

S2.1 Unit description

The Binomial and Poisson distributions; continuous random variables; continuous distributions; samples; hypothesis tests.

S2.2 Assessment information

Prerequisites

A knowledge of the specification for S1 and its prerequisites and associated formulae, together with a knowledge of differentiation and integration of polynomials, binomial coefficients in connection with the binomial distribution and the evaluation of the exponential function is assumed and may be tested.

Examination

The examination will consist of one 1½ hour paper. The paper will contain about seven questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π , x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , $x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Students are expected to know any other formulae which might be required by the specification and which are not included in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Students will be expected to know and be able to recall and use the following formulae:

For the continuous random variable X having probability density function $f(x)$,

$$P(a < X \leq b) = \int_a^b f(x) \, dx.$$

$$f(x) = \frac{dF(x)}{dx}.$$

1 The Binomial and Poisson distributions**What students need to learn:**

The binomial and Poisson distributions.

Students will be expected to use these distributions to model a real-world situation and to comment critically on their appropriateness. Cumulative probabilities by calculation or by reference to tables.

Students will be expected to use the additive property of the Poisson distribution — eg if the number of events per minute $\sim \text{Po}(\lambda)$ then the number of events per 5 minutes $\sim \text{Po}(5\lambda)$.

The mean and variance of the binomial and Poisson distributions.

No derivations will be required.

The use of the Poisson distribution as an approximation to the binomial distribution.

2 Continuous random variables

What students need to learn:

The concept of a continuous random variable.

The probability density function and the cumulative distribution function for a continuous random variable.

Use of the probability density function $f(x)$, where

$$P(a < X \leq b) = \int_a^b f(x) \, dx.$$

Use of the cumulative distribution function

$$F(x_0) = P(X \leq x_0) = \int_{-\infty}^{x_0} f(x) \, dx.$$

The formulae used in defining $f(x)$ will be restricted to simple polynomials which may be expressed piecewise.

Relationship between density and distribution functions.

$$f(x) = \frac{dF(x)}{dx}$$

Mean and variance of continuous random variables.

Mode, median and quartiles of continuous random variables.

3 Continuous distributions

What students need to learn:

The continuous uniform (rectangular) distribution.

Including the derivation of the mean, variance and cumulative distribution function.

Use of the Normal distribution as an approximation to the binomial distribution and the Poisson distribution, with the application of the continuity correction.

4 Hypothesis tests

What students need to learn:

Population, census and sample. Sampling unit, sampling frame.

Students will be expected to know the advantages and disadvantages associated with a census and a sample survey.

Concepts of a statistic and its sampling distribution.

Concept and interpretation of a hypothesis test. Null and alternative hypotheses.

Use of hypothesis tests for refinement of mathematical models.

Critical region.

Use of a statistic as a test statistic.

One-tailed and two-tailed tests.

Hypothesis tests for the parameter p of a binomial distribution and for the mean of a Poisson distribution.

Students are expected to know how to use tables to carry out these tests. Questions may also be set not involving tabular values. Tests on sample proportion involving the normal approximation will not be set.