IYGB – MMS PAPER H – QUESTION 1

a) USING A CALCULATOR IN STATISTICS MODE, THE P.M.C.C BASED ON 7 OBS

$$r = 0.913$$

b) AGAIN FROM A STATISTICAL CALCULATOR

$a = -147.6$
$b = 50.1$

$\Rightarrow \quad y = 50.1x - 147.6$
$\Rightarrow \quad y = 50.1 \times 8 - 147.6$
$\Rightarrow \quad y \simeq 253 \leftarrow$ "k"

AS 8 LIES IN THE RANGE $5 \leq x \leq 17$, WHICH WAS USED TO PRODUCE THE REGRESSION LINE AND r INDICATES STRONG CORRELATION, THE ESTIMATE SHOULD BE VERY RELIABLE

IYGB - MMS PAPER H - QUESTION 2

a)

X = NUMBER OF DEFECTIVE HOOKS

$X \sim B(60, 0.05)$

$H_0 : p = 0.05$

$H_1 : p < 0.05$, WHERE p REPRESENTS THE PROPORTION OF ALL THE DEFECTIVE HOOKS FROM THE PRODUCTION

TESTING AT 10% LEVEL OF SIGNIFICANCE ON THE BASIS THAT $x = 1$

$P(X \leq 1)$ = CALCULATOR OR $P(X=0) + P(X=1)$

$= 0.1916$

$= 19.16\%$

$> 10\%$

THERE IS NO SIGNIFICANT EVIDENCE TO SUPPORT THE CLAIM MADE BY THE MAKERS OF THE NEW MACHINERY

NO SUFFICIENT EVIDENCE TO REJECT H_0

b)

