IYGB - MMS PAPER L - QUESTION 1

a) EXPLANATORY (INDEPENDENT VARIABLE) IS THE TEMPERATURE

AS IT IS SUBJECT TO "NATURAL" VARIATION, IF WE HAVE NO CONTROL OVER IT

IT IS THE TEMPERATURE WHICH AFFECTS THE SALES AND NOT THE OTHER WAY ROUND

THE "ICE CREAM SALES" IS THE RESPONSE VARIABLE

b) USING A STATISTICAL CALCULATOR

$r = 0.933805... \simeq 0.934$

$N = -12.9 + 7.22T$

c) a IS THE "y INTERCEPT"

I.E THE NUMBER OF ICE CREAMS EXPECTED TO BE SOLD IF THE TEMPERATURE IS ZERO (0°C)

b IS THE "GRADIENT"

NO OF EXTRA ICE CREAMS EXPECTED TO BE SOLD, PER °C TEMPERATURE RISE

c) IF $T = 18$

$N = -12.9 + 7.22 \times 18 \simeq 117$

AS $T = 18$ IS WITHIN THE RANGE OF VALUES OF T WHICH WAS USED TO CREATE THE REGRESSION LINE, THE ESTIMATE SHOULD BE RELIABLE (ALSO NOTE THE "HIGH" PMCC)

IF $T = 37$

$N = -12.9 + 7.22 \times 37 \simeq 254$

AS $T = 37$ IS JUST OUTSIDE THE HIGHEST VALUE OF T WHICH WAS USED TO CREATE THE REGRESSION LINE, AND THE P.M.C.C IS VERY STRONG, THE ESTIMATE COULD BE RELIABLE

IF $T = 45$

$N = -12.9 + 7.22 \times 45 \simeq 312$

AS $T = 45$ IS "WAY ABOVE" THE HIGHEST VALUE OF T WHICH WAS USED TO CREATE THE REGRESSION LINE, THE ESTIMATE WOULD BE UNRELIABLE (EXTRAPOLATION)

IYGB - MMS PAPER L - QUESTION 2

WRITING THESE PROBABILITIES INTO A TABLE

x	1	2	3	4	......	12
$P(X=x)$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	......	$\frac{1}{12}$

IE A DISCRETE UNIFORM DISTRIBUTION

NOW WE WORK AS FOLLOWS

$$P(X+2 < 3X-4 \leq 2X+7)$$
$$= P(2 < 2X-4 \leq X+7) \quad \text{(subtract } X\text{)}$$
$$= P(6 < 2X \leq X+11) \quad \text{(add 4)}$$

NOW SPLIT INTO 2 SEPARATE INEQUALITIES (IGNORE "PROBABILITY P")

- $6 < 2X$
  $2X > 6$
  $X > 3$
- $2X \leq 11 + X$
  $X \leq 11$

COMBINING WE HAVE

$$P(3 < X \leq 11)$$
$$= P(X = 4,5,6,\ldots 11)$$
$$= \frac{1}{12} \times 8$$
$$= \frac{2}{3}$$

