Unit C3 Core Mathematics 3

A2 compulsory unit for GCE Mathematics and GCE Pure Mathematics

C3.1 Unit description

Algebra and functions; trigonometry; exponentials and logarithms; differentiation; numerical methods.

C3.2 Assessment information

Prerequisites and preamble	**Prerequisites** A knowledge of the specifications for C1 and C2, their preambles, prerequisites and associated formulae, is assumed and may be tested. **Preamble** Methods of proof, including proof by contradiction and disproof by counter-example, are required. At least one question on the paper will require the use of proof.
Examination	The examination will consist of one 1½ hour paper. It will contain about seven questions of varying length. The mark allocations per question which will be stated on the paper. All questions should be attempted.
Calculators	Students are expected to have available a calculator with at least the following keys: $+, -, \times, \div, \pi, x^2, \sqrt{x}, \frac{1}{x}, x^y, \ln x, e^x, x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Formulae which students are expected to know are given below and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

This section lists formulae that students are expected to remember and that may not be included in formulae booklets.

Trigonometry

$\cos^2 A + \sin^2 A \equiv 1$

$\sec^2 A \equiv 1 + \tan^2 A$

$\text{cosec}^2 A \equiv 1 + \cot^2 A$

$\sin 2A \equiv 2 \sin A \cos A$

$\cos 2A \equiv \cos^2 A - \sin^2 A$

$\tan 2A \equiv \dfrac{2 \tan A}{1 - \tan^2 A}$

Differentiation

function	derivative
$\sin kx$	$k \cos kx$
$\cos kx$	$-k \sin kx$
e^{kx}	ke^{kx}
$\ln x$	$\dfrac{1}{x}$
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x)\,g(x)$	$f'(x)\,g(x) + f(x)\,g'(x)$
$f(g(x))$	$f'(g(x))\,g'(x)$

1 Algebra and functions

What students need to learn:

Simplification of rational expressions including factorising and cancelling, and algebraic division.	Denominators of rational expressions will be linear or quadratic, eg $\frac{1}{ax+b}$, $\frac{ax+b}{px^2+qx+r}$, $\frac{x^3+1}{x^2-1}$.
Definition of a function. Domain and range of functions. Composition of functions. Inverse functions and their graphs.	The concept of a function as a one-one or many-one mapping from $\mathbb{R}$ (or a subset of $\mathbb{R}$) to $\mathbb{R}$. The notation $\mathrm{f}: x \mapsto$ and $\mathrm{f}(x)$ will be used. Students should know that fg will mean 'do g first, then f'. Students should know that if f^{-1} exists, then $\mathrm{f}^{-1}\mathrm{f}(x) = \mathrm{f}\mathrm{f}^{-1}(x) = x$.
The modulus function.	Students should be able to sketch the graphs of $y = \lvert ax+b \rvert$ and the graphs of $y = \lvert \mathrm{f}(x) \rvert$ and $y = \mathrm{f}(\lvert x \rvert)$, given the graph of $y = \mathrm{f}(x)$.
Combinations of the transformations $y = \mathrm{f}(x)$ as represented by $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = \mathrm{f}(ax)$.	Students should be able to sketch the graph of, for example, $y = 2\mathrm{f}(3x)$, $y = \mathrm{f}(-x) + 1$, given the graph of $y = \mathrm{f}(x)$ or the graph of, for example, $y = 3 + \sin 2x$, $y = -\cos\left(x + \frac{\pi}{4}\right)$. The graph of $y = \mathrm{f}(ax + b)$ will *not* be required.

2 Trigonometry

What students need to learn:

Knowledge of secant, cosecant and cotangent and of arcsin, arccos and arctan. Their relationships to sine, cosine and tangent. Understanding of their graphs and appropriate restricted domains.

Angles measured in both degrees and radians.

Knowledge and use of $\sec^2 \theta = 1 + \tan^2 \theta$ and $\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$.

Knowledge and use of double angle formulae; use of formulae for $\sin (A \pm B)$, $\cos (A \pm B)$ and $\tan (A \pm B)$ and of expressions for $a \cos \theta + b \sin \theta$ in the equivalent forms of $r \cos (\theta \pm \alpha)$ or $r \sin (\theta \pm \alpha)$.

To include application to half angles. Knowledge of the t $(\tan \frac{1}{2}\theta)$ formulae will *not* be required.

Students should be able to solve equations such as $a \cos \theta + b \sin \theta = c$ in a given interval, and to prove simple identities such as $\cos x \cos 2x + \sin x \sin 2x \equiv \cos x$.

3 Exponentials and logarithms

What students need to learn:

The function e^x and its graph.

To include the graph of $y = e^{ax + b} + c$.

The function $\ln x$ and its graph; $\ln x$ as the inverse function of e^x.

Solution of equations of the form $e^{ax + b} = p$ and $\ln (ax + b) = q$ is expected.

4 Differentiation

What students need to learn:

Differentiation of e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$ and their sums and differences.	
Differentiation using the product rule, the quotient rule and the chain rule.	Differentiation of $\operatorname{cosec} x$, $\cot x$ and $\sec x$ are required. Skill will be expected in the differentiation of functions generated from standard forms using products, quotients and composition, such as $2x^4 \sin x$, $\frac{e^{3x}}{x}$, $\cos x^2$ and $\tan^2 2x$.
The use of $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$.	Eg finding $\frac{dy}{dx}$ for $x = \sin 3y$.

5 Numerical methods

What students need to learn:

Location of roots of $f(x) = 0$ by considering changes of sign of $f(x)$ in an interval of x in which $f(x)$ is continuous.	
Approximate solution of equations using simple iterative methods, including recurrence relations of the form $x_{n+1} = f(x_n)$.	Solution of equations by use of iterative procedures for which leads will be given.