LOOKING AT THE "BOTTOM TAIL" AT 5%

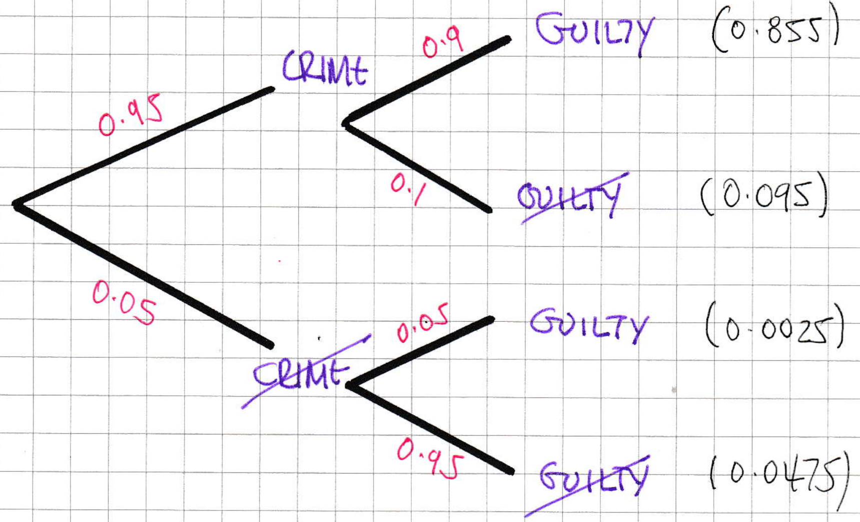
$P(X \leq 0) = 0.0461 = 4.61\% < 5\%$

$P(X \leq 1) = 0.1916 = 19.16\% > 5\%$

$\therefore$ CRITICAL REGION $= \{0\}$

IYGB - MMS PAPER H - QUESTION 3

a) DRAWING A VENN DIAGRAM



FROM THE TREE DIAGRAM, $P(\text{GUILTY}) = 0.855 + 0.0025$

$= 0.8575 \quad \left(\frac{343}{400}\right)$

b) USING THE "CONDITIONAL" FORMULA

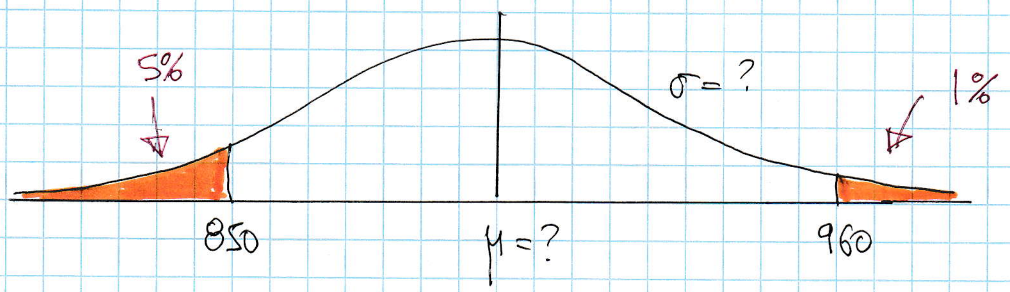
$$P(\text{CRIME} \mid \text{GUILTY}) = \frac{P(\text{CRIME} \cap \text{GUILTY})}{P(\text{GUILTY})}$$

$$= \frac{0.855}{0.8575}$$

$$= 0.9971 \quad \left(\frac{342}{343}\right)$$

IYGB - MMS PAPER H - QUESTION 4

a) PUTTING THE INFORMATION IN A DIAGRAM



X = weekly mileages
$X \sim N(\mu, \sigma^2)$

- $P(X<850) = 5\%$

$\Rightarrow P(X>850) = 95\%$

$\Rightarrow P\left(Z > \frac{850-\mu}{\sigma}\right) = 0.95$

↓ INVERSION

$\Rightarrow \frac{850-\mu}{\sigma} = -\Phi^{-1}(0.95)$

$\Rightarrow \frac{850-\mu}{\sigma} = -1.6449$

$\Rightarrow 850-\mu = -1.6449\sigma$

$\Rightarrow \boxed{850 + 1.6449\sigma = \mu}$

- $P(X>960) = 1\%$

$\Rightarrow P(X<960) = 99\%$

$\Rightarrow P\left(Z < \frac{960-\mu}{\sigma}\right) = 0.99$

↓ INVERSION

$\Rightarrow \frac{960-\mu}{\sigma} = +\Phi^{-1}(0.99)$

$\Rightarrow \frac{960-\mu}{\sigma} = 2.3263$

$\Rightarrow 960-\mu = 2.3263\sigma$

$\Rightarrow \boxed{960 - 2.3263\sigma = \mu}$

SOLVING SIMULTANEOUSLY

$\Rightarrow 850 + 1.6449\sigma = 960 - 2.3263\sigma$

$\Rightarrow 3.9712\sigma = 110$

$\Rightarrow \sigma = 27.69943594\ldots$

$\Rightarrow \sigma \approx 28$

& $\mu = 895.5628022\ldots$

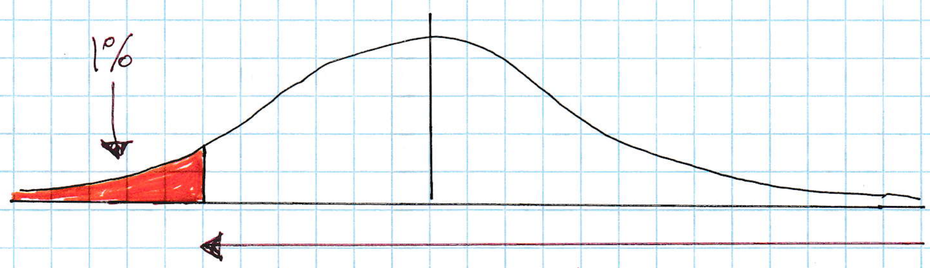
$\mu \simeq 896$

IYGB - MMS PAPER H - QUESTION 4

b) SETTING UP HYPOTHESES

- H_0 : $\mu = 896$
- H_1 : $\mu < 896$, WHERE μ IS THE MEAN OF ALL WEEKLY MILEAGES (POPULATION MEAN)

$n = 4$, $\bar{x}_4 = 863$, $\sigma = 28$, 1% SIGNIFICANCE



CRITICAL VALUE $\Phi^{-1}(0.99) = -2.3263$

- Z-STATISTIC $= \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{863 - 896}{28/\sqrt{4}} = -2.357142\ldots$

