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# Mark Scheme (Results)
# Summer 2008

GCE

## GCE Mathematics (6665/01)

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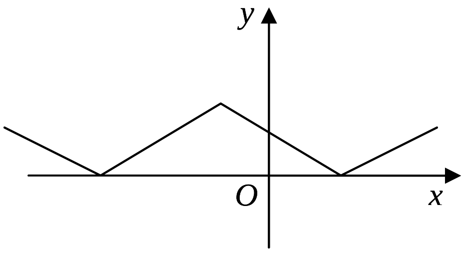
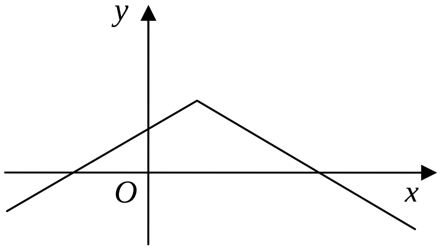
# June 2008
# 6665 Core Mathematics C3
# Mark Scheme

| Question Number | Scheme | Marks |
|---|---|---|
| **1.** | (a) $e^{2x+1} = 2$ | |
| | $2x + 1 = \ln 2$ | M1 |
| | $x = \frac{1}{2}(\ln 2 - 1)$ | A1 (2) |
| | (b) $\frac{dy}{dx} = 8e^{2x+1}$ | B1 |
| | $x = \frac{1}{2}(\ln 2 - 1) \Rightarrow \frac{dy}{dx} = 16$ | B1 |
| | $y - 8 = 16\left(x - \frac{1}{2}(\ln 2 - 1)\right)$ | M1 |
| | $y = 16x + 16 - 8\ln 2$ | A1 (4) |
| | | **[6]** |

| Question Number | Scheme | Marks |
|---|---|---|
| **2.** | (a) $R^2 = 5^2 + 12^2$ | M1 |
| | $R = 13$ | A1 |
| | $\tan\alpha = \frac{12}{5}$ | M1 |
| | $\alpha \approx 1.176$ cao | A1 (4) |
| | (b) $\cos(x-\alpha) = \frac{6}{13}$ | M1 |
| | $x - \alpha = \arccos\frac{6}{13} = 1.091 \ldots$ | A1 |
| | $x = 1.091 \ldots + 1.176 \ldots \approx 2.267 \ldots$ awrt 2.3 | A1 |
| | $x - \alpha = -1.091 \ldots$ accept … = 5.19 … for M | M1 |
| | $x = -1.091 \ldots + 1.176 \ldots \approx 0.0849 \ldots$ awrt 0.084 or 0.085 | A1 (5) |
| | (c)(i) $R_{\max} = 13$ ft their $R$ | B1 ft |
| | (ii) At the maximum, $\cos(x-\alpha) = 1$ *or* $x - \alpha = 0$ | M1 |
| | $x = \alpha = 1.176 \ldots$ awrt 1.2, ft their $\alpha$ | A1ft (3) **[12]** |

| Question Number | Scheme | | Marks | |
|---|---|---|---|---|
| **3.** | (a)  |  shape<br>Vertices correctly placed | B1<br>B1 | <br>(2) |
| | (b)  |  shape<br>Vertex and intersections with axes correctly placed | B1<br>B1 | <br>(2) |
| | (c) $P:(-1, 2)$ | | B1 | |
| | $Q:(0, 1)$ | | B1 | |
| | $R:(1, 0)$ | | B1 | (3) |
| | (d) $x > -1$; $2 - x - 1 = \frac{1}{2}x$ | | M1 A1 | |
| | Leading to $x = \frac{2}{3}$ | | A1 | |
| | $x < -1$; $2 + x + 1 = \frac{1}{2}x$ | | M1 | |
| | Leading to $x = -6$ | | A1 | (5)<br>**[12]** |

| Question Number | Scheme | | Marks | |
|---|---|---|---|---|
| **4.** | (a) $x^2 - 2x - 3 = (x-3)(x+1)$ | | B1 | |
| | $f(x) = \dfrac{2(x-1)-(x+1)}{(x-3)(x+1)}$ $\left( or\ \dfrac{2(x-1)}{(x-3)(x+1)} - \dfrac{x+1}{(x-3)(x+1)} \right)$ | | M1 A1 | |
| | $= \dfrac{x-3}{(x-3)(x+1)} = \dfrac{1}{x+1}$ * | cso | A1 | (4) |
| | (b) $\left(0, \dfrac{1}{4}\right)$ | Accept $0 < y < \dfrac{1}{4}$, $0 < f(x) < \dfrac{1}{4}$ etc. | B1 B1 | (2) |
| | (c) Let $y = f(x)$ $\quad y = \dfrac{1}{x+1}$ | | | |
| | $x = \dfrac{1}{y+1}$ | | | |
| | $yx + x = 1$ | | | |
| | $y = \dfrac{1-x}{x}$ | or $\dfrac{1}{x} - 1$ | M1 A1 | |
| | $f^{-1}(x) = \dfrac{1-x}{x}$ | | | |
| | Domain of $f^{-1}$ is $\left(0, \dfrac{1}{4}\right)$ | ft their part (b) | B1 ft | (3) |
| | (d) $fg(x) = \dfrac{1}{2x^2 - 3 + 1}$ | | | |
| | $\dfrac{1}{2x^2 - 2} = \dfrac{1}{8}$ | | M1 | |
| | $x^2 = 5$ | | A1 | |
| | $x = \pm\sqrt{5}$ | both | A1 | (3) |
| | | | | **[12]** |

