IYGB - MMS PAPER K - QUESTION 1

a) FORMING A TABLE OF MIDPOINTS

WEIGHT	FREQUENCY	MIDPOINTS
$1 \leq w < 3$	15 (15)	2
$3 \leq w < 5$	31 (46)	4
$5 \leq w < 6$	45 (91)	5.5
$6 \leq w < 6.5$	37	6.25
$6.5 \leq w < 7$	21	6.75
$7 \leq w < 10$	15	8.5

OBTAIN SUMMARY STATISTICS

$\sum x = 902$ • $\sum x^2 = 5403.125$ • $n = 164$

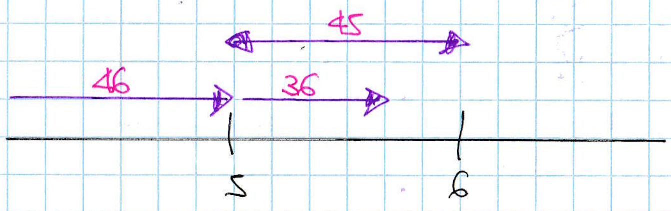
• MEAN $\bar{x} = \frac{\sum x}{n} = \frac{902}{164} = 5.5$

• $\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{5403.125}{164} - 5.5^2}$

$\sigma = 1.64191478... \approx 1.64$

b) LOOKING AT THE DIAGRAM BELOW

$Q_2 = \frac{1}{2} \times 164 = 82^{ND}$ OBS, WHICH LIES IN THE CLASS $5 \leq w \leq 6$



$Q_2 \simeq 5 + \frac{36}{45} \times 1 \simeq 5.80$

HENCE

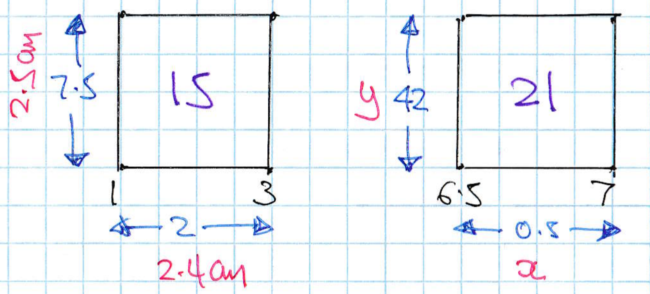
MEAN < MEDIAN < (MODE)

5.50   5.80

∴ NEGATIVE SKEW

IYGB - MMS PAPER K - QUESTION 1

c) LOOKING AT THE DIAGRAM BELOW



- $\frac{2}{2.4} = \frac{0.5}{x}$

  $2x = 1.2$

  $x = 0.6$

- $\frac{2.5}{7.5} = \frac{y}{42}$

  $7.5y = 105$

  $y = 14$

$\therefore$ BASE 0.6 cm & HEIGHT 14 cm

d) LOWER BOUND $= Q_1 - 1.5(Q_3 - Q_1) = 4.68 - (6.43 - 4.68) \times 1.5 = 2.055 > 1$

POSSIBLY OUTLIERS AT THE BOTTOM

UPPER BOUND $= Q_3 + 1.5(Q_3 - Q_1) = 6.43 + (6.43 - 4.68) \times 1.5 = 9.055 < 10$

POSSIBLY OUTLIERS AT THE TOP END

e) ALTHOUGH THE DATA IS CONTINUOUS THERE IS NEGATIVE SKEW, SO A NORMAL DISTRIBUTION MIGHT NOT BE APPROPRIATE, AS THE NORMAL DISTRIBUTION HAS ZERO SKEW

IYGB - MMS PAPER E - QUESTION 2

NUMBER OF SHOPLIFTING INCIDENTS	17	20	23	11	35	32	21
NUMBER OF SECURITY GUARDS EMPLOYED	6	6	5	7	4	3	5

a) FROM CALCULATOR IN STAT MODE

$$r = -0.932$$

b) SETTING HYPOTHESES

$H_0 : \rho = 0$

$H_1 : \rho \neq 0$

WHERE ρ REPRESENTS THE PMCC OF ALL PAIRINGS BETWEEN SHOPLIFTING INCIDENTS AND NUMBER OF SECURITY GUARDS (POPULATION)