IYGB - MMS PAPER L - QUESTION 3

QUOTA SAMPLING

- ADVANTAGES
  - QUICK & SIMPLE
  - COST EFFICIENT
  - NO DANGER OF OVER-REPRESENTATION IN SMALLER SAMPLES

- DISADVANTAGES
  - NON RANDOM
  - IMPOSSIBLE TO DETERMINE SAMPLING ERRORS

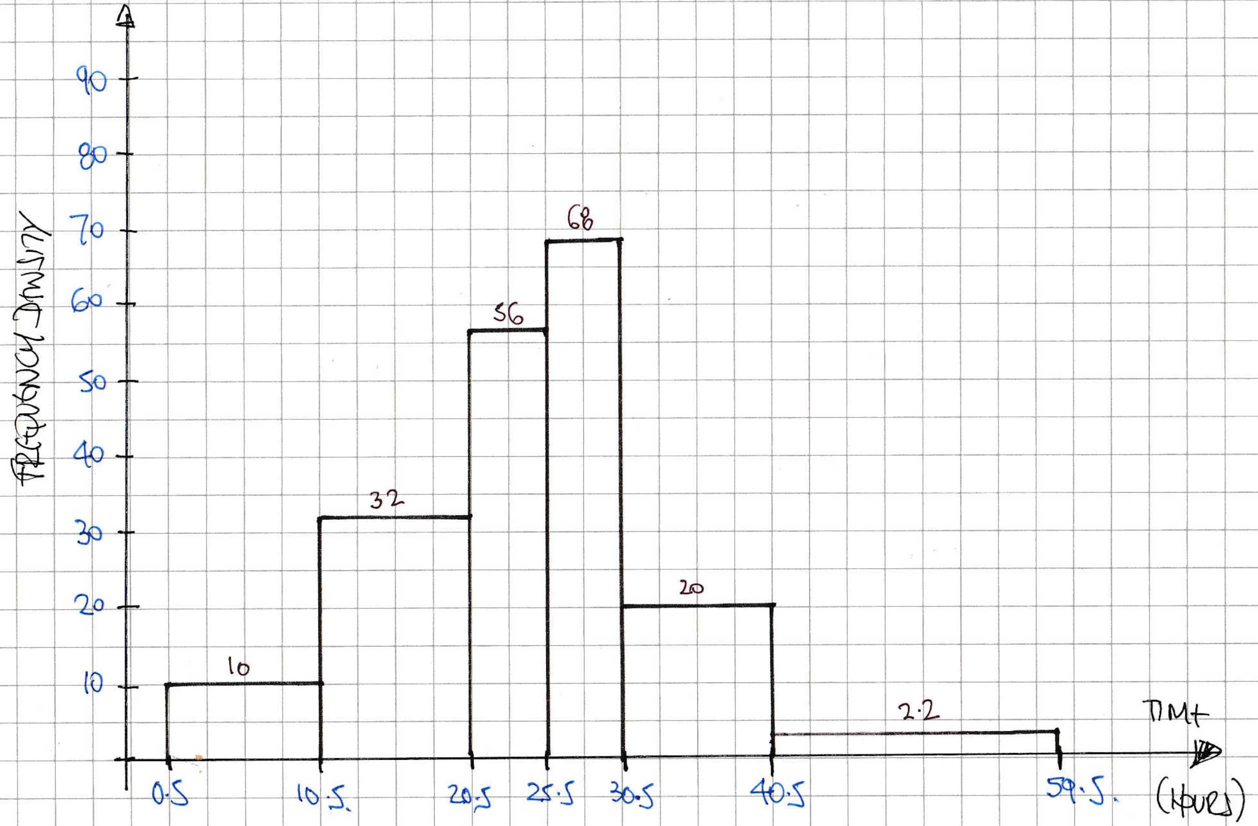
ETC

IYGB - MMS PAPER L - QUESTION 4

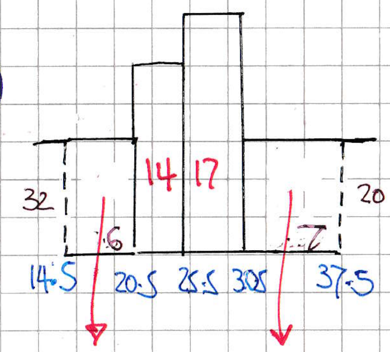
a)

HOURS (NEAREST HOUR)	1-10	11-20	21-25	26-30	31-40	41-59
FREQUENCY	5	16	14	17	10	2
CLASS WIDTH	10	10	5	5	10	19
FREQUENCY DENSITY	$5 \div 10 = 0.5$	$16 \div 10 = 1.6$	$14 \div 5 = 2.8$	$17 \div 5 = 3.4$	$10 \div 10 = 1$	$2 \div 19 = 0.11$
SCALED FREQUENCY DENSITY	10	32	56	68	20	2.2

$\times 20$



b)



$$\frac{32 \times 6}{20} = 9.6 \qquad \frac{20 \times 7}{20} = 7$$

LOOKING AT THE DIAGRAM ABOVE

THE REQUIRED ESTIMATE IS

$$9.6 + 14 + 17 + 7 = 47.6$$

$\therefore$ APPROX 48

IYGB

c) AS DATA TOTAL IS LARGE, SAY $Q_2 = \frac{1}{2} \times 64 = 32^{ND}$ OBS, WHICH LIES IN 21–25 CLASS

USING AREA INSTEAD OF FREQUENCY WE ARE LOOKING FOR A CUT AT $32 \times 20 = 640$ UNITS OF AREA

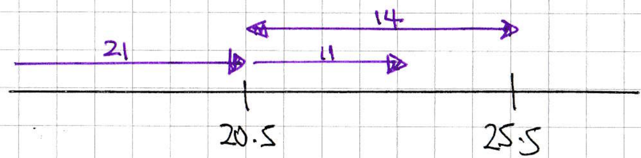
$$\Rightarrow (10 \times 10) + (32 \times 10) + 56x = 640$$

$$\Rightarrow 56x = 220$$

$$\Rightarrow x = 3.93$$

$$\therefore Q_2 = 20.5 + 3.93 \simeq 24.43$$

"ALTERNATIVE BY LINEAR INTERPOLATION"



$$Q_2 \simeq 20.5 + \frac{11}{14} \times 5 \approx 24.43$$

IYGB - MMS PAPER L - QUESTION 5

a)

X = NUMBER OF STUDENTS WHICH GET DRIVEN BACK HOME
$X \sim B(36, 0.15)$

SETTING HYPOTHESES

$H_0 : p = 0.15$

$H_1 : p \neq 0.15$, WHERE p IS THE PROPORTION OF STUDENTS WHO GET DRIVEN IN GENERAL

CRITICAL REGION REQUIRED, AT 6%, TWO TAILED

FROM CALCULATOR

CRITICAL
$P(X \leq 1) = 0.0212 = 2.12\% < 3\%$
$P(X \leq 2) = 0.0776 = 7.76\% > 3\%$

$\vdots \quad \vdots \quad \vdots$

$P(X \geq 10) = 1 - P(X \leq 9) = 1 - 0.9649 = 0.0351 = 3.51\% > 3\%$
CRITICAL
$P(X \geq 11) = 1 - P(X \leq 10) = 1 - 0.9863 = 0.0137 = 1.37\% < 3\%$

