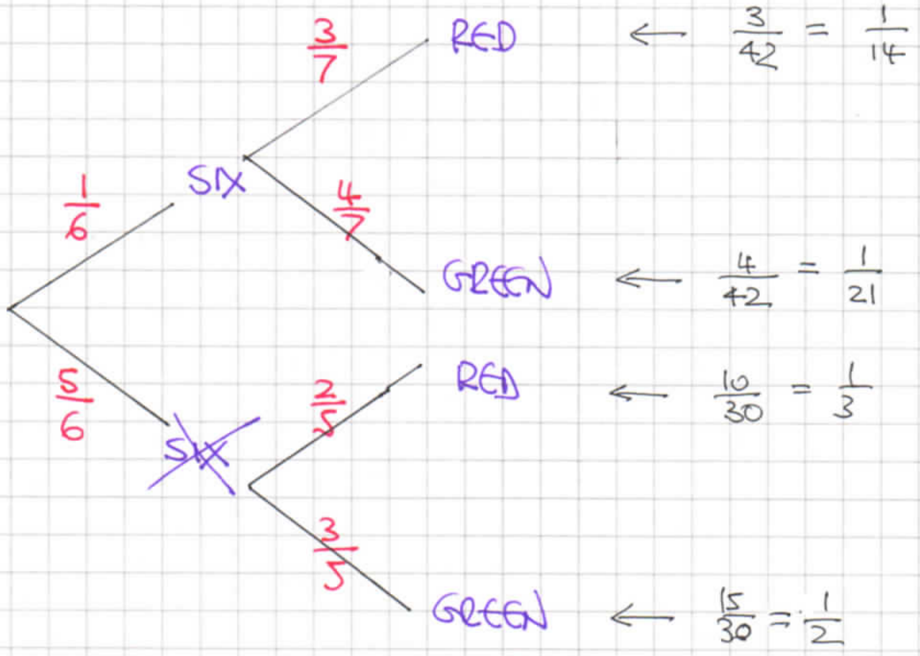
IYGB – MMS PAPER N – QUESTION 1

DRAWING A TREE DIAGRAM



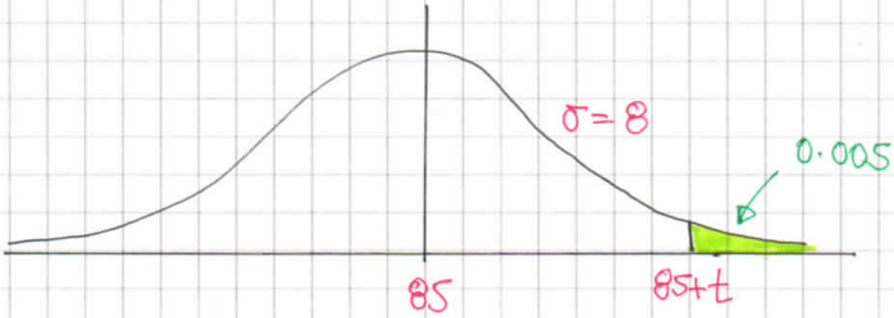
a) $P(\text{Red}) = \frac{1}{14} + \frac{1}{3} = \frac{17}{42}$

b) $P(\text{six} \mid \text{Red}) = \frac{P(\text{six} \cap \text{red})}{P(\text{Red})} = \frac{\frac{1}{14}}{\frac{17}{42}} = \frac{3}{17}$

IYGB - MMS - PAPER N - QUESTION 2

X = FLIGHT TIME (minutes)

$X \sim N(85, 8^2)$



LOOKING AT THE DIAGRAM ABOVE

$$P(X > 85 + t) = 0.005$$
$$P(X < 85 + t) = 0.995$$
$$P\left(Z < \frac{85 + t - 85}{8}\right) = 0.995$$
$$P\left(Z < \frac{t}{8}\right) = 0.995$$

$\big\downarrow$ INVERSION (+)

$$\frac{t}{8} = +\Phi^{-1}(0.995)$$
$$\frac{t}{8} = 2.5758$$
$$t \approx 20.6064$$

$\therefore$ $t = 21$, CORRECT TO THE NEAREST MINUTE

IYGB - MMS PAPER N - QUESTION 3

EGGS LAID IN A WEEK	NUMBER OF WEEKS	
52	1	(1)
53	4	(5)
54	7	(12)
55	10	(22)
56	11	(33)
57	8	(41)
58	5	(46)
59	1	(47)

a) FROM CALCULATOR IN STAT MODE

$$\sum x = 2613 \qquad \sum x^2 = 145391 \qquad n = 47$$

$$\text{MEAN, } \bar{x} = \frac{\sum x}{n} = \frac{2613}{47} \simeq 55.6$$

$$\text{S.D, } \sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{145391}{47} - 55.6...^2} \approx 1.59$$

b) $n = 47$ (ODD, SO $n+1$ RULE APPLIES)