THE CRITICAL VALUES FOR $n=7$, AT 1% (TWO TAILED) ARE ± 0.8745

AS $-0.932 < -0.8745$ THERE IS EVIDENCE OF (NEGATIVE) CORRELATION

SUFFICIENT EVIDENCE TO REJECT H_0

c) CORRELATION DOES NOT IMPLY CAUSE, SO THE STATEMENT COULD BE TRUE OR UNTRUE

IYGB - MMS PAPER K - QUESTION 3

WORK WITH OR WITHOUT A TREE DIAGRAM

- $P(X=0) = P(\text{no girl}) = P(B,B,B) = \frac{6}{10}\times\frac{5}{9}\times\frac{4}{8} = \frac{120}{720}$
- $P(X=3) = P(\text{all girls}) = P(G,G,G) = \frac{4}{10}\times\frac{3}{9}\times\frac{2}{8} = \frac{24}{720}$
- $P(X=1) = P(\text{1 girl \& 2 boys})$

$$= P(GBB) + P(B,G,B) + P(B,B,G)$$

$$= \left(\frac{4}{10}\times\frac{6}{9}\times\frac{5}{8}\right) + \left(\frac{6}{10}\times\frac{4}{9}\times\frac{5}{8}\right) + \left(\frac{6}{10}\times\frac{5}{9}\times\frac{4}{8}\right)$$

$$= \frac{120}{720}\times 3$$

$$= \frac{360}{720}$$

- $P(X=2) = 1 - P(X=0,1,3) = 1 - \left(\frac{120}{720}+\frac{24}{720}+\frac{360}{720}\right) = \frac{216}{720}$

HENCE WE OBTAIN

x	0	1	2	3
$P(X=x)$	$\frac{120}{720}$	$\frac{360}{720}$	$\frac{216}{720}$	$\frac{24}{720}$
	$\frac{5}{30}$	$\frac{15}{30}$	$\frac{9}{30}$	$\frac{1}{30}$
	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

$\frac{\div 24}{\div 24}$

OR IN ITS SIMPLEST FORM

IYGB – MMS PAPER K – QUESTION 4

RANDOM SAMPLING

SELECTING MEMBERS OF A POPULATION FOR A SURVEY WHERE EACH MEMBER HAS EQUAL CHANCE OF BEING PICKED

EXAMPLE

WE NEED A RANDOM SAMPLE OF 50 STUDENTS FROM A COLLEGE WITH A POPULATION OF 838

"USE RANDOM BUTTON"

- ASSIGN EACH STUDENT A NUMBER, 001 to 838
- GENERATE RANDOM NUMBERS ON A CALCULATOR E.G

0.137, 0.905, 0.461, 0.552, 0.011, 0.137, 0.708, ...

0.137	0.905	0.461	0.552	0.011	0.137	0.708
↓	↓	↓	↓	↓	↓	↓
PICK 137TH	IGNORE 905 > 838	PICK 461TH	PICK 552ND	PICK 11TH	IGNORE REPEAT	PICK 708TH

- CONTINUE UNTIL 50 STUDENTS ARE SELECTED iYGB - MMS PAPER K - QUESTION 5

X = NUMBER OF PEOPLE WHO FAVOUR SUNDAY

$X \sim B(15, 0.25)$

$H_0: \ p = 0.25$

$H_1: \ p \neq 0.25$, WHERE p IS THE PREFERENCE FOR SUNDAY FOR ALL PEOPLE

TESTING AT 5% (TWO TAILED) ON THE BASIS THAT $x = 7$

$$P(X \geq 7) = 1 - P(X \leq 6)$$

$$= 1 - 0.943379\ldots$$

$$= 0.0566203\ldots$$

$$= 5.66\%$$

$$> 2.5\% \quad \leftarrow \text{TWO TAILED AT } 5\%$$

THERE IS NO SIGNIFICANT EVIDENCE TO SUPPORT THE VALIDITY OF THE STATEMENT

INSUFFICIENT EVIDENCE TO REJECT H_0 //

IYGB - MMS PAPER K - QUESTION 6

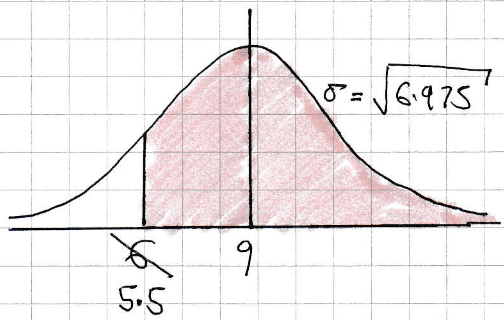
X = NUMBER OF WORKERS WHICH LIVE WITHIN 30 MILES

