore the use of a '$+c$'. 2nd M1: Note that this method mark is dependent upon the award of the first M1 mark in part (b). Substitutes 5 and -4 (and not 4 if the candidate has stated $x=-4$ in part (a).) (or the limits the candidate has found from part(a)) into an "integrated function" and subtracts, either way round. Allow one slip! 3rd M1: Area of triangle = $\frac{1}{2}$(their x_2 – their x_1)(their y_2) or Area of triangle = $\int_{x_1}^{x_2} x+4\ \{dx\}$. Where x_1 = their -4, x_2 = their 5 and y_2 = their y usually found in part (a). 4th M1: Area under curve – Area under triangle, where both Area under curve > 0 and Area under triangle > 0 and Area under curve > Area under triangle. 3rd A1: 121.5 or $\frac{243}{2}$ oe **cao.**	

Question Number	Scheme	Marks
Aliter **9.**(b) **Way 2**	Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$ Area of $R = \int_{-4}^{5} (-x^2 + 2x + 24) - (x+4)\,\mathrm{d}x$ — 3rd M1: Uses integral of $(x+4)$ with correct ft limits. 4th M1: Uses "curve" – "line" function with correct ft limits.	
	$= -\frac{x^3}{3} + \frac{x^2}{2} + 20x \{+c\}$ — M: $x^n \to x^{n+1}$ for any one term. A1 at least two out of three terms Correct answer (Ignore + c).	M1 A1ft A1
	$\left[-\frac{x^3}{3} + \frac{x^2}{2} + 20x\right]_{-4}^{5} = (......) - (......)$ — Substitutes 5 and -4 (or ***their limits*** from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{\left(-\frac{125}{3} + \frac{25}{2} + 100\right) - \left(\frac{64}{3} + 8 - 80\right) = \left(70\frac{5}{6}\right) - \left(-50\frac{2}{3}\right)\right\}$	
	See above working to decide to award 3rd M1 mark here: ***See above working to decide to award 4th M1 mark here:***	M1 M1
	So area of R is $= 121.5$ — 121.5	A1 oe **cao**
		[7] **11**
(b)	1st M1 for an attempt to integrate meaning that $x^n \to x^{n+1}$ for at least one of the terms. Note that $20 \to 20x$ is sufficient for M1. 1st A1 at least two out of three terms correctly ft. Note this accuracy mark is ft in Way 2. 2nd A1 for correct integration only and no follow through. Ignore the use of a $'+c'$. Allow 2nd A1 also for $-\frac{x^3}{3} + \frac{2x^2}{2} + 24x - \left(\frac{x^2}{2} + 4x\right)$. Note that $\frac{2x^2}{2} - \frac{x^2}{2}$ or $24x - 4x$ only counts as one integrated term for the 1st A1 mark. Do not allow any extra terms for the 2nd A1 mark. 2nd M1: Note that this method mark is dependent upon the award of the first M1 mark in part (b). Substitutes 5 and -4 (and not 4 if the candidate has stated $x = -4$ in part (a).) (or the limits the candidate has found from part(a)) into an "integrated function" and subtracts, either way round. Allow one slip! 3rd M1: Uses the integral of $(x+4)$ with correct ft limits of their x_1 and their x_2 (usually found in part (a)) $\{$where $(x_1, y_1) = (-4, 0)$ and $(x_2, y_2) = (5, 9).\}$ This mark is usually found in the first line of the candidate's working in part (b). 4th M1: Uses "curve" – "line" function with correct ft (usually found in part (a)) limits. Subtraction must be correct way round. This mark is usually found in the first line of the candidate's working in part (b). Allow $\int_{-4}^{5} (-x^2 + 2x + 24) - x + 4 \{\mathrm{d}x\}$ for this method mark. 3rd A1: 121.5 oe **cao.** **<u>Note: SPECIAL CASE for this alternative method</u>** Area of $R = \int_{-4}^{5} (x^2 - x - 20)\,\mathrm{d}x = \left[\frac{x^3}{3} - \frac{x^2}{2} - 20x\right]_{-4}^{5} = \left(\frac{125}{3} - \frac{25}{2} - 100\right) - \left(-\frac{64}{3} - 8 + 80\right)$ The working so far would score SPEICAL CASE M1A1A1M1M1M0A0. The candidate may then go on to state that $= \left(-70\frac{5}{6}\right) - \left(50\frac{2}{3}\right) = -\frac{243}{2}$ If the candidate then multiplies their answer by -1 then they would gain the 4th M1 and 121.5 would gain the final A1 mark.	