$\therefore$ C.R $= \{0, 1\} \cup \{11, 12, 13, \ldots, 36\}$

b) IF WE REJECT AS CLOSE AS POSSIBLE TO 3% IN EACH TAIL

2.12% IS CLOSER TO 3% THAN 7.76%

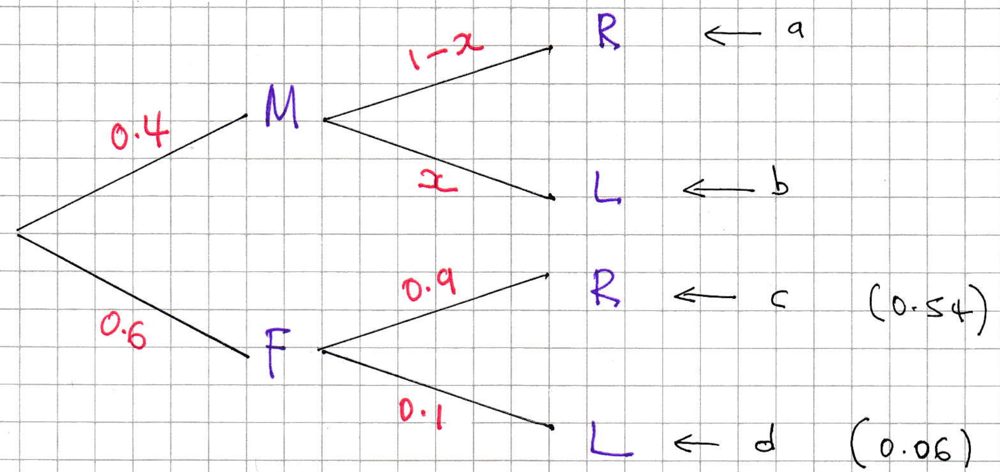
BUT

3.51% IS CLOSER TO 3% THAN 1.37%

$\therefore$ C.R $= \{0, 1\} \cup \{10, 11, 12, \ldots, 36\}$

IYGB – MMS PAPER L – QUESTION 6

a) DRAWING A TREE DIAGRAM WITH GENDER FIRST



"0.11 OF THE STUDENTS IS LEFT HANDED" $\Rightarrow$ $b + d = 0.11$

$\Rightarrow 0.4x + 0.06 = 0.11$

$\Rightarrow 0.4x = 0.05$

$\Rightarrow x = 0.125$

b) $P(F \cap R) = 0.6 \times 0.9 = 0.54$

c) $P(F \mid L) = \dfrac{P(F \cap L)}{P(L)} = \dfrac{0.6 \times 0.1}{0.11} = \dfrac{0.06}{0.11} = \dfrac{6}{11} \simeq 0.545$

d) $P(L \mid M) = x = 0.125$

OR

$$= \frac{P(L \cap M)}{P(M)} = \frac{\cancel{0.4} \times 0.125}{\cancel{0.4}} = 0.125$$

IYGB - MMS PAPER L - QUESTION 7

$X \sim N(425, 20^2)$

a) I) $P(X > 455)$

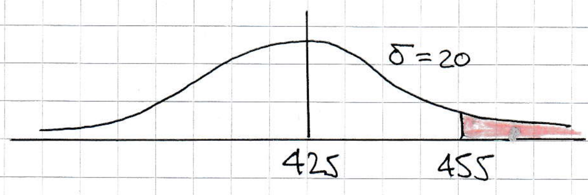
$= 1 - P(X < 455)$

$= 1 - P\left(Z < \frac{455 - 425}{20}\right)$

$= 1 - \Phi(1.5)$

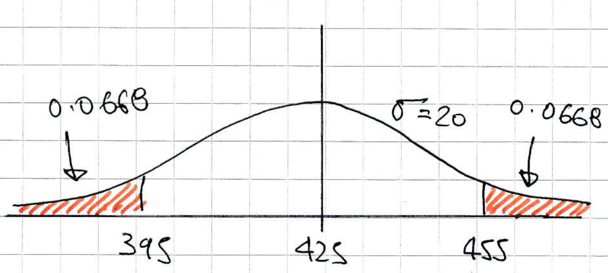
$= 1 - 0.9332$

$= 0.0668$



II) $P(395 < X < 455)$

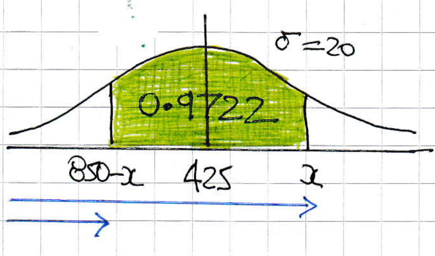
= BY SYMMETRY & USING THE PREVIOUS PART...