$X \sim B(40, 0.225)$

- MEAN $= E(X) = np = 40 \times 0.225 = 9$
- VARIANCE $= Var(X) = np(1-p) = 9 \times (1 - 0.225) = 6.975$

APPROXIMATE BY $Y \sim N(9, 6.975)$

$$P(X > 5)$$
$$= P(X \geqslant 6)$$
$$= P(Y > 5.5)$$
$$= P\left(Z > \frac{5.5 - 9}{\sqrt{6.975}}\right)$$
$$= \Phi(-1.32524\ldots)$$
$$= 0.90745\ldots \quad \text{(CALCULATOR FIGURE)}$$
$$\approx 0.907 \quad \text{(3 sf)}$$

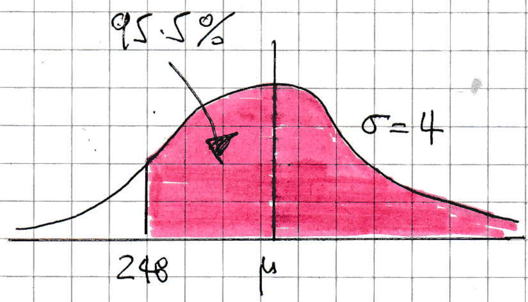


IYGB – MMS PAPER K – QUESTION 7

W = weights of packs of cheese

$W \sim N(\mu, 4^2)$

a) LOOKING AT A DIAGRAM



$\Rightarrow P(W > 248) = 95.5\%$

$\Rightarrow P\left(Z > \frac{248-\mu}{4}\right) = 0.9550$

INVERSION

$\frac{248-\mu}{4} = -\Phi^{-1}(0.9550)$

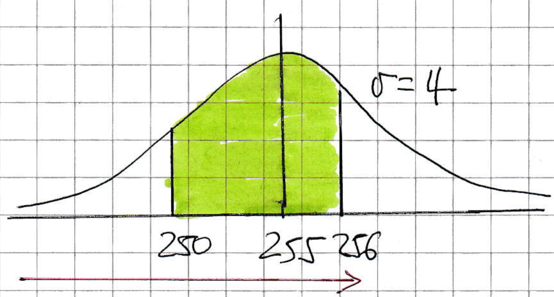
$\frac{248-\mu}{4} = -1.695$

$248 - \mu = -6.78$

$\mu = 254.78$

$\mu = 255$

b) USING THE MEAN FROM PART (a)



$P(250 < W < 256)$

$= P(W < 256) - P(W < 250)$

$= P(W < 256) - [1 - P(W > 250)]$

$= P(W < 256) + P(W > 250) - 1$

$= P\left(Z < \frac{256-255}{4}\right) + P\left(Z > \frac{250-255}{4}\right) - 1$

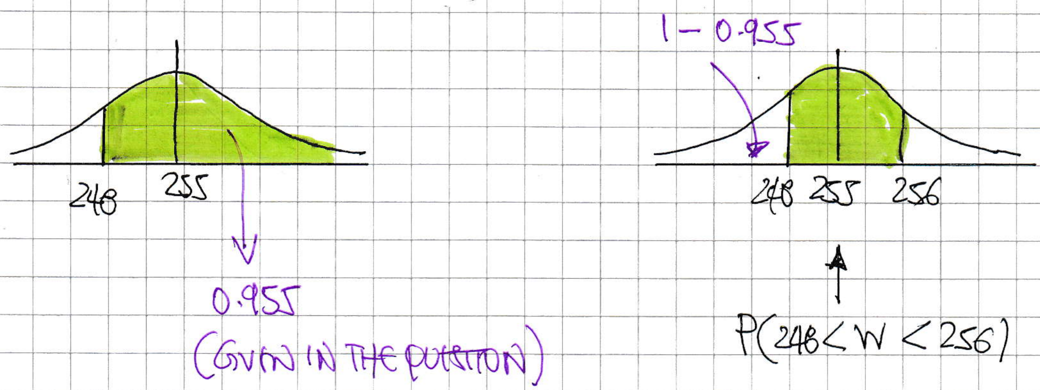
$= \Phi(0.25) + \Phi(-1.25) - 1$

$= 0.5987 + 0.8944 - 1 = 0.4931$

IYGB - MMS PAPER K - QUESTION 7

c) LOOKING AT THIS CONDITIONAL PROBABILITY WITHOUT THE STANDARD FORMULA

$$P(W<256 \mid W>248) = ?$$



$P(248<W<256)$

$= P(W<256) - P(W<248)$

$= 0.5987 - 0.0450$

FOUND IN (b)