- $Q_1 = \frac{1}{4}(47+1) = 12^{TH}$ OBS
- $Q_2 = \frac{1}{2}(47+1) = 24^{TH}$ OBS
- $Q_3 = \frac{3}{4}(47+1) = 36^{TH}$ OBS

$\therefore Q_1 = 54$

$Q_2 = 56$

$Q_3 = 57$

c) THIS IS CODING $y = x - 45$

- MEAN, $\bar{y} = \bar{x} - 45 = 10.6$
- S.D, $\sigma_y = \sigma_x = 1.59$ (UNCHANGED)

IYGB - MMS PAPER N - QUESTION 3

d)

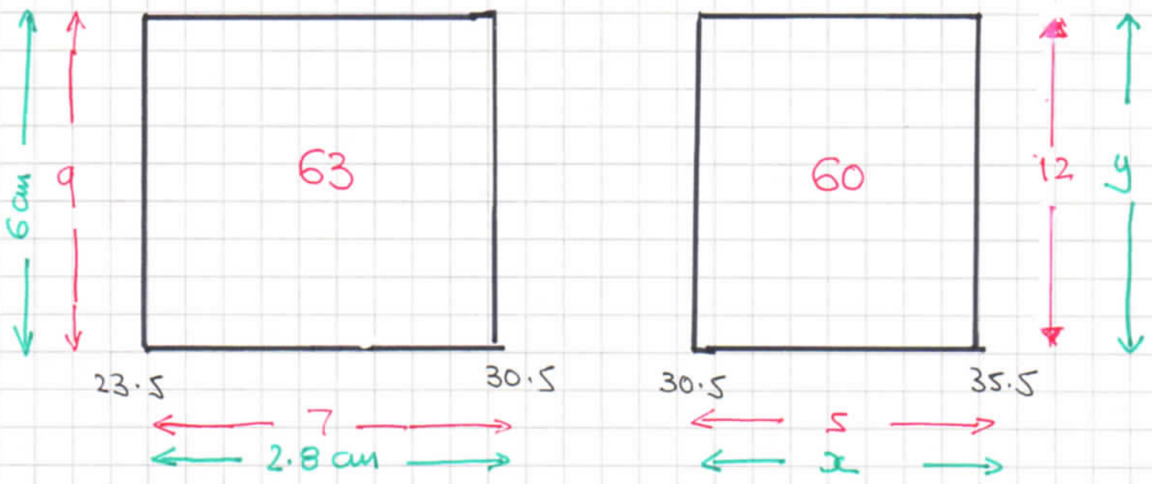
MEDIAN = 54
MEAN = 55.6
MODE = 56

$\left.\begin{array}{l}\text{MEDIAN} = 54\\ \text{MEAN} = 55.6\\ \text{MODE} = 56\end{array}\right\}$ APPROXIMATELY EQUAL, SO VERY LITTLE SKEW

DATA CANNOT BE MODELLED BY A NORMAL DISTRIBUTION AS THE DATA IS DISCRETE AND NOT GROUPED

IYGB – MMS PAPER N – QUESTION 4

DRAWING TWO RECTANGLES FROM THE HISTOGRAM (NOT TO SCALE)



HISTOGRAM PARTICULARS & NUMBERS

ACTUAL DRAWING MEASUREMENTS

LOOKING AT THE RATIOS, SEPARATELY FOR THE BASE TO THAT OF THE HEIGHT, WE OBTAIN

- $\frac{x}{2.8} = \frac{5}{7}$

  $7x = 14$

  $x = 2$

- $\frac{y}{12} = \frac{6}{9}$

  $9y = 72$

  $y = 8$

$\therefore$ BASE = 2 cm & HEIGHT = 8 cm

IYGB – MMS PAPER N – QUESTION 5

- START BY FINDING THE SAMPLE MEAN

$$\bar{x}_{10} = \frac{127.0 + 124.6 + 122.8 + \dots + 121.8}{10} = \frac{1244}{10} = 124.4$$

- SET THE HYPOTHESES

$H_0: \mu = 125$

$H_1: \mu \neq 125$

$\bar{x}_{10} = 124.4$

$\sigma = 1.1$

$n = 10$

- THUS WE NOW HAVE

$P(\bar{x}_{10} < 124.4)$

$= 1 - P(\bar{x}_{10} > 124.4)$

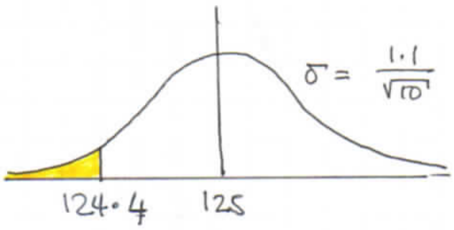
$= 1 - P\left(z > \frac{124.4 - 125}{\frac{1.1}{\sqrt{10}}}\right)$