Question Number	Scheme		Marks
Aliter **9.** (a) **Way 2**	Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$		
	$\{\text{Curve} = \text{Line}\} \Rightarrow y = -(y-4)^2 + 2(y-4) + 24$	Eliminating x correctly.	B1
	$y^2 - 9y \{= 0\} \Rightarrow y(y-9)\{= 0\} \Rightarrow y = ...$	Attempt to solve a resulting quadratic to give y = their values.	M1
	So, $y = 0, 9$	Both $y = 0$ and $y = 9$.	A1
	So corresponding y-values are $x = -4$ and $x = 5$.	See notes below.	B1ft
			[4]
	2nd B1ft: For correctly substituting their values of y in equation of line or parabola to give ***both correct ft*** x-values.		
9. (b)	**<u>Alternative Methods for obtaining the M1 mark for use of limits:</u>**		

There are two alternative methods can candidates can apply for finding "162".

<u>Alternative 1:</u>

$$\int_{-4}^{0}(-x^2 + 2x + 24)\,\mathrm{d}x + \int_{0}^{5}(-x^2 + 2x + 24)\,\mathrm{d}x$$

$$= \left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x\right]_{-4}^{0} + \left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x\right]_{0}^{5}$$

$$= (0) - \left(\frac{64}{3} + 16 - 96\right) + \left(-\frac{125}{3} + 25 + 120\right) - (0)$$

$$= \left(103\frac{1}{3}\right) - \left(-58\frac{2}{3}\right) = 162$$

<u>Alternative 2:</u>

$$\int_{-4}^{6}(-x^2 + 2x + 24)\,\mathrm{d}x - \int_{5}^{6}(-x^2 + 2x + 24)\,\mathrm{d}x$$

$$= \left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x\right]_{-4}^{6} - \left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x\right]_{5}^{6}$$

$$= \left\{\left(-\frac{216}{3} + 36 + 144\right) - \left(\frac{64}{3} + 16 - 96\right)\right\} - \left\{\left(-\frac{216}{3} + 36 + 144\right) - \left(-\frac{125}{3} + 25 + 120\right)\right\}$$

$$= \left\{(108) - \left(-58\frac{2}{3}\right)\right\} - \left\{(108) - \left(103\frac{1}{3}\right)\right\}$$

$$= \left(166\frac{2}{3}\right) - \left(4\frac{2}{3}\right) = 162$$

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Appendix

List of Abbreviations

- dM1 denotes a method mark which is dependent upon the award of the previous method mark.
- ft or $\sqrt{\ }$ denotes "follow through"
- cao denotes "correct answer only"
- aef denotes "any equivalent form"
- cso denotes "correct solution only"
- AG or * denotes "answer given" (in the question paper.)
- awrt denotes "anything that rounds to"
- aliter denotes "alternative methods"

Extra Solutions

If a candidate makes more than one attempt at any question:

- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

Question Number	Scheme		Marks
***Aliter* 4.** (c) **Way 2**	$(x+2)^2 + (y-1)^2 = 16$, centre $(x_1, y_1) = (-2, 1)$ and radius $r = 4$.		
	$d_1 = \sqrt{4^2 - 2^2} = \sqrt{12}$	Applying $\sqrt{\text{their } r^2 - \lvert\text{their } x_1\rvert^2}$	M1
		$\sqrt{12}$	A1 aef
	Hence, $y = 1 \pm \sqrt{12}$	Applies $y =$ their $y_1 \pm$ their d	M1
	So, $y = 1 \pm 2\sqrt{3}$	$1 \pm 2\sqrt{3}$	A1 **cao cso**
			[4]
	Special Case: Award Final SC: M1A1 M1A0 if candidate achieves any one of either $y = 1 + 2\sqrt{3}$ or $y = 1 - 2\sqrt{3}$ or $y = 1 + \sqrt{12}$ or $y = 1 - \sqrt{12}$.		

Question Number	Scheme		Marks
***Aliter* 8.** (a) **Way 2**	$2x^2\left(\frac{L-12x}{4}\right) = 81$	$2x^2\left(\frac{L-12x}{4}\right) = 81$	B1 oe
	$\Rightarrow x^2(L-12x) = 162 \Rightarrow L = 12x + \frac{162}{x^2}$	Rearranges their equation to make y the subject.	M1
		Correct solution only. **AG.**	A1 **cso**
			[3]

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