$= 0.5537$

$\therefore$ REQUIRED PROBABILITY $= \dfrac{0.5537}{0.955} = \underline{0.5798}$

d) LET Y = NUMBER OF PACKS WHICH WEIGH OVER 248 GRAMS

$\Longrightarrow Y \sim B(10, 0.955)$

$\Longrightarrow P(Y=6) = \binom{10}{6}(0.955)^6(0.045)^4 = \underline{0.000653}$

IYGB – MMS PAPER K – QUESTION 8

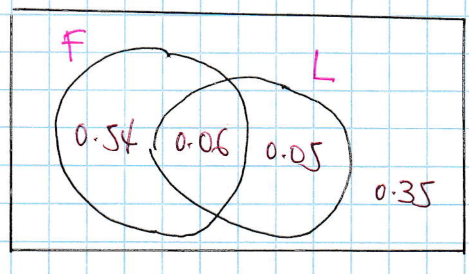
P(Female) = 0.6 , P(Left handed) = 0.11 , P(Left handed | Female) = 0.1

a) USING $P(L|F) = \frac{P(L \cap F)}{P(F)}$

$$0.1 = \frac{P(L \cap F)}{0.6}$$

$$P(L \cap F) = 0.06$$

THE VENN DIAGRAM CAN NOW BE COMPLETED



b) FROM PART (a) OR THE VENN DIAGRAM

$$P(F \cap L') = 0.54$$

c) $P(F|L) = \frac{P(F \cap L)}{P(L)} = \frac{0.06}{0.11} = \frac{6}{11} \simeq 0.545$

d) $P(L|F') = \frac{P(L \cap F')}{P(F')} = \frac{0.05}{0.40} = \frac{1}{8} = 0.125$

IYGB - MMS PAPER K - QUESTION 9

X = NO OF DEFECTIVE HOOKS IN A BOX

$X \sim B(50, 0.05)$

$$P(X>5) = P(X \geq 6) = 1 - P(X \leq 5) = 1 - 0.96222\ldots$$

$$= 0.037776\ldots$$

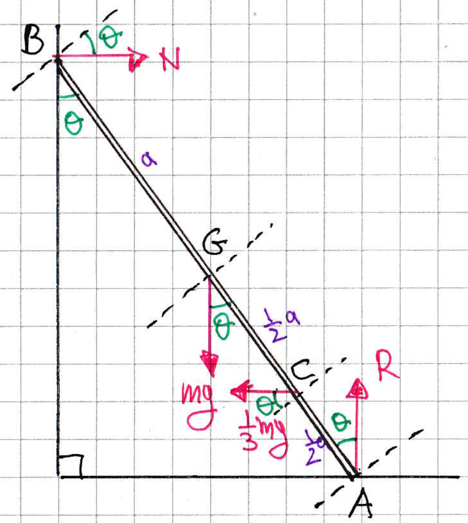
Y = A BOX WITH MORE THAN 5 DEFECTIVE HOOKS

$Y \sim B(10, 0.037776)$

$$P(Y=5) = \binom{10}{5}(0.037776)^5(0.96222)^5 = 0.000016$$

IYGB - MMS PAPER K - QUESTION 10

STARTING WITH A DETAILED DIAGRAM



$(\uparrow)$: $R = mg$

$(\rightarrow)$: $N = \frac{1}{3}mg$

$\overset{\curvearrowright}{A}$: $\left(\frac{1}{3}mg\cos\theta \times \frac{1}{2}a\right) + \left(mg\sin\theta \times a\right) = \left(N\cos\theta\right) \times 2a$

TIDYING UP THE MOMENT EQUATION

$\Rightarrow \frac{1}{6}mga\cos\theta + mga\sin\theta = 2Na\cos\theta$

$\Rightarrow \frac{1}{6}mg\cos\theta + mga\sin\theta = 2\left(\frac{1}{3}mg\right)a\cos\theta$

$\Rightarrow \frac{1}{6}mga\cos\theta + mga\sin\theta = \frac{2}{3}mga\cos\theta$