$= 1 - \Phi(-1.724878\dots)$

$= 1 - 0.9577$

$= 0.0423$

$= 4.23\%$



- AS THE TEST IS TWO TAILED THE p – VALUE IS $2 \times 4.23\% = 8.46\%$

COMPARING WITH 5%, THERE IS NO SIGNIFICANT EVIDENCE THAT THE MEAN SERVING SPEED HAS CHANGED – NOT SUFFICIENT EVIDENCE TO REJECT H_0

IYGB - MMS PAPER N - QUESTION 6

X = NUMBER OF PATIENTS WHO FAIL TO TURN UP

$X \sim B(20, 0.2)$

a) P(All patients turn up) $= P(X=0) = \binom{20}{0}(0.2)^0(0.8)^{20}$

$= \underline{0.0115}$

b) P(more than 3 will not turn up) $= P(X>3) = P(X \geqslant 4)$

$= 1 - P(X \leqslant 3) = 1 - 0.411448...$

$= \underline{0.5886}$

Y = NUMBER OF PATIENTS WHO FAIL TO TURN UP OUT OF 21

$Y \sim B(21, 0.2)$

c) P(DOCTOR WILL BE ABLE TO SEE ALL THE PATIENTS WHO TURN UP)

$= P(Y \geqslant 1)$

$= 1 - P(Y=0)$

$= 1 - \binom{21}{0}(0.2)^0(0.8)^{21}$

$= 1 - 0.009223...$

$= \underline{0.9908}$

d) W = NUMBER OF PATIENTS WHO FAIL TO TURN UP OUT OF 25

$W \sim B(25, 0.2)$

P(DOCTOR WILL NOT BE ABLE TO SEE ONE PATIENT)

= P(21 PATIENTS SHOWED UP)

= P(4 PATIENTS DID NOT SHOW UP)

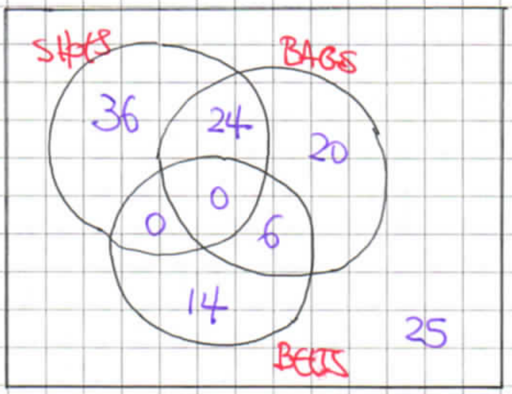
$= P(W=4)$

$= \binom{25}{4}(0.2)^4(0.8)^{21}$

$= \underline{0.1867}$

IYGB - MMS PAPER N - QUESTION 7

a) FILL IN IN A VENN DIAGRAM



- $P(\text{SHOES}) = \frac{60}{125} = 0.48$
- $P(\text{BELT}) = \frac{20}{125} = 0.16$

- $P(\text{EXACTLY TWO ITEMS}) = \frac{24+6}{125} = \frac{30}{125} = \frac{6}{25} = 0.24$

b) $P(\text{SHOES} \mid \text{BAG}) = \frac{24}{24+20+6} = \frac{24}{50} = \frac{12}{25} = 0.48$

c) $P(\text{BELT} \mid \text{BAG}) = \frac{6}{24+20+6} = \frac{6}{50} = \frac{3}{25} = 0.12$

d) AS $P(\text{SHOES} \mid \text{BAG}) = P(\text{SHOES}) = 0.48$, THE EVENTS "BUY SHOES" & "BUY BAG" ARE INDEPENDENT

ALTERNATIVE

$P(\text{SHOES} \cap \text{BAG}) = \frac{24}{125} = 0.192$

$P(\text{SHOES}) = \frac{60}{125} = 0.48$

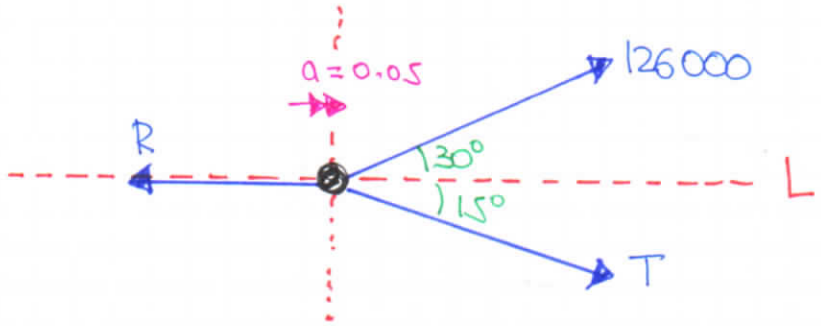
$P(\text{BAG}) = \frac{50}{125} = 0.4$

$P(\text{SHOES}) \times P(\text{BAG}) = 0.48 \times 0.4 = 0.192 = P(\text{SHOES} \cap \text{BAG})$

$\therefore$ INDEPENDENT.