$= 1 - 2 \times 0.0668$

$= 0.8664$

b) $P(850 - x < X < x) = 0.9722$



$\Rightarrow P(X < x) - P(X < 850 - x) = 0.9722$

$\Rightarrow P(X < x) - [1 - P(X > 850 - x)] = 0.9722$

$\Rightarrow P(X < x) + P(X > 850 - x) - 1 = 0.9722$

$\Rightarrow P(X < x) + P(X > 850 - x) = 1.9722$

$\Rightarrow P\left(Z < \frac{x - 425}{20}\right) + P\left(Z > \frac{850 - x - 425}{20}\right) = 1.9722$

$\Rightarrow P\left(Z < \frac{x - 425}{20}\right) + P\left(Z > \frac{425 - x}{20}\right) = 1.9722$

$\Rightarrow P\left(Z < \frac{x - 425}{20}\right) + P\left(Z > -\frac{x - 425}{20}\right) = 1.9722$

$\Rightarrow \Phi\left(\frac{x - 425}{20}\right) + \Phi\left(-\frac{x - 425}{20}\right) = 1.9722$

IYGB − MMS PAPER L − QUESTION 7

$$\Longrightarrow \quad 2\Phi\left(\frac{x-425}{20}\right) = 1.9722 \qquad \Phi(-y) \equiv \Phi(y)$$

$$\Longrightarrow \quad \Phi\left(\frac{x-425}{20}\right) = 0.9861$$

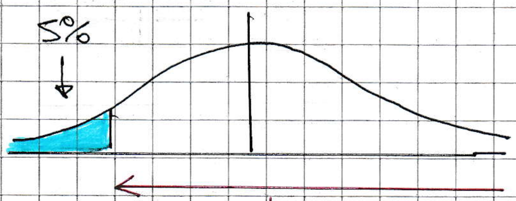
$$\Longrightarrow \quad \frac{x-425}{20} = \Phi^{-1}(0.9861)$$

$$\Longrightarrow \quad \frac{x-425}{20} = 2.2$$

$$\Longrightarrow \quad x = 469$$

c) COLLECTING ALL INFORMATION FOR THE TEST

$H_0 : \mu = 425$
$H_1 : \mu < 425$, WHERE μ IS THE POPULATION MEAN
$n = 12$ $\sigma = 20$ $\bar{x} = 417$, 5% SIGNIFICANCE, ONE TAILED TEST



$\Phi^{-1}(0.95) = -1.6449$

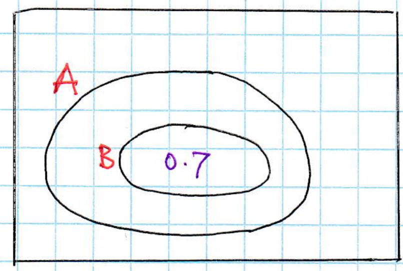
$$Z \text{ STATISTIC} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{417 - 425}{20/\sqrt{12}} = -1.3856$$

AS $-1.3856 > -1.6449$ THERE IS NO SIGNIFICANT EVIDENCE THAT μ IS LESS THAN 425, AT THE 5% SIGNIFICANCE LEVEL

NO ENOUGH EVIDENCE TO REJECT H_0

IYGB – MMS PAPER L – QUESTION 8

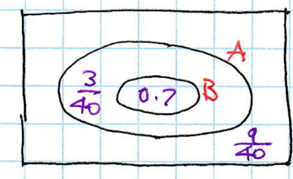
$P(A|B) = 1 \quad \bullet \quad P(A|B') = \frac{1}{4} \quad \bullet \quad P(B) = \frac{7}{10}$



$$P(A|B') = \frac{P(A \cap B')}{P(B')}$$

$$\Rightarrow \frac{1}{4} = \frac{P(A \cap B')}{0.3}$$

$$\Rightarrow P(A \cap B') = \frac{3}{40}$$



FINALLY $P(B'|A) = \frac{P(B' \cap A)}{P(A)}$

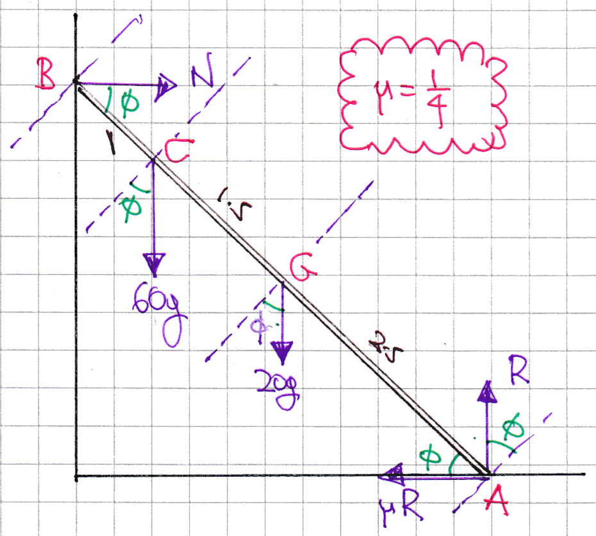
$$= \frac{\frac{3}{40}}{\frac{3}{40} + 0.7}$$

$$= \frac{\frac{3}{40}}{\frac{31}{40}}$$

$$= \frac{3}{31}$$

IYGB – MMS PAPER L – QUESTION 9

STARTING WITH A DIAGRAM



$(\uparrow)$: $R = 60g + 20g = 80g$

$(\rightarrow)$: $N = \mu R$

$\overset{\curvearrowright}{A}$: $(20g\cos\phi \times 2.5) + (60g\cos\phi \times 4) = N\sin\phi \times 5$

