



1.

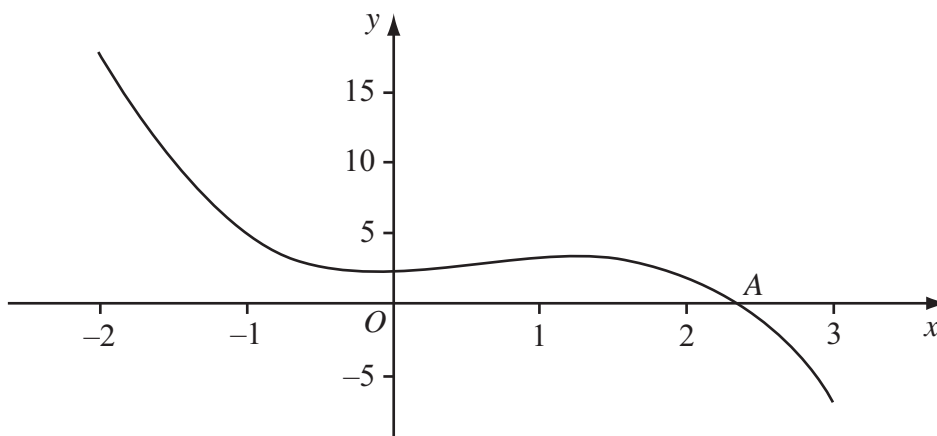


Figure 1

Figure 1 shows part of the curve with equation  $y = -x^3 + 2x^2 + 2$ , which intersects the  $x$ -axis at the point  $A$  where  $x = \alpha$ .

To find an approximation to  $\alpha$ , the iterative formula

$$x_{n+1} = \frac{2}{(x_n)^2} + 2$$

is used.

(a) Taking  $x_0 = 2.5$ , find the values of  $x_1, x_2, x_3$  and  $x_4$ .  
Give your answers to 3 decimal places where appropriate.

(3)

(b) Show that  $\alpha = 2.359$  correct to 3 decimal places.

(3)

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3. Rabbits were introduced onto an island. The number of rabbits,  $P$ ,  $t$  years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{5}t}, \quad t \in \mathbb{R}, t \geq 0$$

- (a) Write down the number of rabbits that were introduced to the island. (1)
- (b) Find the number of years it would take for the number of rabbits to first exceed 1000. (2)
- (c) Find  $\frac{dP}{dt}$ . (2)
- (d) Find  $P$  when  $\frac{dP}{dt} = 50$ . (3)

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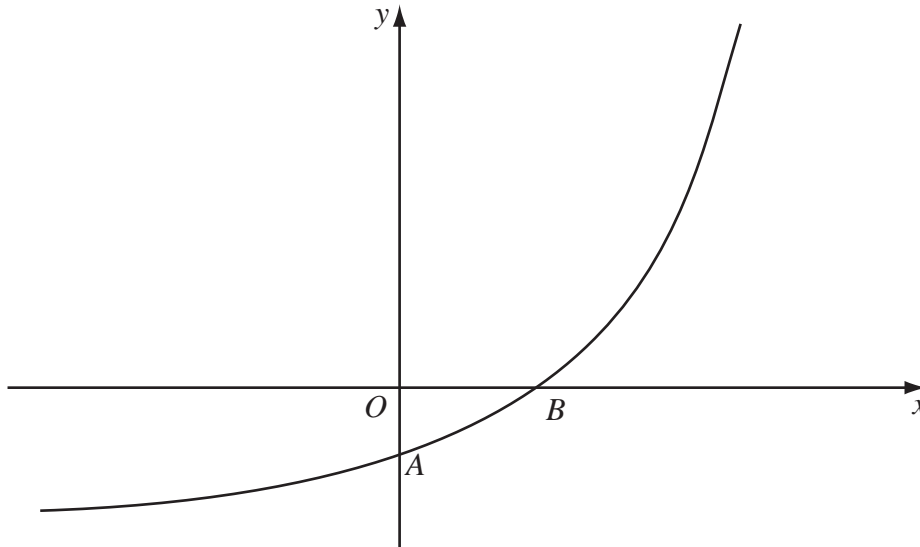
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5.



**Figure 2**

Figure 2 shows a sketch of part of the curve with equation  $y = f(x)$ ,  $x \in \mathbb{R}$ .

The curve meets the coordinate axes at the points  $A(0, 1-k)$  and  $B(\frac{1}{2} \ln k, 0)$ , where  $k$  is a constant and  $k > 1$ , as shown in Figure 2.

On separate diagrams, sketch the curve with equation

(a)  $y = |f(x)|$ , (3)

(b)  $y = f^{-1}(x)$ . (2)

Show on each sketch the coordinates, in terms of  $k$ , of each point at which the curve meets or cuts the axes.

Given that  $f(x) = e^{2x} - k$ ,

(c) state the range of  $f$ , (1)

(d) find  $f^{-1}(x)$ , (3)

(e) write down the domain of  $f^{-1}$ . (1)



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**Question 5 continued**





6. (a) Use the identity  $\cos(A+B) = \cos A \cos B - \sin A \sin B$ , to show that

$$\cos 2A = 1 - 2 \sin^2 A \tag{2}$$

The curves  $C_1$  and  $C_2$  have equations

$$C_1: y = 3 \sin 2x$$

$$C_2: y = 4 \sin^2 x - 2 \cos 2x$$

(b) Show that the  $x$ -coordinates of the points where  $C_1$  and  $C_2$  intersect satisfy the equation

$$4 \cos 2x + 3 \sin 2x = 2 \tag{3}$$

(c) Express  $4 \cos 2x + 3 \sin 2x$  in the form  $R \cos(2x - \alpha)$ , where  $R > 0$  and  $0 < \alpha < 90^\circ$ , giving the value of  $\alpha$  to 2 decimal places. (3)

(d) Hence find, for  $0 \leq x < 180^\circ$ , all the solutions of

$$4 \cos 2x + 3 \sin 2x = 2$$

giving your answers to 1 decimal place. (4)

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7. The function  $f$  is defined by

$$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}, \quad x \in \mathbb{R}, x \neq -4, x \neq 2$$

(a) Show that  $f(x) = \frac{x-3}{x-2}$  (5)

The function  $g$  is defined by

$$g(x) = \frac{e^x - 3}{e^x - 2}, \quad x \in \mathbb{R}, x \neq \ln 2$$

(b) Differentiate  $g(x)$  to show that  $g'(x) = \frac{e^x}{(e^x - 2)^2}$  (3)

(c) Find the exact values of  $x$  for which  $g'(x) = 1$  (4)

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**Question 7 continued**

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