IYGB - MMS PAPER N - QUESTION 8

START WITH A DIAGRAM IN ORDER TO RESOLVE FORCES



RESOLVING FORCES "VERTICALLY" & HORIZONTALLY

$(\uparrow)\quad 126000 \sin 30 = T \sin 15°$ (EQUILIBRIUM)

$(\rightarrow)\quad 126000 \cos 30 + T\cos 15° - R = 800000 \times 0.05$ ($F = ma$)

SOLVE BY SUBSTITUTION

$$T = \frac{126000 \sin 30}{\sin 15°} = 63000\left(\sqrt{6}+\sqrt{2}\right) \simeq 243413.3082\ldots$$

$$\Rightarrow 126000\cos 30° + T\cos 15° - 800000 \times 0.05 = R$$

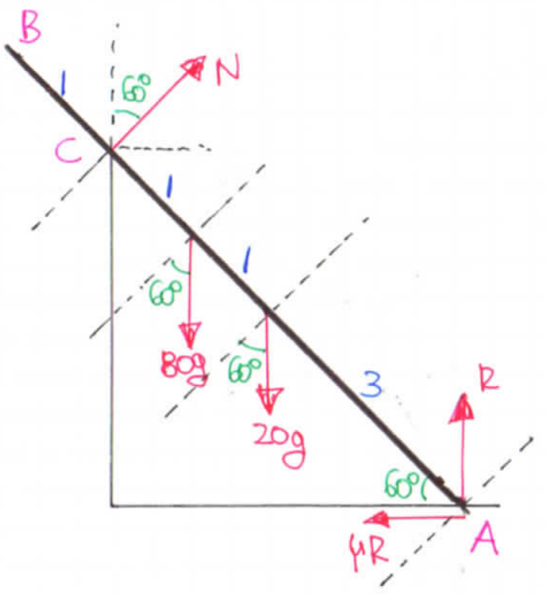
$$\Rightarrow 63000\sqrt{3} + 63000\left(\sqrt{6}+\sqrt{2}\right)\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) - 40000 = R$$

$$\Rightarrow R = 304238.4018$$

$$\Rightarrow R \simeq 304000\text{ N}$$

(3 s.f.)

IYGB - MMS PAPER N - QUESTION 9



- STARTING WITH A GOOD DIAGRAM IN ORDER TO FORM SOME EQUATIONS

$(\uparrow)$: $R + N\cos 60 = 80g + 20g$ (I)

$(\rightarrow)$: $\mu R = N\sin 60°$ (II)

$(\overset{\curvearrowright}{A})$: $(80g\cos 60)\times 4 + (20g\cos 60)\times 3 = N\times 5$ (III)