| Question Number | Scheme | | Marks | |
|---|---|---|---|---|
| **5.** | (a) $\sin^2\theta + \cos^2\theta = 1$ | | | |
| | $\div \sin^2\theta$ $\quad \dfrac{\sin^2\theta}{\sin^2\theta} + \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta}$ | | M1 | |
| | $1 + \cot^2\theta = \operatorname{cosec}^2\theta$ ∗ | cso | A1 | (2) |
| | *Alternative for* (a) | | | |
| | $1 + \cot^2\theta = 1 + \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta}$ | | M1 | |
| | $= \operatorname{cosec}^2\theta$ ∗ | cso | A1 | |
| | (b) $2\left(\operatorname{cosec}^2\theta - 1\right) - 9\operatorname{cosec}\theta = 3$ | | M1 | |
| | $2\operatorname{cosec}^2\theta - 9\operatorname{cosec}\theta - 5 = 0$ *or* $5\sin^2\theta + 9\sin\theta - 2 = 0$ | | M1 | |
| | $(2\operatorname{cosec}\theta + 1)(\operatorname{cosec}\theta - 5) = 0$ *or* $(5\sin\theta - 1)(\sin\theta + 2) = 0$ | | M1 | |
| | $\operatorname{cosec}\theta = 5$ *or* $\sin\theta = \dfrac{1}{5}$ | | A1 | |
| | $\theta = 11.5°, 168.5°$ | | A1 A1 | (6) |
| | | | | **[8]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **6.** | (a)(i) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{3x}(\sin x + 2\cos x)\right) = 3\mathrm{e}^{3x}(\sin x + 2\cos x) + \mathrm{e}^{3x}(\cos x - 2\sin x)$ | | M1 A1 A1 (3) |
| | $\left(= \mathrm{e}^{3x}(\sin x + 7\cos x)\right)$ | | |
| | (ii) $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^3 \ln(5x+2)\right) = 3x^2 \ln(5x+2) + \frac{5x^3}{5x+2}$ | | M1 A1 A1 (3) |
| | (b) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x+1)^2(6x+6) - 2(x+1)(3x^2+6x-7)}{(x+1)^4}$ | | M1 $\frac{\text{A1}}{\text{A1}}$ |
| | $= \frac{(x+1)(6x^2+12x+6-6x^2-12x+14)}{(x+1)^4}$ | | M1 |
| | $= \frac{20}{(x+1)^3}$ * | cso | A1 (5) |
| | (c) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\frac{60}{(x+1)^4} = -\frac{15}{4}$ | | M1 |
| | $(x+1)^4 = 16$ | | M1 |
| | $x = 1, -3$ | both | A1 (3) |
| | | | **[14]** |
| | *Note*: The simplification in part (b) can be carried out as follows | | |
| | $\frac{(x+1)^2(6x+6) - 2(x+1)(3x^2+6x-7)}{(x+1)^4}$ | | |
| | $= \frac{(6x^3+18x^2+18x+6) - (6x^3+18x^2-2x-14)}{(x+1)^4}$ | | |
| | $= \frac{20x+20}{(x+1)^4} = \frac{20(x+1)}{(x+1)^4} = \frac{20}{(x+1)^3}$ | | M1 A1 |

| Question Number | Scheme | | Marks | |
|---|---|---|---|---|
| **7.** | (a) $\text{f}(1.4) = -0.568 \ldots < 0$ | | | |
| | $\text{f}(1.45) = 0.245 \ldots > 0$ | | M1 | |
| | Change of sign (and continuity) $\Rightarrow \alpha \in (1.4, 1.45)$ | | A1 | (2) |
| | (b) $3x^3 = 2x + 6$ | | | |
| | $x^3 = \frac{2x}{3} + 2$ | | | |
| | $x^2 = \frac{2}{3} + \frac{2}{x}$ | | M1 A1 | |
| | $x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}$ ∗ | cso | A1 | (3) |
| | (c) $x_1 = 1.4371$ | | B1 | |
| | $x_2 = 1.4347$ | | B1 | |
| | $x_3 = 1.4355$ | | B1 | (3) |
| | (d) Choosing the interval $(1.4345, 1.4355)$ or appropriate tighter interval. | | M1 | |
| | $\text{f}(1.4345) = -0.01$ ... | | | |
| | $\text{f}(1.4355) = 0.003 \ldots$ | | M1 | |
| | Change of sign (and continuity) $\Rightarrow \alpha \in (1.4345, 1.4355)$ | | | |
| | $\Rightarrow \alpha = 1.435$, correct to 3 decimal places ∗ | cso | A1 | (3) **[11]** |
| | *Note*: $\alpha = 1.435\,304\,553 \ldots$ | | | |