TIDY THE MOMENT EQUATION

$\Rightarrow 50g\cos\phi + 240g\cos\phi = 5N\sin\phi$

$\Rightarrow 290g\cos\phi = 5(\mu R)\sin\phi$

$\Rightarrow 290g\cos\phi = 5\left(\frac{1}{4} \times 80g\right)\sin\phi$

$\Rightarrow 290g\cos\phi = 100g\sin\phi$

$\Rightarrow 29\cos\phi = 10\sin\phi$

$\Rightarrow \dfrac{\sin\phi}{\cos\phi} = \dfrac{29}{10}$

$\Rightarrow \tan\phi = 2.9$

$\Rightarrow \phi = \arctan(2.9)$

$\Rightarrow \phi \simeq 71°$

AND USING "$N = \mu R$"

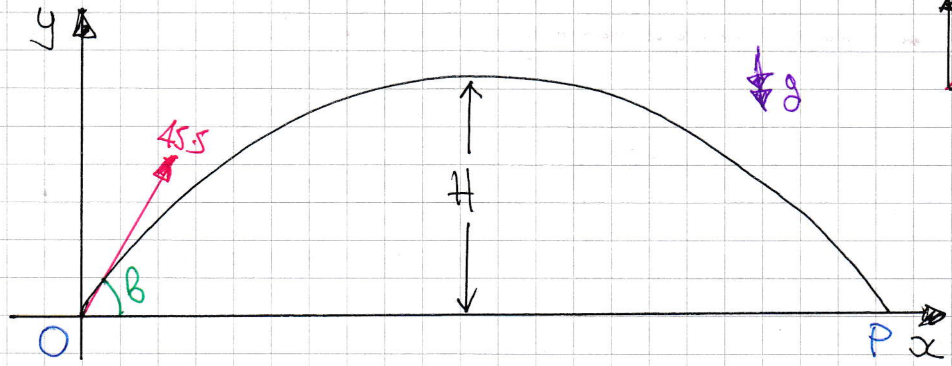
μR = FRICTIONAL FORCE

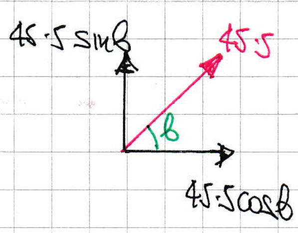
$\frac{1}{4}(80g)$ = FRICTION

FRICTION $= 20g$

FRICTION $= 196$ N

IYGB - MMS PAPER L - QUESTION 10





$\tan\theta = \frac{12}{5}$

$\sin\theta = \frac{12}{13}$

$\cos\theta = \frac{5}{13}$

a) LOOKING AT THE VERTICAL MOTION FROM O TO P

$u = 45.5\sin\theta = 42 \text{ ms}^{-1}$

$a = -9.8 \text{ ms}^{-2}$

$s = 0$

$t = ?$

v

$s = ut + \frac{1}{2}at^2$

$0 = 42t + \frac{1}{2}(-9.8)t^2$

$0 = 42t - 4.9t^2$

$0 = t(42 - 4.9t)$

$t = \cancel{0}$ or $\frac{60}{7}$

b) LOOKING AT THE HORIZONTAL MOTION

"DISTANCE = SPEED × TIME"

$|OP| = 45.5\cos\theta \times \frac{60}{7}$

$|OP| = 45.5 \times \frac{5}{13} \times \frac{60}{7}$

$|OP| = 150 \text{ m}$

USING SYMMETRY OR DIRECTLY

$u = 42 \text{ ms}^{-1}$

$a = -9.8 \text{ ms}^{-2}$

$s =$

$t = \frac{30}{7}$ (SYMMETRY)