- STARTING WITH THE "MOMENTS" EQUATION

$\Rightarrow 320g\cos 60 + 60g\cos 60 = 5N$

$\Rightarrow N = 38g$

- ELIMINATING R BETWEEN THE FIRST TWO EQUATIONS

$\Rightarrow \mu(100g - N\cos 60°) = N\sin 60$

$\Rightarrow \mu(100g - 38g\cos 60°) = 38g\sin 60$

$\Rightarrow \mu(100 - 38\cos 60°) = 38\sin 60$

$\Rightarrow 81\mu = 19\sqrt{3}$

$\Rightarrow \mu = \dfrac{19\sqrt{3}}{81}$ // $\simeq 0.406$

IYGB – MMS PAPER N – QUESTION 10

$$v = t^2 + kt + 3.2 \quad , \quad t \geqslant 0$$

a) THE MINIMUM MUST SATISFY $\frac{dv}{dt} = 0$

$\Longrightarrow 2t + k = 0$

$\Longrightarrow 2 \times 2.4 + k = 0$

$\Longrightarrow k = -4.8$

b) SOLVING $v = 0$

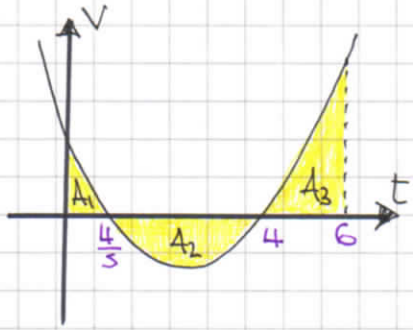
$\Longrightarrow 0 = t^2 - 4.8t + 3.2$

$\Longrightarrow 0 = 5t^2 - 24t + 16$

$\Longrightarrow 0 = (5t - 4)(t - 4)$

$\Longrightarrow t = \begin{cases} 4 \\ 4/5 \end{cases}$

c) SKETCHING THE VELOCITY TIME GRAPH



- $\int v \, dt = \int t^2 - 4.8t + 3.2 \, dt$

$= \frac{1}{3}t^3 - 2.4t^2 + 3.2t + C$

- $A_1 = \left[\frac{1}{3}t^3 - 2.4t^2 + 3.2t \right]_0^{0.8}$

$A_1 = \left(\frac{64}{375} - \frac{192}{125} + \frac{64}{25} \right) - (0)$

$A_1 = \frac{448}{375}$

IYGB - MMS PAPER N - QUESTION 10

- $A_2 = \left[\frac{1}{3}t^3 - 2.4t^2 + 3.2t\right]_{0.8}^{4}$

$A_2 = \left(\frac{64}{3} - \frac{192}{5} + \frac{64}{5}\right) - \left(\frac{448}{375}\right)$

$A_2 = -\frac{64}{15} - \frac{448}{375} = -\frac{2048}{375}$

- $A_3 = \left[\frac{1}{3}t^3 - 2.4t^2 + 3.2t\right]_{4}^{6}$

$A_3 = \left(72 - \frac{432}{5} + \frac{96}{5}\right) - \left(-\frac{64}{15}\right)$

$A_3 = \frac{136}{15}$

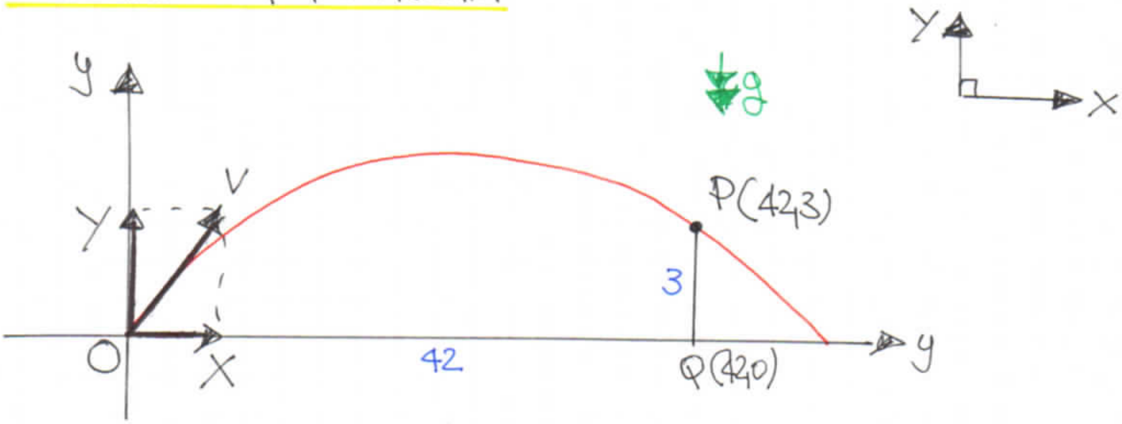
$\therefore$ TOTAL DISTANCE $= \frac{448}{375} + \frac{2048}{375} + \frac{136}{15}$

$= \frac{5896}{375}$

$\simeq 15.72\text{ m}$

IYGB - MMS PAPER N - QUESTION 11

STARTING WITH A DIAGRAM



- HORIZONTALLY

$42 = X \times 2.5$

$X = 16.8 \text{ ms}^{-1}$

- VERTICALLY $\left(s = ut + \frac{1}{2}at^2\right)$

$3 = Y(2.5) + \frac{1}{2}(-9.8)(2.5)^2$

$3 = 2.5Y - 30.625$

$2.5Y = 33.625$

$Y = 13.45 \text{ ms}^{-1}$

NOW LOOKING AT THE VELOCITIES AT P

- HORIZONTALLY

16.8 ms^{-1}

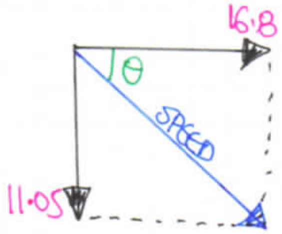
(UNCHANGED)

- VERTICALLY $(v = u + at)$

$V = 13.45 - 9.8 \times 2.5$

$V = -11.05 \text{ ms}^{-1}$

SPEED AND DIRECTION AT P, LOOKING AT THE DIAGRAM BELOW



- SPEED $= \sqrt{(11.05)^2 + (16.8)^2} \approx 20.11 \text{ ms}^{-1}$

- $\tan\theta = \dfrac{11.05}{16.8}$

$\theta \approx 33.3°$

BELOW THE HORIZONTAL

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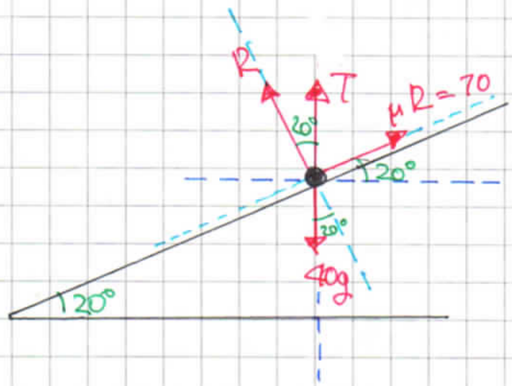
START WITH A DETAILED DIAGRAM