- AS $-2.357142\ldots < -2.3263$ THERE IS SIGNIFICANT EVIDENCE, AT 1% LEVEL, TO SUPPORT THE SALES REP'S BELIEF

- THERE IS SUFFICIENT EVIDENCE TO REJECT H_0

IYGB - MMS PAPER H - QUESTION 5

a) STRATIFIED SAMPLING GUARANTEES REPRESENTATION OF ALL GROUPS WHILE SIMPLE RANDOM SAMPLING DOES NOT

WITH STRATIFIED SAMPLING YOU CAN FURTHER ANALYSE WITHIN A CERTAIN GROUP WHILE WITH SIMPLE RANDOM SAMPLING YOU CANNOT

b) STRATIFIED SAMPLING IS NOT BIASED, WHILE QUOTA SAMPLING IS

YOU CAN ESTIMATE SAMPLING ERRORS IN STRATIFIED SAMPLING WHILE YOU CANNOT WITH QUOTA SAMPLING

IYGB - MMS PAPER H - QUESTION 6

a) FORMING A TABLE OF MIDPOINTS

DISTANCE	MIDPOINTS	FREQUENCY
3-5	4	12 (12)
6-7	6.5	14 (26)
8	8	19 (45)
9-11	10	13
12-17	14.5	6

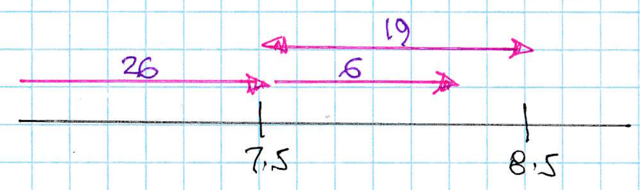
OBTAIN SUMMARY STATISTICS

$\sum x = 508$ • $\sum x^2 = 4561$ • $n = 64$

• $\bar{x} = \frac{\sum x}{n} = \frac{508}{64} = 7.9375 = 7.94$

• $\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = 2.87432\ldots \simeq 2.87$

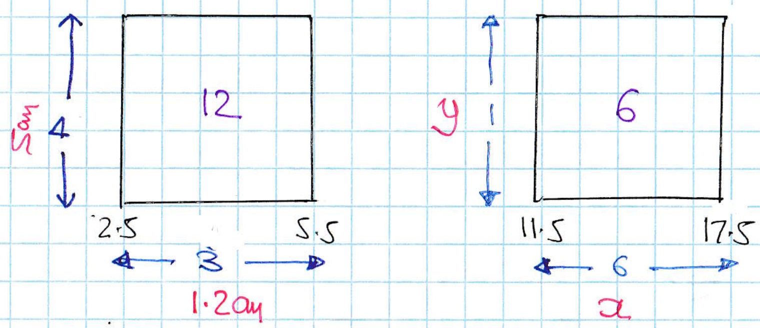
b) MEDIAN (Q_2) IS $\frac{1}{2} \times 64 = 32^{ND}$ OBS WHICH LIES IN "8"



$Q_2 \simeq 7.5 + \frac{6}{19} \times 1$

$Q_2 \simeq 7.82$

c) LOOKING AT THE DIAGRAM BELOW



• $\frac{3}{1.2} = \frac{6}{x}$

$3x = 7.2$

$x = 2.4$

• $\frac{5}{4} = \frac{y}{1}$

$4y = 5$

$y = 1.25$

∴ BASE 2.4 cm & HEIGHT 1.25 cm

d) USING THE QUARTILES FOR OUTLIERS

• LOWER BOUND $= Q_1 - \frac{3}{2}(Q_3 - Q_1)$

$= 6.07 - 1.5 \times (9.19 - 6.07)$

$= 1.39$

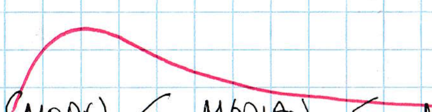
IYGB – MMS PAPER H – QUESTION 6

• UPPER BOUND $= Q_3 + 1.5(Q_3 - Q_1) = 9.19 + 1.5(9.19 - 6.07) = 13.87$

$1.39 < 2.5 \Rightarrow$ NO OUTLIERS AT THE BOTTOM END

$11.5 < 13.87 < 17.5 \Rightarrow$ ALMOST CERTAIN TO HAVE OUTLIERS AT THE TOP END

e)