$\Rightarrow \frac{1}{6}\cos\theta + \sin\theta = \frac{2}{3}\cos\theta$

$\Rightarrow \cos\theta + 6\sin\theta = 4\cos\theta$

$\Rightarrow 6\sin\theta = 3\cos\theta$

$\Rightarrow 2\sin\theta = \cos\theta$

$\Rightarrow \frac{2\sin\theta}{\cos\theta} = \frac{\cos\theta}{\cos\theta}$

$\Rightarrow 2\tan\theta = 1$

$\Rightarrow \tan\theta = \frac{1}{2}$

AS REQUIRED

IYGB - MMS PAPER L - QUESTION 11

a) LOOKING AT THE DECELERATING PART

$u = 14$
$a = -0.5$
$s =$
$t = 2$
$v = ?$

$v = u + at \implies v = 14 + (-0.5)\times 2$

$v = 13$

NOW AREA IS 100

$$\left(\tfrac{1}{2}\times T\times 14\right) + (8-T)\times 14 + \tfrac{1}{2}(14+13)\times 2 = 100$$

$$7T + 112 - 14T + 27 = 100$$

$$39 = 7T$$

$$T = \frac{39}{7} = 5\frac{4}{7}$$

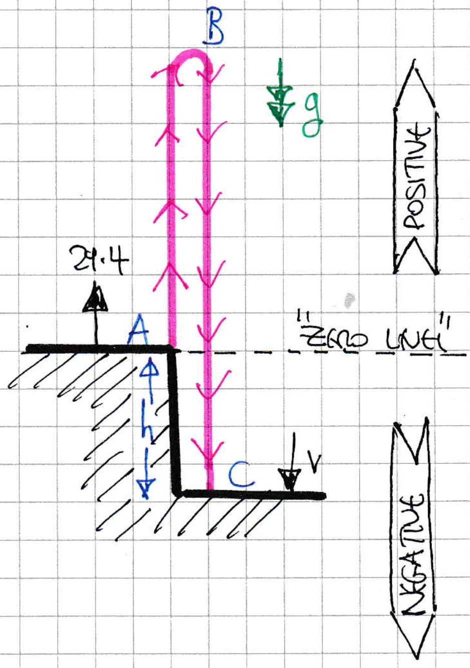
b) ACCELERATION = GRADIENT

$$a = \frac{\Delta v}{\Delta t} = \frac{14}{39/7} = \frac{98}{39} = 2.51\ \text{ms}^{-2}$$

NGB – MMS PAPER L – QUESTION 12

a) LOOKING AT THE DIAGRAM & CONSIDERING THE ENTIRE JOURNEY

$u = +29.4 \text{ ms}^{-1}$
$a = -9.8 \text{ ms}^{-2}$
$s = ?$
$t = 6 \text{ s}$
$v = -?$



USING "$s = ut + \frac{1}{2}at^2$"

$$\Rightarrow s = 29 \times 6 + \frac{1}{2}(-9.8) \times 6^2$$
$$\Rightarrow s = 174 - 176.4$$
$$\Rightarrow s = -2.4 \text{ m}$$

IE 2.4 BELOW THE LEVEL OF PROJECTION

$\therefore h = 2.4$

b) USING "$v = u + at$"

$$\Rightarrow v = 29 + (-9.8) \times 6$$
$$\Rightarrow v = 29 - 58.8$$
$$\Rightarrow v = -29.8 \text{ ms}^{-1}$$

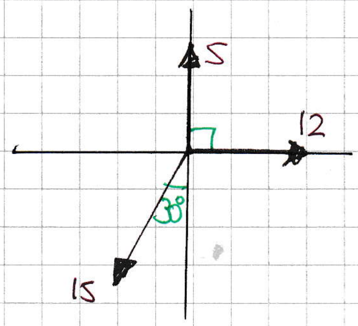
IE 29.8 ms^{-1} DOWN-WARDS $\therefore v = 29.8$

IYGB - MMS PAPER K - QUESTION 13

a) REDUCING THE SYSTEM OF 3 FORCES INTO 2 FORCES

- NET FORCE TO THE "RIGHT" ($\rightarrow$)

$$12 - 15\sin 30° = 4.5$$

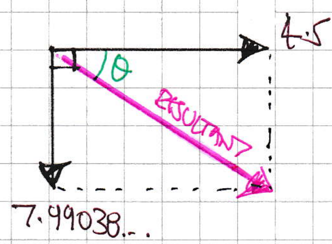


- NET FORCE "UPWARDS" ($\uparrow$)