RESOLVING FORCES HORIZONTALLY, INSTEAD OF THE MORE STANDARD METHOD OF RESOLVING PARALLEL/PERPENDICULAR TO THE PLANE

$$R \sin 20 = \mu R \cos 20°$$
$$\sin 20 = \mu \cos 20$$
$$\mu = \tan 20°$$
$$\mu = 0.364$$

ALTERNATIVE

RESOLVING PARALLEL & PERPENDICULAR TO THE PLANE

$(\parallel)$: $\mu R + T\sin 20 = 40g \sin 20$
$$70 + T\sin 20 = 40g \sin 20$$
$$T\sin 20° = 40g\sin 20 - 70$$
$$T = 40g - \frac{70}{\sin 20°}$$

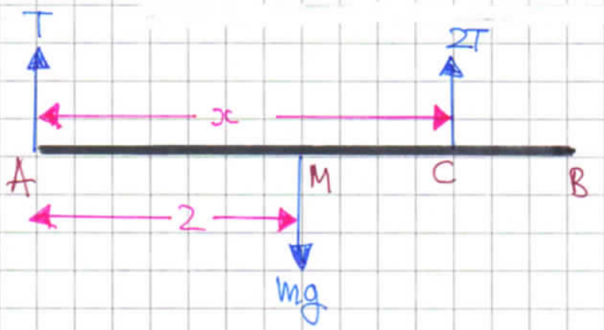
$(\perp)$: $R + T\cos 20 = 40g\cos 20°$
$$R = 40g\cos 20° - T\cos 20°$$
$$R = 40g\cos 20° - \left(40g - \frac{70}{\sin 20°}\right)\cos 20°$$
$$R = \cancel{40g\cos 20°} - \cancel{40g\cos 20°} + \frac{70\cos 20}{\sin 20°}$$

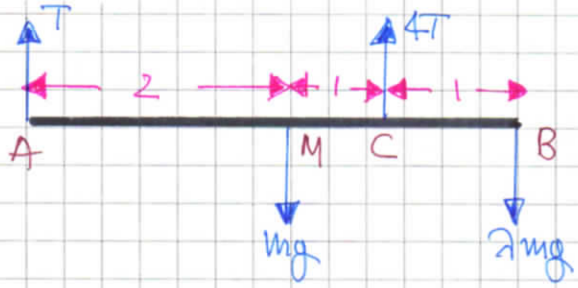
FINALLY $\mu R = 70$
$$\mu = \frac{70}{R}$$
$$\mu = \frac{70}{\frac{70\cos 20°}{\sin 20°}}$$
$$\mu = \frac{\sin 20°}{\cos 20°} = \tan 20° = 0.364$$

AS BEFORE

IYGB – MMS PAPER N – QUESTION 13

DRAW A DIAGRAM OF EACH OF THE TWO DESCRIBED SITUATIONS





TAKING MOMENTS ABOUT A IN EACH OF THE ABOVE SITUATIONS

$\overset{\curvearrowright}{A}$: $mg \times 2 = 2T \times x$

$\Rightarrow mg = Tx$

BUT $3T = mg$

$\Rightarrow 3T = Tx$

$\Rightarrow x = 3$

PUTTING $x=3$ INTO THE DIAGRAM

$\overset{\curvearrowright}{A}$: $mg \times 2 + \lambda mg \times 4 = 4T \times 3$

$\Rightarrow (2+4\lambda)mg = 12T$

BUT $5T = mg + \lambda mg$

$5T = (\lambda+1)mg$

$mg = \frac{5T}{\lambda+1}$

$\Rightarrow (2+4\lambda)\left(\frac{5T}{\lambda+1}\right) = 12T$

$\Rightarrow \frac{5(2+4\lambda)}{\lambda+1} = 12$

$\Rightarrow 10 + 20\lambda = 12\lambda + 12$

$\Rightarrow 8\lambda = 2$

$\Rightarrow \lambda = \frac{1}{4}$ //

IYGB - MMS PAPER N - QUESTION 14

a) VELOCITY OF A : $\underline{v}_A = 4\underline{i} - 7\underline{j}$

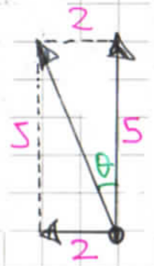
SPEED OF A : $|\underline{v}_A| = |4\underline{i} - 7\underline{j}|$