(MODE) < MEDIAN < MEAN

7.82   7.94

POSITIVE SKEW $\Rightarrow$ NOT APPROPRIATE TO BE MODELLED BY A NORMAL DISTRIBUTION

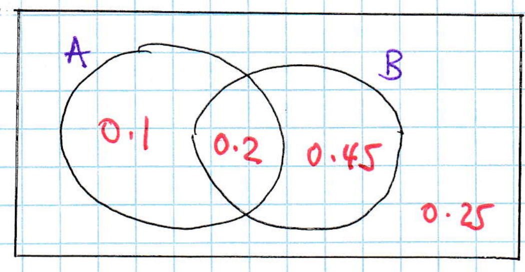
IYGB – MMS PAPER – QUESTION 7

a) $P(A) = 0.3 \quad \bullet \quad P(A \cap B') = 0.1 \quad \bullet \quad P(A \cup B') = 0.55$

$$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\Rightarrow P(A \cup B') = P(A) + P(B') - P(A \cap B')$$
$$\Rightarrow 0.55 = 0.3 + P(B') - 0.1$$
$$\Rightarrow P(B') = 0.35$$

$$\therefore P(B) = 1 - P(B') = 1 - 0.35 = 0.65$$

b)



c) I) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.2}{0.65} = \dfrac{20}{65} = \dfrac{4}{13}$

II) $P(B'|A') = \dfrac{P(B' \cap A')}{P(A')} = \dfrac{0.25}{0.70} = \dfrac{25}{70} = \dfrac{5}{14}$

IYGB – MMS PAPER H – QUESTION 8

a) START WITH A TREE DIAGRAM OR SOME KIND OF SORTER

X = NO OF BLUE DISCS PICKED

$P(X=1) = P(\text{Red}-\text{Red}-\text{blue}) \times 3 \text{ WAYS} = \left(\frac{2}{5}\times\frac{1}{4}\times\frac{3}{3}\right)\times 3 = \frac{3}{10}$

$P(X=2) = P(\text{Red}-\text{Blue}-\text{Blue}) \times 3 \text{ WAYS} = \left(\frac{2}{5}\times\frac{3}{4}\times\frac{2}{3}\right)\times 3 = \frac{6}{10}$

$P(X=3) = P(\text{blue}-\text{blue}-\text{blue}) \times 1 \text{ WAY} = \frac{3}{5}\times\frac{2}{4}\times\frac{1}{3} = \dots = \frac{1}{10}$

b) ORGANISING OUTCOMES

3, 3, 3, 3 $\Longrightarrow$ $\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10} = 0.0001$

3, 3, 3, 2
3, 3, 2, 3
3, 2, 3, 3
2, 3, 3, 3
$\Longrightarrow$ $\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}\times\frac{6}{10}\times 4 \text{ WAYS} = 0.0024$

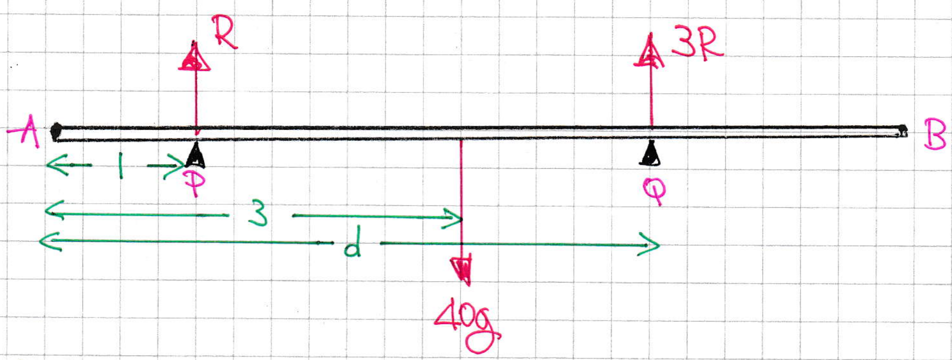
3 3 2 2
3 2 3 2
3 2 2 3
2 3 3 2
2 3 2 3
2 2 3 3
$\Longrightarrow$ $\frac{1}{10}\times\frac{1}{10}\times\frac{6}{10}\times\frac{6}{10}\times 6 \text{ WAYS} = 0.0216$