$$5 - 15\cos 30 = 5 - \frac{15}{2}\sqrt{3}$$

$$\simeq -7.99038\ldots$$

RECOMBINING USING A NEW DIAGRAM



RESULTANT MAGNITUDE BY PYTHAGORAS

$$\text{RESULTANT} = \sqrt{4.5^2 + 7.99038^2}$$

$$\simeq 9.17039\ldots$$

$$\simeq 9.17\ \text{N}$$

b) BY SIMPLE TRIGONOMETRY LOOKING AT THE PREVIOUS DIAGRAM

$$\tan\theta = \frac{7.99038\ldots}{4.5} \qquad \therefore\ \theta \simeq 60.612°\ldots$$

$$\therefore\ \text{REQUIRED ANGLE} = 90 + \theta$$

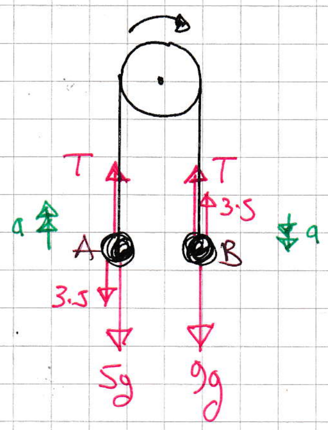
$$= 151° \quad (3\text{ sf})$$

c) BY INSPECTION

- MAX MAGNITUDE $= 12 + 5 + 15 = 32\ \text{N}$ — ALL ACTING IN THE SAME DIRECTION
- MIN MAGNITUDE $= 0\ \text{N}$ — WHEN THEY "CLOSE" A TRIANGLE

IYGB – MMS PAPER K – QUESTION 14

a) STARTING WITH A DIAGRAM, AND CONSIDERING EACH PARTICLE SEPARATELY



(A): $T - 3.5 - 5g = 5a$

(B) $9g - T - 3.5 = 9a$

ADDING THE EQUATIONS

$\Rightarrow 4g - 7 = 14a$

$\Rightarrow 14a = 32.2$

$\Rightarrow a = 2.3 \text{ ms}^{-2}$

FINDING THE TENSION

$T - 3.5 - 5g = 5a$

$T - 3.5 - 49 = 11.5$

$T = 64 \text{ N}$

b) HAVING FOUND THE ACCELERATION

u	=	0 ms^{-1}
a	=	2.3 ms^{-2}
s	=	1.75 m
t	=	
v	=	?

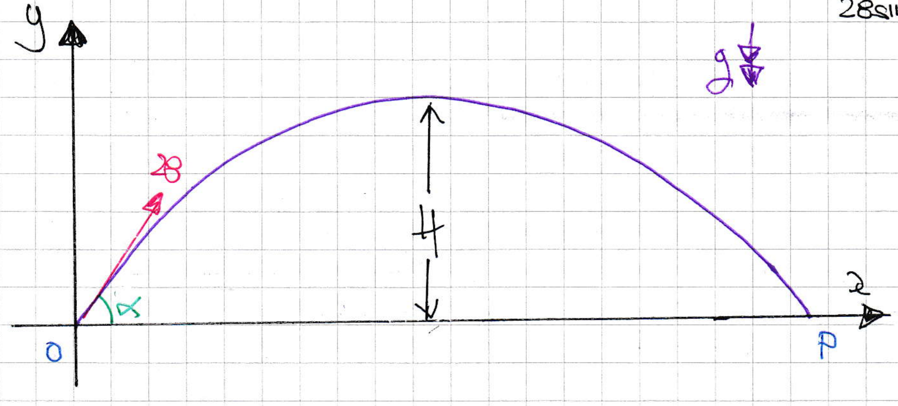
$v^2 = \cancel{u^2} + 2as$

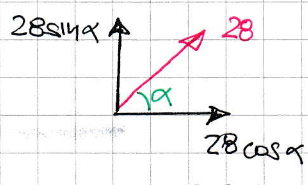
$v^2 = 2 \times 2.3 \times 1.75$

$v^2 = 8.05$

$v \approx 2.84 \text{ ms}^{-1}$

IYGB - MMS PAPER K - QUESTION 15





$\tan\alpha = \frac{3}{4}$

$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

a) LOOKING AT THE VERTICAL MOTION, USING "$v = u + at$"