$= \sqrt{4^2 + (-7)^2}$

$= \sqrt{16 + 49}$

$= \sqrt{65} \approx 8.06 \text{ km h}^{-1}$

b) LOOKING AT THE DIAGRAM BELOW



$\tan\theta = \frac{2}{5}$

$\theta = 21.8°$

$\therefore$ BEARING $= 360 - 21.8°$

$= 338.2°$

c) USING $\underline{r} = \underline{r}_0 + \underline{v}t$

$\underline{r}_A = (-2\underline{i} + \underline{j}) + (4\underline{i} - 7\underline{j})t$

$\underline{r}_B = (10\underline{i} - 3\underline{j}) + (-2\underline{i} + 5\underline{j})t$

$\underline{r}_A = (4t-2)\underline{i} + (1-7t)\underline{j}$

$\underline{r}_B = (10-2t)\underline{i} + (5t-3)\underline{j}$

$\Rightarrow \underline{r}_A - \underline{r}_B = \left[(4t-2)\underline{i} + (1-7t)\underline{j}\right] - \left[(10-2t)\underline{i} + (5t-3)\underline{j}\right]$

$= (6t-12)\underline{i} + (4-12t)\underline{j}$

d) DISTANCE $= |\underline{r}_A - \underline{r}_B|$

$d = |(6t-12)\underline{i} + (4-12t)\underline{j}|$

$d = \sqrt{(6t-12)^2 + (4-12t)^2}$

$d = \sqrt{\begin{matrix} 36t^2 - 144t + 144 \\ 144t^2 - 96t + 16 \end{matrix}}$

$d = \sqrt{180t^2 - 240t + 160}$

$d^2 = 180t^2 - 240t + 160$

AS REQUIRED

IYGB - MMS PAPER N - QUESTION 14

e) WHEN $d = 10$

$$10^2 = 180t^2 - 240t + 160$$
$$100 = 180t^2 - 240t + 160$$
$$180t^2 - 240t + 60 = 0$$
$$3t^2 - 4t + 1 = 0$$
$$(3t-1)(t-1)$$
$$t = \begin{cases} 1 \\ \frac{1}{3} \end{cases}$$

I.E 1 HOUR LATER OR $\frac{1}{3} \times 60 = 20$ MINUTES LATER

f) LET $f(t) = d^2 = 180t^2 - 240t + 160$ (WHICH HAS A MIN)

$$f'(t) = 360t - 240$$

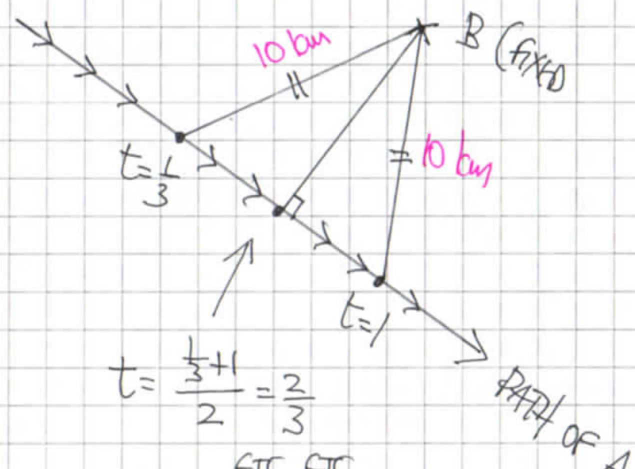
SOLVING FOR ZERO

$$360t - 240 = 0$$
$$360t = 240$$
$$t = \frac{2}{3}$$

USING $d^2 = 180t^2 - 240t + 160$

$$d = \sqrt{180 \times \left(\tfrac{2}{3}\right)^2 - 240\left(\tfrac{2}{3}\right) + 160}$$
$$d = \sqrt{80 - 160 + 160}$$
$$d = \sqrt{80}$$
$$d \simeq 8.94 \text{ km}$$

