IYGB - MP1 PAPER B - QUESTION 1

a) $\sqrt{24} + \sqrt{6} = \sqrt{4}\sqrt{6} + \sqrt{6} = 2\sqrt{6} + \sqrt{6} = 3\sqrt{6}$

b) $(2+\sqrt{3})(4-\sqrt{12}) = 8 - 2\sqrt{12} + 4\sqrt{3} - \sqrt{3}\sqrt{12}$

$= 8 - 2\sqrt{4}\sqrt{3} + 4\sqrt{3} - \sqrt{36}$

$= 8 - 2\times 2\sqrt{3} + 4\sqrt{3} - 6$

$= 8 - \cancel{4\sqrt{3}} + \cancel{4\sqrt{3}} - 6$

$= 2$

IYGB – MP1 PAPER B – QUESTION 2

SOLVING THE RATIONAL INEQUALITY BY TRANSFORMING IT INTO A QUADRATIC

$$\Rightarrow \frac{4x+1}{x-1} > 3$$

$$\Rightarrow \frac{(4x+1)(x-1)}{(x-1)(x-1)} > 3$$

$$\Rightarrow \frac{(4x+1)(x-1)}{(x-1)^2} > 3$$

$$\Rightarrow (4x+1)(x-1) > 3(x-1)^2$$

$$\Rightarrow (4x+1)(x-1) - 3(x-1)^2 > 0$$

$$\Rightarrow (x-1)\big[(4x+1) - 3(x-1)\big] > 0$$

$$\Rightarrow (x-1)\big[4x+1-3x+3\big] > 0$$

$$\Rightarrow (x-1)(x+4) > 0$$

C.V. $\begin{cases} 1 \\ -4 \end{cases}$

$x < -4$ OR $x > 1$ //

IYGB − MP1 PAPER B − QUESTION 3

PROCEED AS FOLLOWS

$$f(n) = n^2+n+2 = n(n+1)+2$$

NOW $n(n+1)$ IS THE PRODUCT OF 2 CONSECUTIVE INTEGERS WHICH MUST BE EVEN, AS ONE OF THESE INTEGERS MUST BE EVEN

LET $n(n+1) = 2m$, m BEING AN INTEGER

$$\ldots = n(n+1)+2 = 2m+2 = 2(m+2)$$

INDEED EVEN

ALTERNATIVE ARGUMENTS BASED ON EXHAUSTION ARE ALSO VALID

IYGB - MP1 PAPER B - QUESTION 4

a) SOLVING BY FACTORIZATION & RECOGNIZING THAT IT IS A PERFECT SQUARE

$\Rightarrow f(x) = 0$

$\Rightarrow 4x^2 + 20x + 25 = 0$

$\Rightarrow (2x+5)^2$

$\Rightarrow x = -\frac{5}{2}$

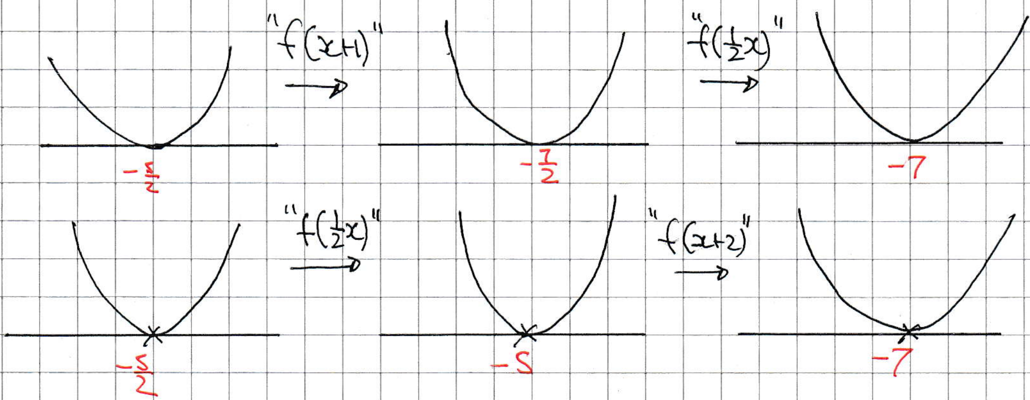
b) $f\left(\frac{1}{2}x + 1\right)$ REPRESENTS

- EITHER

TRANSLATION TO THE LEFT BY 1 UNIT, FOLLOWED BY A HORIZONTAL STRETCH BY SCALE FACTOR 2 (FIRST 3 FIGURES)