$\Rightarrow 0 = 28\sin\alpha - gt$

$\Rightarrow 0 = 28 \times \frac{3}{5} - 9.8t$

$\Rightarrow 9.8t = 16.8$

$\Rightarrow t = \frac{12}{7}\,\text{s}$ // $\approx 1.71\,\text{s}$

b) BY SYMMETRY, THE FLIGHT TIME IS TWICE THE ANSWER OF PART a

I.E $\frac{24}{7} \approx 3.43\,\text{s}$

USING DISTANCE = SPEED × TIME (HORIZONTALLY)

$|OP| = (28\cos\alpha) \times \frac{24}{7}$

constant throughout

$|OP| = 28 \times 0.8 \times \frac{24}{7}$

$|OP| = 76.8\,\text{m}$ //

c) LOOKING AT THE VERTICAL MOTION AGAIN

$u = 28\sin\alpha = 16.8$
$a = -9.8$
$s =$
$t = 1$
$v =$

$s = ut + \frac{1}{2}at^2$

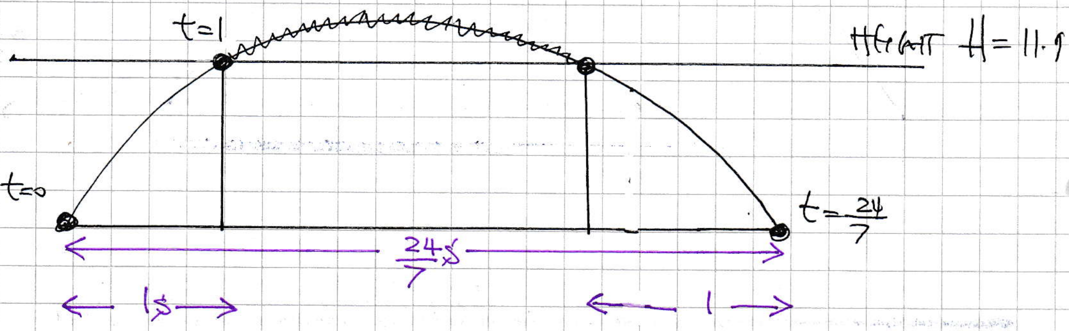
$s = 16.8 \times 1 + \frac{1}{2}(-9.8) \times 1^2$

$s = 11.9\,\text{m}$

i.e. $H = 11.9$ //

IYGB - MMS PAPER E - QUESTION 15

d) LOOKING AT A DIAGRAM



REQUIRED TIME $= \frac{24}{7} - (2 \times 1)$ ← SYMMETRY

$= \frac{10}{7}$ s

ALTERNATIVE

LOOKING AT THE VERTICAL MOTION

$$s = ut + \frac{1}{2}at^2$$

$$11.9 = 16.8t + \frac{1}{2}(-9.8)t^2$$

$$11.9 = 16.8t - 4.9t^2$$

$$4.9t^2 - 16.8t + 11.9 = 0$$

$$(t-1)(4.9t - 11.9) = 0$$

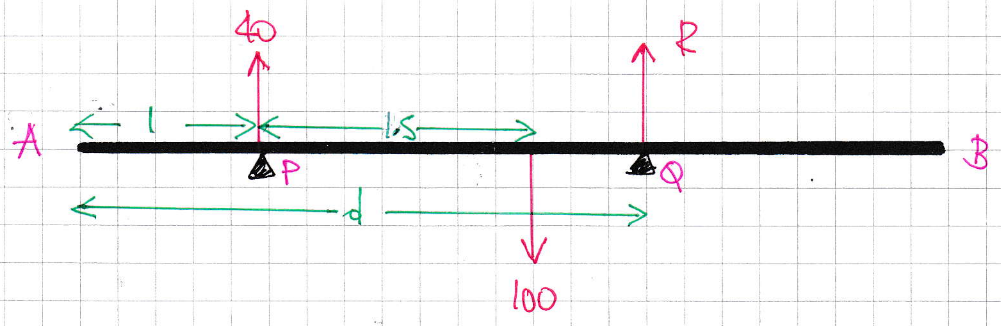
$$t = \begin{cases} 1 \\ \frac{17}{7} \end{cases}$$