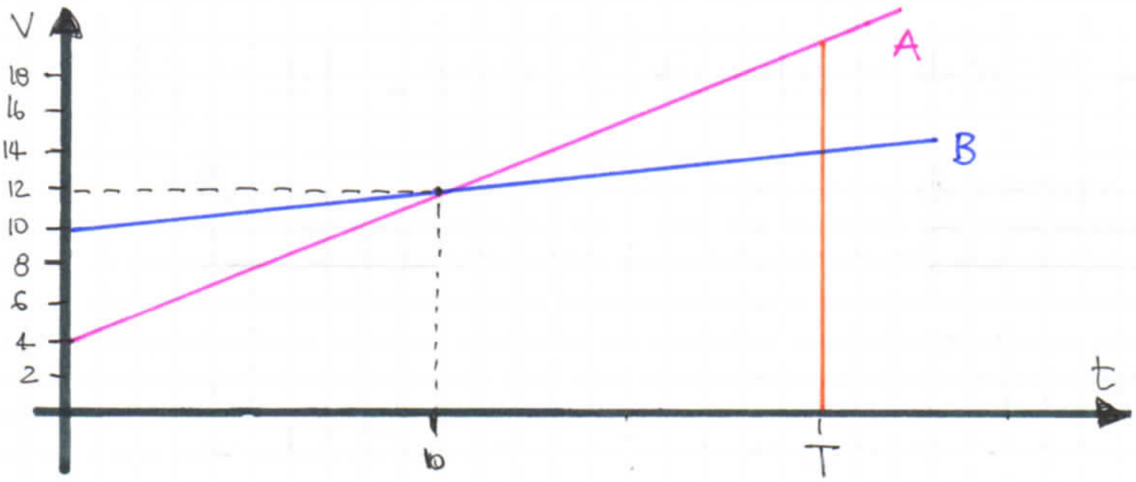
WE CAN ALSO USE GEOMETRICAL CONSIDERATIONS, AS FOLLOWS



$$t = \frac{\frac{1}{3}+1}{2} = \frac{2}{3}$$

ETC ETC

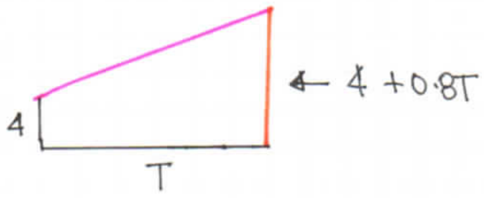
IYGB - MMS PAPER N - QUESTION 15



- "GRADIENT A" $= \frac{\Delta v}{\Delta t} = \frac{12-4}{10} = 0.8$ ← ACCELERATION OF A
- "GRADIENT B" $= \frac{\Delta v}{\Delta t} = \frac{12-10}{10} = 0.2$ ← ACCELERATION OF B

- "EQUATION OF A" : $V = 4 + 0.8t$
- "EQUATION OF B" : $V = 10 + 0.2t$

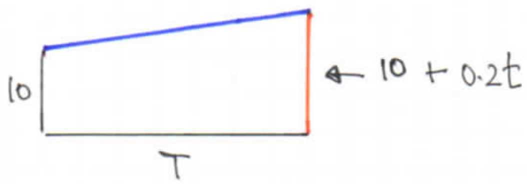
- NEXT SUPPOSE THAT $t=T$ THE "FRONT END" OF B IS 350 m BEHIND THE "BACK END" OF A



$\Rightarrow$ DISTANCE COVERED BY "A" IS GIVEN BY

$$\frac{4 + (4+0.8T)}{2} \times T$$

$$= \frac{(8+0.8T)T}{2} = 4T + 0.4T^2$$



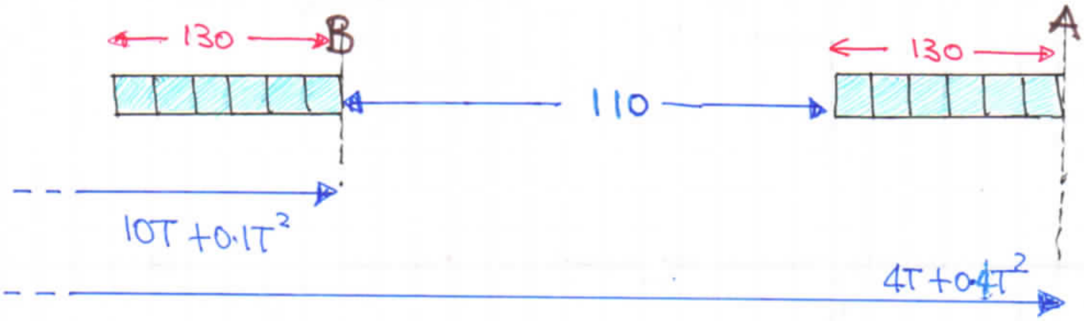
$\Rightarrow$ DISTANCE COVERED BY "B" IS GIVEN BY

$$\frac{10 + (10+0.2t)}{2} \times T = \frac{(20+0.2T)T}{2}$$

$$= 10T + 0.1T^2$$

IYGB - MMS PAPER N - QUESTION 15

- FINALLY WE NEED TO ALLOW FOR THE LENGTH OF THE TRAINS



$$\Longrightarrow (10T + 0.1T^2) + 110 + 130 = 4T + 0.4T^2$$

$$\Longrightarrow 10T + 0.1T^2 + 240 = 4T + 0.4T^2$$

$$\Longrightarrow 0 = 0.3T^2 - 6T - 240$$

$$\Longrightarrow 3T^2 - 60T - 2400 = 0$$

$$\Longrightarrow T^2 - 20T - 800 = 0$$

$$\Longrightarrow (T - 40)(T + 20) = 0$$

$$\Longrightarrow T = \begin{cases} \cancel{-20} \\ 40 \end{cases}$$