3 3 3 1
3   1 3
3 1 3 3
1 3 3 3
$\Longrightarrow$ $\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}\times\frac{3}{10}\times 4 \text{ WAYS} = 0.0012$

ADDING GIVES 0.0253

IYGB - MMS PAPER H - QUESTION 9

DRAWING A DIAGRAM



RESOLVING VERTICALLY

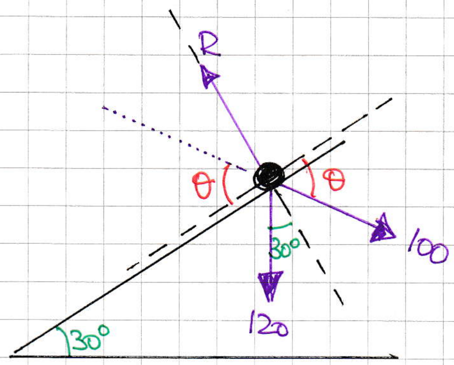
$$R + 3R = 40g$$
$$4R = 40g$$
$$R = 10g$$

TAKING MOMENTS ABOUT A

$$\overset{\curvearrowright}{A}: \ (R \times 1) + (3R \times d) = 40g \times 3$$
$$10g + 30g \times d = 120g$$
$$10 + 30d = 120$$
$$30d = 110$$
$$d = \frac{11}{3} = 3\tfrac{2}{3}\text{ m}$$

IYGB - MMS PAPER H - QUESTION 10

START WITH A DETAILED DIAGRAM — MODEL THE PUSHING FORCE AS A "PULLING" FORCE



RESOLVING PARALLEL & PERPENDICULAR TO THE PLANE

$$(\parallel): \quad 120\sin 30 = 100\cos\theta \quad \text{—(I)}$$

$$(\perp): \quad R = 120\cos 30 + 100\sin\theta \quad \text{—(II)}$$

FROM THE FIRST EQUATION (I)

$$\Rightarrow \cos\theta = \frac{120\sin 30}{100}$$

$$\Rightarrow \cos\theta = 0.6$$

NOW WE HAVE FROM (II)

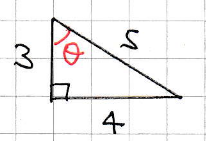
$$\Rightarrow R = 120\cos 30 + 100\sin\theta$$

$$\Rightarrow R = 120 \times \frac{\sqrt{3}}{2} + 100 \times \frac{4}{5}$$

$$\Rightarrow R = 60\sqrt{3} + 80$$

$$\Rightarrow R \simeq 184\,\text{N}$$

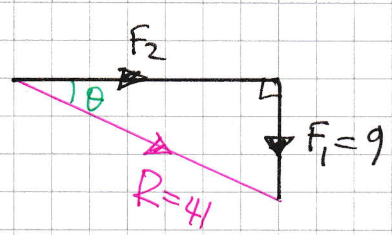
$\cos\theta = 0.6 = \frac{3}{5}$



$\sin\theta = \frac{4}{5}$

IYGB – MMS PAPER H – QUESTION 11

a) LOOKING AT THE DIAGRAM



$\Rightarrow |F_2|^2 + |F_1|^2 = R^2$

$\Rightarrow |F_2|^2 + 9^2 = 41^2$

$\Rightarrow |F_2|^2 + 81 = 1681$

$\Rightarrow |F_2|^2 = 1600$

$\Rightarrow |F_2| = 40\text{ N}$

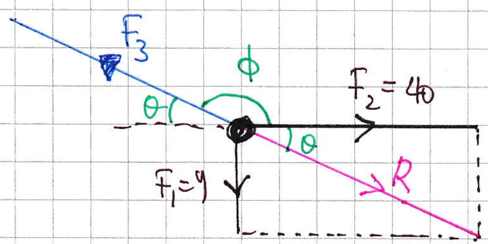
b) BY SIMPLE TRIGONOMETRY

$\tan\theta = \frac{9}{40}$ OR $\sin\theta = \frac{9}{41}$ OR $\cos\theta = \frac{40}{41}$