$v = 0$

$s = ut + \frac{1}{2}at^2$

$s = 42 \times \frac{30}{7} + \frac{1}{2}(-9.8)\left(\frac{30}{7}\right)^2$

$s = 90 \text{ m}$

$v^2 = u^2 + 2as$

$0^2 = 42^2 + 2(-9.8)s$

$19.6s = 1764$

$s = 90 \text{ m}$

IYGB - MMS PAPER L - QUESTION 10

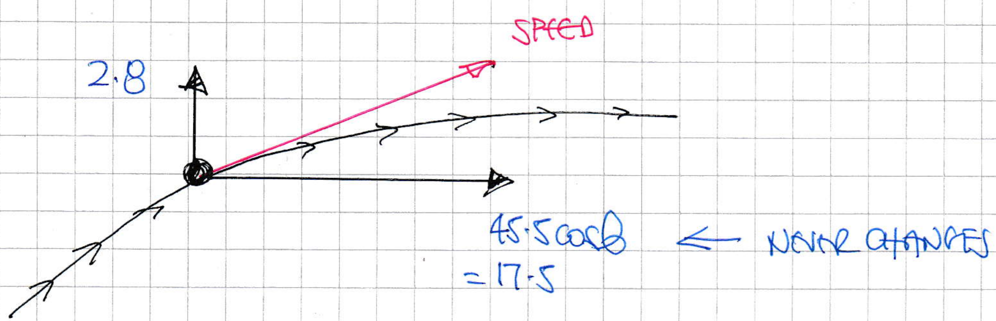
c) LOOKING AT VERTICAL MOTION

$u = 42 \text{ ms}^{-1}$
$a = -9.8 \text{ ms}^{-2}$
s
$t = 4 \text{ s}$
$v = ?$

$v = u + at$

$v = 42 - 9.8 \times 4$

$v = 2.8 \text{ ms}^{-1}$

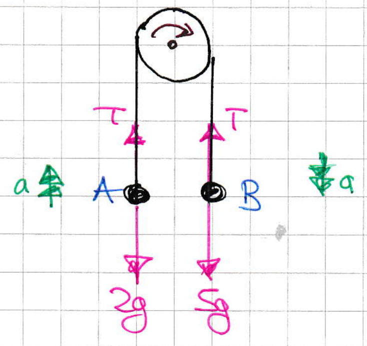


$\text{SPEED} = \sqrt{2.8^2 + 17.5^2}$

$\text{SPEED} \approx 17.72 \text{ ms}^{-1}$

IYGB — MMS PAPER L — QUESTION 11

a) CONSIDERING THE MOTION OF EACH PARTICLE



$$(A):\quad T - 2g = 2a$$
$$(B):\quad 5g - T = 5a$$

ADDING THE EQUATIONS

$$3g = 7a$$
$$a = \frac{3}{7}g = 4.2\ \mathrm{ms^{-1}}$$

USING $T - 2g = 2a$

$$T = 2\times 4.2 + 2g$$
$$T = 28\,\mathrm{N}$$

b) FIND THEIR COMMON SPEED WHEN B HITS THE GROUND

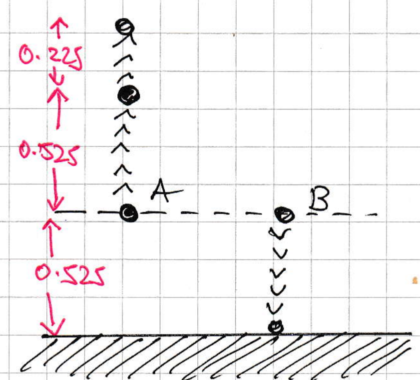
$u = 0$
$a = 4.2$
$s = ?$
$t = 0.5$
$v = ?$

$v = \cancel{u} + at$
$v = 4.2\times 0.5$
$v = 2.1\ \mathrm{ms^{-1}}$

ALSO

$s = \cancel{ut} + \frac{1}{2}at^2$
$s = \frac{1}{2}(4.2)\times 0.5^2$
$s = 0.525\,\mathrm{m}$

ONCE B HITS THE GROUND, THE STRING GOES SLACK, SO A IS "FREE" TO MOVE UNDER GRAVITY



$u = 2.1\ \mathrm{ms^{-1}}$
$a = -9.8\ \mathrm{ms^{-2}}$
$s = ?$
t
$v = 0$

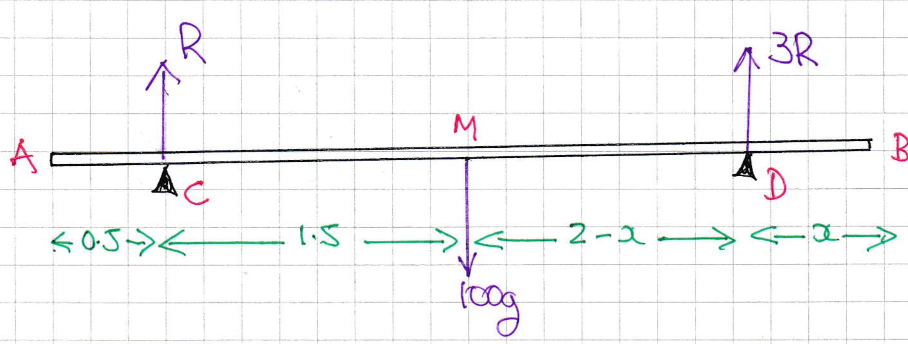
$v^2 = u^2 + 2as$
$0 = 2.1^2 + 2(-9.8)s$
$19.6s = 4.41$
$s = 0.225\,\mathrm{m}$

REQUIRED DISTANCE IS

$$0.525 + 0.525 + 0.225 = 1.275\,\mathrm{m}$$

IYGB - MMS PAPER L - QUESTION 12

a)



RESOLVING VERTICALLY

$R + 3R = 100g$

$4R = 100g$

$R = 25g$

(NOT ACTUALLY NEEDED IF WE TAKE MOMENTS ABOUT M)