- OR

HORIZONTAL STRETCH BY SCALE FACTOR 2, FOLLOWED BY TRANSLATION TO THE LEFT BY 2 UNITS (LAST 3 FIGURES)



$\therefore$ REQUIRED SOLUTION IS $x = -7$

IYGB – MP1 PAPER B – QUESTION 5

a) $$y = 6x^3 + Ax^2 - 6x + B, \quad x \in \mathbb{R}$$

USING GIVEN INFORMATION (WRITE $y = f(x)$)

• $(x-5)$ IS A FACTOR

$\Rightarrow f(5) = 0$

$\Rightarrow 6(5)^3 + A(5)^2 - 6(5) + B = 0$

$\Rightarrow 750 + 25A - 30 + B = 0$

$\Rightarrow 25A + B = -720$

• $(x-1)$ GIVES REMAINDER -24

$f(1) = -24$

$\cancel{6} + A - \cancel{6} + B = -24$

$A + B = -24$

$B = -24 - A$

$25A + (-18 - A) = -720$

$24A = -696$

$A = -29$

$B = -24 - (-29)$

$B = 5$

b) BY LONG DIVISION

$$\begin{array}{r|l} & 6x^2 + x - 1 \\ \hline x-5 & 6x^3 - 29x^2 - 6x + 5 \\ & -6x^3 + 30x^2 \\ \hline & \qquad\quad x^2 - 6x + 5 \\ & \qquad -x^2 + 5x \\ \hline & \qquad\qquad -x + 5 \\ & \qquad\qquad +x - 5 \\ \hline & \qquad\qquad\quad 0 \end{array}$$

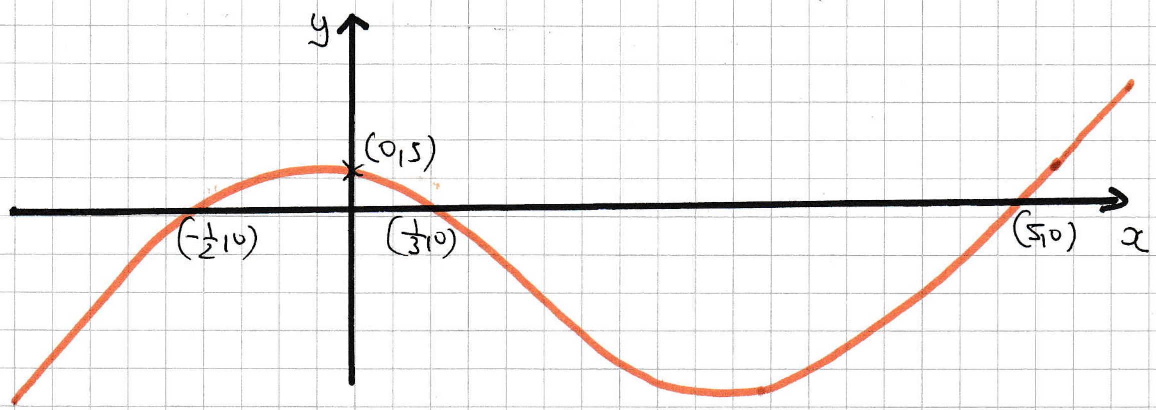
IYGB - MP1 PAPER B - QUESTION 5

$$\therefore \quad y = (x-5)(6x^2+x-1)$$

$$y = (x-5)(3x-1)(2x+1)$$

c) COLLECTING INFORMATION

- $+x^3 \Longrightarrow \sim$
- $x=0 \Longrightarrow y=5$ IE $(0,5)$
- $y=0 \Longrightarrow x = \begin{cases} -\frac{1}{2} \\ \frac{1}{3} \\ 5 \end{cases}$ IE $(-\frac{1}{2},0)$, $(\frac{1}{3},0)$, $(5,0)$