$\theta = 12.7°$ 3 s.f.

c) BY INSPECTION, $|F_3| = 41$, SO THE TRIANGLE CLOSES

d) LOOKING AT A NEW DIAGRAM



REQUIRED ANGLE IS ϕ

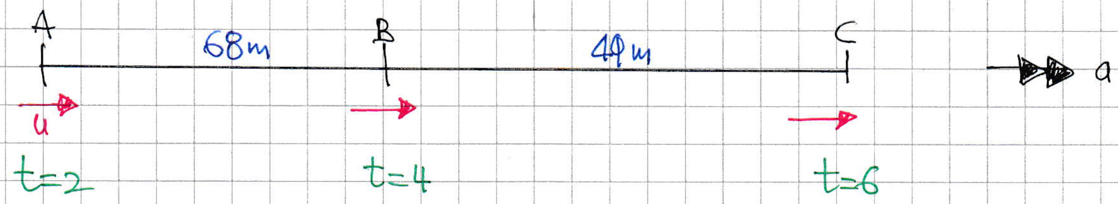
$\phi = 180 - \theta$

$\phi = 180 - 12.7$

$\phi \simeq 167°$

IYGB — MMS PAPER H — QUESTION 12

PUTTING THE INFORMATION INTO A DIAGRAM



LOOKING AT A TO B

$u =$	?
$a =$	?
$s =$	68 m
$t =$	4 s
$v =$	

$$s = ut + \frac{1}{2}at^2$$

$$68 = 4u + \frac{1}{2}a \times 4^2$$

$$68 = 4u + 8a$$

$$17 = u + 2a$$

LOOKING AT A TO C

$u =$	?
$a =$	?
$s =$	117 m
$t =$	6
$v =$	?

$$s = ut + \frac{1}{2}at^2$$

$$117 = 6u + \frac{1}{2}a \times 6^2$$

$$117 = 6u + 18a$$

$$39 = 2u + 6a$$

SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{l} u + 2a = 17 \\ 2u + 6a = 39 \end{array}\right\} \Longrightarrow \quad u = 17 - 2a$$

$$\Downarrow$$

$$2(17 - 2a) + 6a = 39$$

$$34 - 4a + 6a = 39$$

$$2a = 5$$

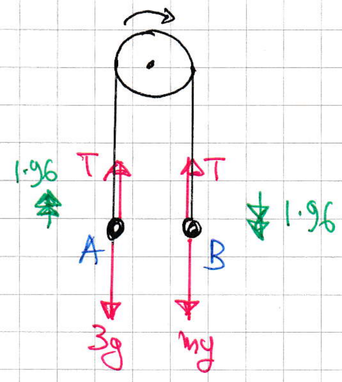
$$a = 2.5 \text{ ms}^{-2}$$

& $u = 17 - 2 \times 2.5$

$$u = 12 \text{ ms}^{-1}$$

IYGB - MMS PAPER H - QUESTION 13

a) STARTING WITH A DIAGRAM & CONSIDERING THE EQUATION OF MOTION FOR EACH PARTICLE SEPARATELY



(A): $T - 3g = 3a$ ["F = ma"]