TAKING MOMENTS ABOUT M

$R \times 1.5 = 3R \times (2 - x)$

$1.5 = 3(2 - x)$

$0.5 = 2 - x$

$x = 1.5 \text{ m}$

b) **REDRAWING THE DIAGRAM**

TAKING MOMENTS ABOUT B

$(2R \times 0.75) + (R \times 3.5) = 100g \times 2$

$1.5R + 3.5R = 200g$

$5R = 200g$

$R = 40g$

RESOLVING VERTICALLY

$R + 2R = 100g + mg$

$3R = 100g + mg$

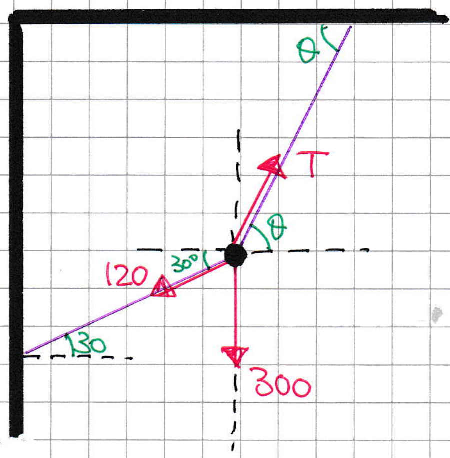
$120g = 100g + mg.$

$20g = mg$

$m = 20 \text{ kg}$

IYGB - MMS PAPER L - QUESTION 13

LOOKING AT THE DIAGRAM



$(\uparrow)$: $T\sin\theta = 300 + 120\sin 30°$
$(\rightarrow)$: $T\cos\theta = 120\cos 30°$

$T\sin\theta = 360$
$T\cos\theta = 60\sqrt{3}$

DIVIDING THE EQUATIONS, SIDE BY SIDE

$$\Longrightarrow \frac{\cancel{T}\sin\theta}{\cancel{T}\cos\theta} = \frac{360}{60\sqrt{3}}$$

$$\Longrightarrow \tan\theta = 2\sqrt{3}$$

$$\Longrightarrow \theta = 73.897°...$$

$$\Longrightarrow \theta \simeq 74°$$

FINALLY TO FIND T

$$\Longrightarrow T\sin\theta = 360$$

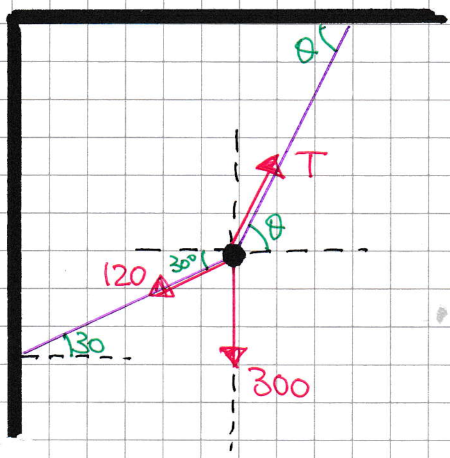
$$\Longrightarrow T = \frac{360}{\sin(73.89...)}$$

$$\Longrightarrow T = 374.6998...$$

$$\Longrightarrow T \simeq 375 \text{ N}$$

IYGB - MMS PAPER L - QUESTION 13

LOOKING AT THE DIAGRAM



$$(\uparrow): \quad T\sin\theta = 300 + 120\sin 30°$$
$$(\rightarrow): \quad T\cos\theta = 120\cos 30°$$

$$T\sin\theta = 360$$
$$T\cos\theta = 60\sqrt{3}$$

DIVIDING THE EQUATIONS, SIDE BY SIDE

$$\Longrightarrow \frac{\cancel{T}\sin\theta}{\cancel{T}\cos\theta} = \frac{360}{60\sqrt{3}}$$

$$\Longrightarrow \tan\theta = 2\sqrt{3}$$

$$\Longrightarrow \theta = 73.897°...$$

$$\Longrightarrow \theta \simeq 74°$$

FINALLY TO FIND T

$$\Longrightarrow T\sin\theta = 360$$

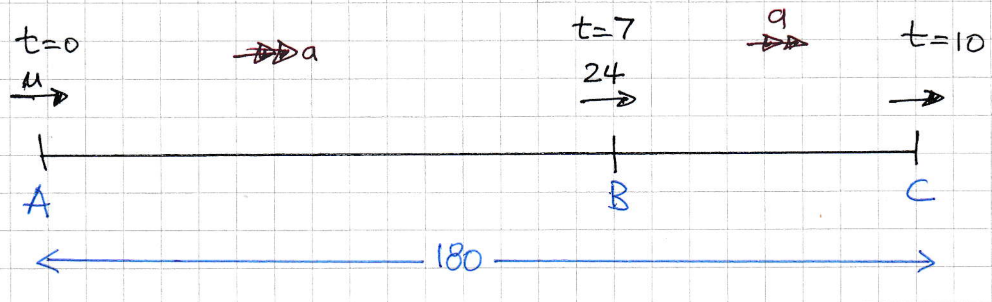
$$\Longrightarrow T = \frac{360}{\sin(73.89...)}$$

$$\Longrightarrow T = 374.6998...$$

$$\Longrightarrow T \simeq 375\text{ N}$$

IYGB – MMS PAPER L – QUESTION 14

LOOKING AT A DIAGRAM



LOOKING AT THE JOURNEY AB

$u = \ ?$
$a = \ ?$
$s =$
$t = 7$
$v = 24$

$v = u + at$

$$24 = u + 7a$$

LOOKING AT THE JOURNEY AC

$u = \ ?$
$a = \ ?$
$s = 180$
$t = 10$
$v =$

$s = ut + \frac{1}{2}at^2$

$180 = 10u + \frac{1}{2}a \times 10^2$

$180 = 10u + 50a$

$$18 = u + 5a$$

SUBTRACTING THE EQUATIONS YIELDS

$$\left.\begin{array}{l} 24 = u + 7a \\ 18 = u + 5a \end{array}\right\} \Rightarrow 6 = 2a$$