∴ REQUIRED TIME

$\frac{17}{7} - 1 = \frac{10}{7}$

IYGB – MMS PAPER L – QUESTION 16

STARTING WITH A DIAGRAM



RESOLVING VERTICALLY

$$40 + R = 100$$

$$R = 60\text{ N}$$

TAKING MOMENTS ABOUT A

$$A: \quad 40 \times 1 + R \times d = 100 \times 2.5$$

$$40 + 60d = 250$$

$$60d = 210$$

$$d = 3.5\text{ m}$$

IYGB - MMS PAPER K - QUESTION 17

a) DIFFERENTIATE VELOCITY TO OBTAIN ACCELERATION

$$v = 3t^2 - t^3$$

$$a = \frac{dv}{dt} = 6t - 3t^2$$

$$a\Big|_{t=2} = (6\times2) - (3\times2^2) = 12 - 12 = 0$$

$\therefore$ ZERO ACCELERATION

b) BY INSPECTION, AT REST WHEN $v=0$, YIELDS $t=0$ & $t=3$

I.E $\quad v = t^2(3-t)$

$$0 = t^2(3-t)$$

$$t = \begin{cases} 0 \\ 3 \end{cases} \qquad \text{IE } T=3$$

INTEGRATE TO OBTAIN DISPLACEMENT

$$v = 3t^2 - t^3$$

$$x = \int 3t^2 - t^3 \, dt$$

$$x = t^3 - \frac{1}{4}t^4 + C$$

APPLY CONDITION $t=2$, $x=4$

$$4 = 2^3 - \frac{1}{4}\times2^4 + C$$
$$4 = 8 - 4 + C$$
$$C = 0$$

$$\therefore x = t^3 - \frac{1}{4}t^4$$

FINALLY WHEN $t = T = 3$

$$x = 3^3 - \frac{1}{4}\times3^4 = 6.75$$

$\therefore$ 6.75 m

IYGB − MMS PAPER K − QUESTION 17

ALTERNATIVE FOR (b)

LOOKING AT THE VELOCITY GRAPH

distance = displacement here as graph up to $t=3$ is above the x axis

$$= \int_0^3 t^2(3-t)\,dt = \int_0^3 3t^2 - t^3\,dt$$

$$= \left[t^3 - \frac{1}{4}t^4\right]_0^3$$

$$= \left(27 - \frac{81}{4}\right) - (0-0)$$

$$= \frac{27}{4}$$

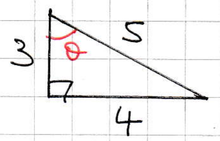
$$= 6.75$$

// AS BEFORE

IYGB - MMS PAPER L - QUESTION 18

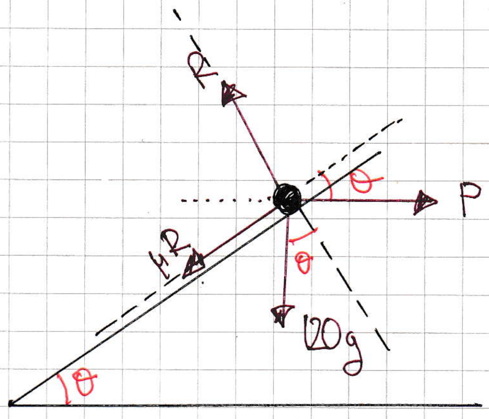
START WITH A DETAILED DIAGRAM, MARKING THE "RUSHING" FORCE AS A "PULLING" FORCE

$\tan\theta = \frac{4}{3}$



$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$



$\mu = \frac{2}{3}$

RESOLVING PARALLEL & PERPENDICULAR TO THE PLANE

$$\left.\begin{aligned} (\parallel):&\quad \mu R + 120g\sin\theta = P\cos\theta \quad —(\mathrm{I}) \\ (\perp):&\quad R = P\sin\theta + 120g\cos\theta \quad —(\mathrm{II}) \end{aligned}\right\} \Longrightarrow$$

$$\left.\begin{aligned} \tfrac{2}{3}R + 120g \times \tfrac{4}{5} &= P \times \tfrac{3}{5} \\ R &= P \times \tfrac{4}{5} + 120g \times \tfrac{3}{5} \end{aligned}\right\} \Longrightarrow$$

$$\left.\begin{aligned} \tfrac{2}{3}R + 96g &= \tfrac{3}{5}P \\ R &= \tfrac{4}{5}P + 72g \end{aligned}\right\} \Longrightarrow$$

BY SUBSTITUTION NOW

$$\Longrightarrow \tfrac{2}{3}\left[\tfrac{4}{5}P + 72g\right] + 96g = \tfrac{3}{5}P$$

$$\Longrightarrow \tfrac{8}{15}P + 48g + 96g = \tfrac{3}{5}P$$

$$\Longrightarrow 144g = \tfrac{1}{15}P$$

$$\Longrightarrow P = 2160g = 21168\,\mathrm{N}$$