$T - 3g = 3 \times 1.96$

$T = 35.28\,N$

b) LOOKING AT THE MOTION OF B

(B): $mg - T = ma$

$mg - ma = T$

$m(g-a) = T$

$m(9.8 - 1.96) = 35.28$

$m = 4.5\,kg$

c) BY KINEMATICS

$u = 0$	
$a = 1.96$	
$s = 1.28$	
t	
$v = ?$	

$v^2 = \cancel{u^2} + 2as$

$v^2 = 2 \times 1.96 \times 1.28$

$v^2 = 5.0176$

$\therefore |v| = 2.24\,ms^{-1}$

d) ONCE B HITS THE FLOOR - "A" IS MOVING UNDER GRAVITY

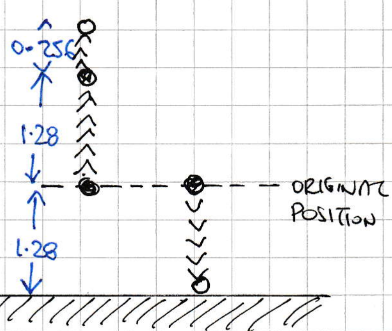
$u = 2.24\,ms^{-1}$	
$a = -9.8\,ms^{-2}$	
$s = ?$	
$t =$	
$v = 0$	

$v^2 = u^2 + 2as$

$0 = 2.24^2 + 2(-9.8)s$

$19.6s = 5.0176$

$s = 0.256$



$\therefore$ MAX HEIGHT $= 1.28 + 1.28 + 0.256 = 2.816\,m$

IYGB MMS PAPER H - QUESTION 14

a) DRAWING A SPEED TIME GRAPH

TOTAL DISTANCE = 1362m

b) "DISTANCE = AREA"

$$(17 \times 30) + \frac{1}{2}(30+12) \times T + (53-T) \times 12 = 1362$$

$$510 + 21T + 636 - 12T = 1362$$

$$9T = 216$$

$$T = 24$$

FINALLY GRADIENT = DECELERATION

$$a = \frac{\Delta v}{\Delta t} = \frac{12-30}{T} = \frac{-18}{24} = -0.75 \text{ ms}^{-2}$$

i.e. DECELERATION 0.75 ms^{-2}

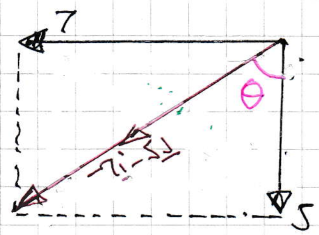
IYGB - MMS PAPER H - QUESTION 15

a) SPEED = MAGNITUDE OF VELOCITY VECTOR

$$\Rightarrow \text{SPEED} = |\underline{v}| = |-7\underline{i} - 5\underline{j}| = \sqrt{49+25} = \sqrt{74}$$

$$\simeq 8.60 \text{ km h}^{-1}$$

b) DRAWING A DIAGRAM



$$\tan\theta = \frac{7}{5}$$

$$\theta = 54.46...^\circ$$

$$\therefore \text{BEARING} = 180 + 54.46...$$

$$\simeq 234^\circ$$

c) OBTAIN A GENERAL EXPRESSION FOR THE POSITION VECTOR OF THE SHIP AS IT WILL BE NEEDED IN PART (d) ALSO

$$\underline{r} = \underline{r}_0 + \underline{v}t$$

$$\underline{r} = (40\underline{i} + 28\underline{j}) + (-7\underline{i} - 5\underline{j})t$$

$$\underline{r} = (40-7t)\underline{i} + (28-5t)\underline{j}$$

$$\underline{r}_4 = (40-7\times4)\underline{i} + (28-5\times4)\underline{j}$$

$$\underline{r}_4 = 12\underline{i} + 8\underline{j}$$

DISTANCE BETWEEN $(-12\underline{i} + \underline{j})$ & $(12\underline{i} + 8\underline{j})$

$$d = \sqrt{(-12-12)^2 + (1-8)^2} = \sqrt{576+49} = 25 \text{ km}$$

d) EAST OF THE LIGHT-HOUSE $\Rightarrow$ SAME $\underline{j}$ & $\underline{i}$ GREATER THAN 12

