Unit C4 Core Mathematics 4

A2 compulsory unit for GCE Mathematics and GCE Pure Mathematics

C4.1 Unit description

Algebra and functions; coordinate geometry in the (x, y) plane; sequences and series; differentiation; integration; vectors.

C4.2 Assessment information

Prerequisites	A knowledge of the specifications for C1, C2 and C3 and their preambles, prerequisites and associated formulae, is assumed and may be tested.
Examination	The examination will consist of one 1½ hour paper. It will contain about seven questions of varying length. The mark allocations per question which will be stated on the paper. All questions should be attempted.
Calculators	Students are expected to have available a calculator with at least the following keys: $+, -, \times, \div, \pi, x^2, \sqrt{x}, \frac{1}{x}, x^y, \ln x, e^x, x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.
Formulae	Formulae which students are expected to know are given overleaf and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage. This section lists formulae that students are expected to remember and that may not be included in formulae booklets.

Integration

function	integral
$\cos kx$	$\frac{1}{k}\sin kx + c$
$\sin kx$	$-\frac{1}{k}\cos kx + c$
e^{kx}	$\frac{1}{k}e^{kx} + c$
$\frac{1}{x}$	$\ln \lvert x \rvert + c, x \neq 0$
$f'(x) + g'(x)$	$f(x) + g(x) + c$
$f'(g(x))\,g'(x)$	$f(g(x)) + c$

Vectors

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \bullet \begin{pmatrix} a \\ b \\ c \end{pmatrix} = xa + yb + zc$$

1 Algebra and functions

What students need to learn:

Rational functions. Partial fractions (denominators not more complicated than repeated linear terms).

Partial fractions to include denominators such as $(ax + b)(cx + d)(ex + f)$ and $(ax + b)(cx + d)^2$.

The degree of the numerator may equal or exceed the degree of the denominator. Applications to integration, differentiation and series expansions.

Quadratic factors in the denominator such as $(x^2 + a)$, $a > 0$, are *not* required.

2 Coordinate geometry in the (x, y) plane

What students need to learn:

Parametric equations of curves and conversion between Cartesian and parametric forms.	Students should be able to find the area under a curve given its parametric equations. Students will *not* be expected to sketch a curve from its parametric equations.

3 Sequences and series

What students need to learn:

Binomial series for any rational n.	For $\lvert x \rvert < \frac{b}{a}$, students should be able to obtain the expansion of $(ax + b)^n$, and the expansion of rational functions by decomposition into partial fractions.

4 Differentiation

What students need to learn:

Differentiation of simple functions defined implicitly or parametrically.	The finding of equations of tangents and normals to curves given parametrically or implicitly is required.
Exponential growth and decay.	Knowledge and use of the result $\frac{\mathrm{d}}{\mathrm{d}x}(a^x) = a^x \ln a$ is expected.
Formation of simple differential equations.	Questions involving connected rates of change may be set.

5 Integration

What students need to learn:

Integration of e^x, $\frac{1}{x}$, $\sin x$, $\cos x$.	To include integration of standard functions such as $\sin 3x$, $\sec^2 2x$, $\tan x$, e^{5x}, $\frac{1}{2x}$. Students should recognise integrals of the form $\int \frac{f'(x)}{f(x)}\, dx = \ln f(x) + c$. Students are expected to be able to use trigonometric identities to integrate, for example, $\sin^2 x$, $\tan^2 x$, $\cos^2 3x$.
Evaluation of volume of revolution.	$\pi \int y^2 \, dx$ is required, but *not* $\pi \int x^2 \, dy$. Students should be able to find a volume of revolution, given parametric equations.
Simple cases of integration by substitution and integration by parts. These methods as the reverse processes of the chain and product rules respectively.	Except in the simplest of cases the substitution will be given. The integral $\int \ln x \, dx$ is required. More than one application of integration by parts may be required, for example $\int x^2 e^x \, dx$.

Simple cases of integration using partial fractions.	Integration of rational expressions such as those arising from partial fractions, eg $\frac{2}{3x+5}$, $\frac{3}{(x-1)^2}$. Note that the integration of other rational expressions, such as $\frac{x}{x^2+5}$ and $\frac{2}{(2x-1)^4}$ is also required (see above paragraphs).
Analytical solution of simple first order differential equations with separable variables.	General and particular solutions will be required.
Numerical integration of functions.	Application of the trapezium rule to functions covered in C3 and C4. Use of increasing number of trapezia to improve accuracy and estimate error will be required. Questions will not require more than three iterations. Simpson's Rule is *not* required.

6 Vectors

What students need to learn:

Vectors in two and three dimensions.	
Magnitude of a vector.	Students should be able to find a unit vector in the direction of $\mathbf{a}$, and be familiar with $\lvert a \rvert$.
Algebraic operations of vector addition and multiplication by scalars, and their geometrical interpretations.	

Position vectors.	$\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{AB} = \mathbf{b} - \mathbf{a}$.
The distance between two points.	The distance d between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by $d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$.
Vector equations of lines.	To include the forms $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ and $\mathbf{r} = \mathbf{c} + t(\mathbf{d} - \mathbf{c})$. Intersection, or otherwise, of two lines.
The scalar product. Its use for calculating the angle between two lines.	Students should know that for $\overrightarrow{OA} = \mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\overrightarrow{OB} = \mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then $\mathbf{a} . \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ and $\cos \angle AOB = \dfrac{\mathbf{a} . \mathbf{b}}{\lvert\mathbf{a}\rvert \, \lvert\mathbf{b}\rvert}$. Students should know that if $\mathbf{a} . \mathbf{b} = 0$, and $\mathbf{a}$ and $\mathbf{b}$ are non-zero vectors, then $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.