$$\Rightarrow a = 3 \text{ ms}^{-2}$$

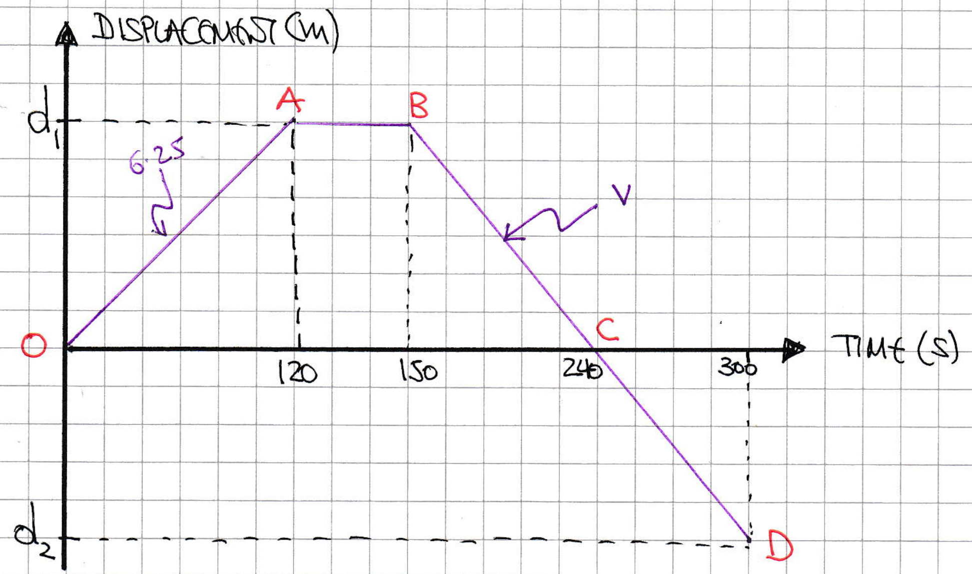
$\&$ $\quad 18 = u + 5a$

$18 = u + 15$

$$u = 3 \text{ ms}^{-1}$$

IYGB - MMS PAPER L - QUESTION 15

LOOKING AT THE DISPLACEMENT - TIME GRAPH



$$\text{VELOCITY} = \frac{\Delta x}{\Delta t} = \text{GRADIENT}$$

$$6.25 = \frac{d_1}{120}$$

$$d_1 = 750$$

BY SIMILAR TRIANGLES

$$\frac{d_1}{240-150} = \frac{|d_2|}{300-240} \implies |d_2| = \frac{2}{3}d_1$$

$$\implies |d_2| = 500$$

AVERAGE SPEED $= \dfrac{2\times 750 + 500}{300} = \dfrac{2000}{300} = \dfrac{20}{3} = 6\tfrac{2}{3}$

$\therefore$ AVERAGE SPEED OF 6.67 ms^{-1}

IYGB - MMS PAPER L - QUESTION 16

a) USING $\underline{r} = \underline{r}_0 + \underline{v}t$

$9\underline{i} + 20\underline{j} = -11\underline{i} - 24\underline{j} + \underline{v} \times 4$

$20\underline{i} + 44\underline{j} = 4\underline{v}$

$\underline{v} = 5\underline{i} + 11\underline{j}$

b) OBTAIN EXPRESSIONS FOR THE POSITION VECTORS OF EACH SHIP, t HOURS AFTER MIDNIGHT.

$\underline{r}_A = (-11\underline{i} - 24\underline{j}) + (5\underline{i} + 11\underline{j})t$

$\underline{r}_B = (5\underline{i} - 10\underline{j}) + (0\underline{i} + 8\underline{j})t$

$\underline{r}_A = (5t - 11)\underline{i} + (11t - 24)\underline{j}$

$\underline{r}_B = 5\underline{i} + (8t - 10)\underline{j}$

OR AS CO-ORDINATES

$A(5t-11, 11t-24)$ & $B(5, 8t-10)$

USING THE DISTANCE FORMULA $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{[(5t-11) - 5]^2 + [(11t-24) - (8t-10)]^2}$

$d = \sqrt{(5t-16)^2 + (3t-14)^2}$

$d^2 = 25t^2 - 160t + 256 + 9t^2 - 84t + 196$

$d^2 = 34t^2 - 244t + 452$

AS REQUIRED

c) LET $d = 10$ & NOTE $t = 2$ IS A SOLUTION

$\Rightarrow 10^2 = 34t^2 - 244t^2 + 452$

$\Rightarrow 0 = 34t^2 - 244t^2 + 352$

$\Rightarrow 17t^2 - 122t + 176 = 0$

$\Rightarrow (t-2)(17t - 88) = 0$

$t = \begin{cases} 2 & \text{(ALREADY KNOWN)} \\ \frac{88}{17} = 5.1764\ldots \end{cases}$

$= 05:11$

0.1764×60

$\simeq 10.58\ldots$

$\simeq 11$