$$\underline{r} = (40-7t)\underline{i} + (28-5t)\underline{j}$$

$$\therefore 28 - 5t = 1$$

$$5t = 27$$

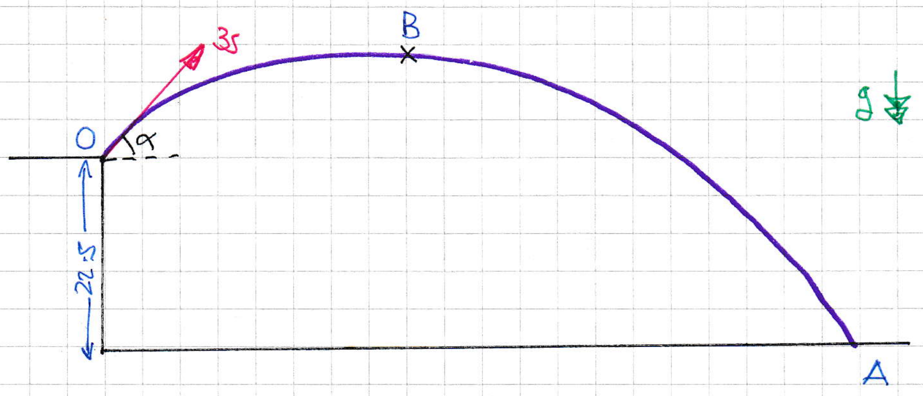
$$t = 5.4$$

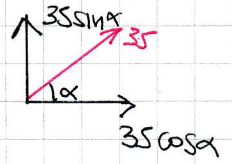
NOW $0.4 \times 60 = 24$

$\therefore$ 17:24

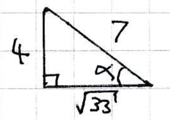
IYGB - MMS PAPER H - QUESTION 16

a)





$\sin\alpha = \frac{4}{7}$



$\cos\alpha = \frac{\sqrt{33}}{7}$

LOOKING AT THE VERTICAL MOTION (O TO B)

$u = 35\sin\alpha$	
$a = -9.8$	
$s = ?$	
$t =$	
$v = 0$	

$v^2 = u^2 + 2as$

$0 = (35\sin\alpha)^2 + 2(-9.8)s$

$0 = \left(35 \times \frac{4}{7}\right)^2 - 19.6s$

$19.6s = 400$

$s = \frac{1000}{49} \simeq 20.408...$

$\therefore$ MAX HEIGHT $= 22.5 + 20.408... = 42.91...$ m

b) LOOKING AT THE VERTICAL MOTION (O TO A)

$u = 35\sin\alpha$	
$a = -9.8$	
$s = -22.5$	
$t = ?$	
$v =$	

$s = ut + \frac{1}{2}at^2$

$-22.5 = (35\sin\alpha)t + \frac{1}{2}(-9.8)t^2$

$-22.5 = 20t - 4.9t^2$

$4.9t^2 - 20t - 22.5 = 0$

$49t^2 - 200t - 225 = 0$

FACTORIZE USING "GIVEN FACT" THAT $t = 5$

$(t-5)(49t+45) = 0$

$t = \begin{cases} 5 \\ \cancel{-\frac{45}{49}} \end{cases}$

IYGB – MMS PAPER H – QUESTION 16

c) LOOKING AT THE VERTICAL MOTION FROM O TO A

u	$= 35\sin\alpha$
a	$= -9.8$
s	$= -22.5$
t	$= 5$
v	

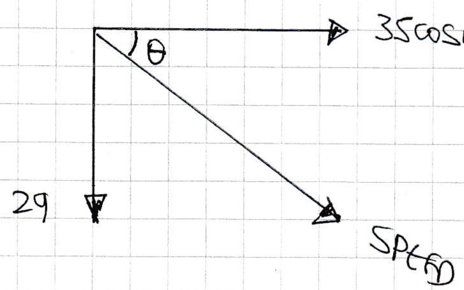
$V = u + at$

$V = 35\sin\alpha + (-9.8)\times 5$

$V = 35\times\frac{4}{7} - 9.8\times 5$

$V = -29$

LOOKING AT "SPEEDS" AT A



- $\text{SPEED} = \sqrt{29^2 + (5\sqrt{33})^2} = \sqrt{1666} \simeq 40.82\ \text{ms}^{-1}$

- $\tan\theta = \frac{29}{5\sqrt{33}}$

  $\theta = 45.3°$ TO THE HORIZONTAL