IYGB – MP1 PAPER B – QUESTION 6

EXPAND & DIFFERENTIATE

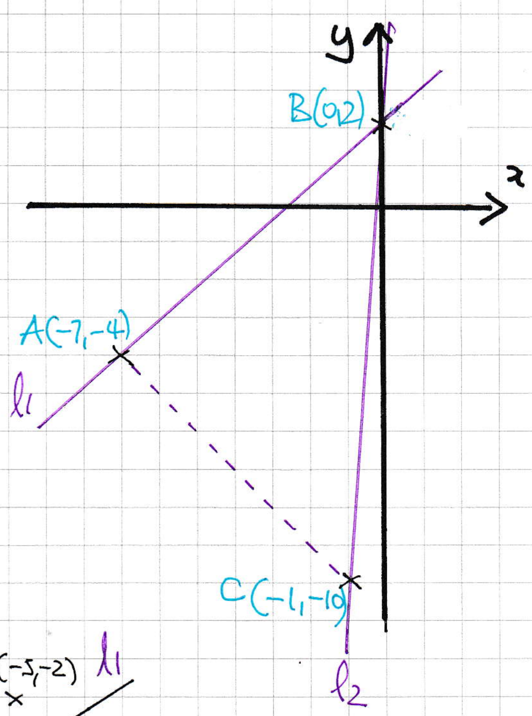
$$\Rightarrow y = 2(x+a)^2$$
$$\Rightarrow y = 2(x^2+2ax+a^2)$$
$$\Rightarrow y = 2x^2+4ax+2a^2$$
$$\Rightarrow \frac{dy}{dx} = 4x+4a$$

BY DIRECT COMPARISON

$$\Rightarrow 4a = 10$$
$$\Rightarrow a = \frac{5}{2}$$

IYGB – MP1 PAPER B – QUESTION 7

a) START WITH A DIAGRAM



DETERMINE THE EQUATION OF l_1

- GRAD AB $= \frac{2-(-4)}{0-(-7)} = \frac{6}{7}$
- AS THE LINE PASSES THROUGH (0,2)

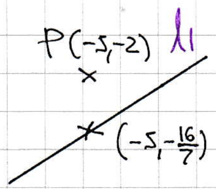
$$l_1: \ y = \frac{6}{7}x + 2$$

NOW CHECK THE GIVEN POINT $P(-5,-2)$

- IF $x = -5$, $y = \frac{6}{7}(-5) + 2$

$$y = -\frac{30}{7} + 2$$

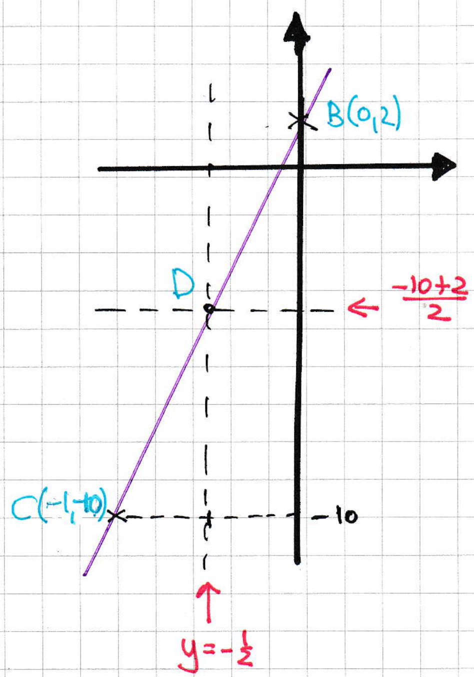
$$y = -\frac{16}{7} < -2$$



AS THE LINE IS LOWER, P IS ABOVE l_1

b) EVIDENTLY $Q(-\frac{1}{2},-5)$ WILL BE INSIDE THE TRIANGLE $\overset{\Delta}{ABC}$ IF IT LIES TO THE "LEFT" OF l_2

WITHOUT WORKING THE EQUATION OF l_2, AND BY CONSIDERING SIMILAR TRIANGLES $D(-\frac{1}{2},-4)$



$\therefore Q(-\frac{1}{2},-5)$ IS OUTSIDE $\overset{\Delta}{ABC}$

IYGB - MP1 PAPER B - QUESTION 8

START BY "LINEARIZING" THE EQUATION

$$\Longrightarrow P = at^b$$
$$\Longrightarrow \log_{10}P = \log_{10}(at^b)$$
$$\Longrightarrow \log_{10}P = \log_{10}a + \log_{10}t^b$$
$$\Longrightarrow \log_{10}P = \log_{10}a + b\log_{10}t$$
$$\Longrightarrow \underbrace{\log_{10}P}_{Y} = \underbrace{b}_{m}(\underbrace{\log_{10}t}_{X}) + \underbrace{\log_{10}a}_{C}$$

FROM THE ~~ABOVE~~ EQUATION IT IS EVIDENT THAT WE NEED TO PLOT

$X = \log_{10}t$ AGAINST $Y = \log_{10}P$

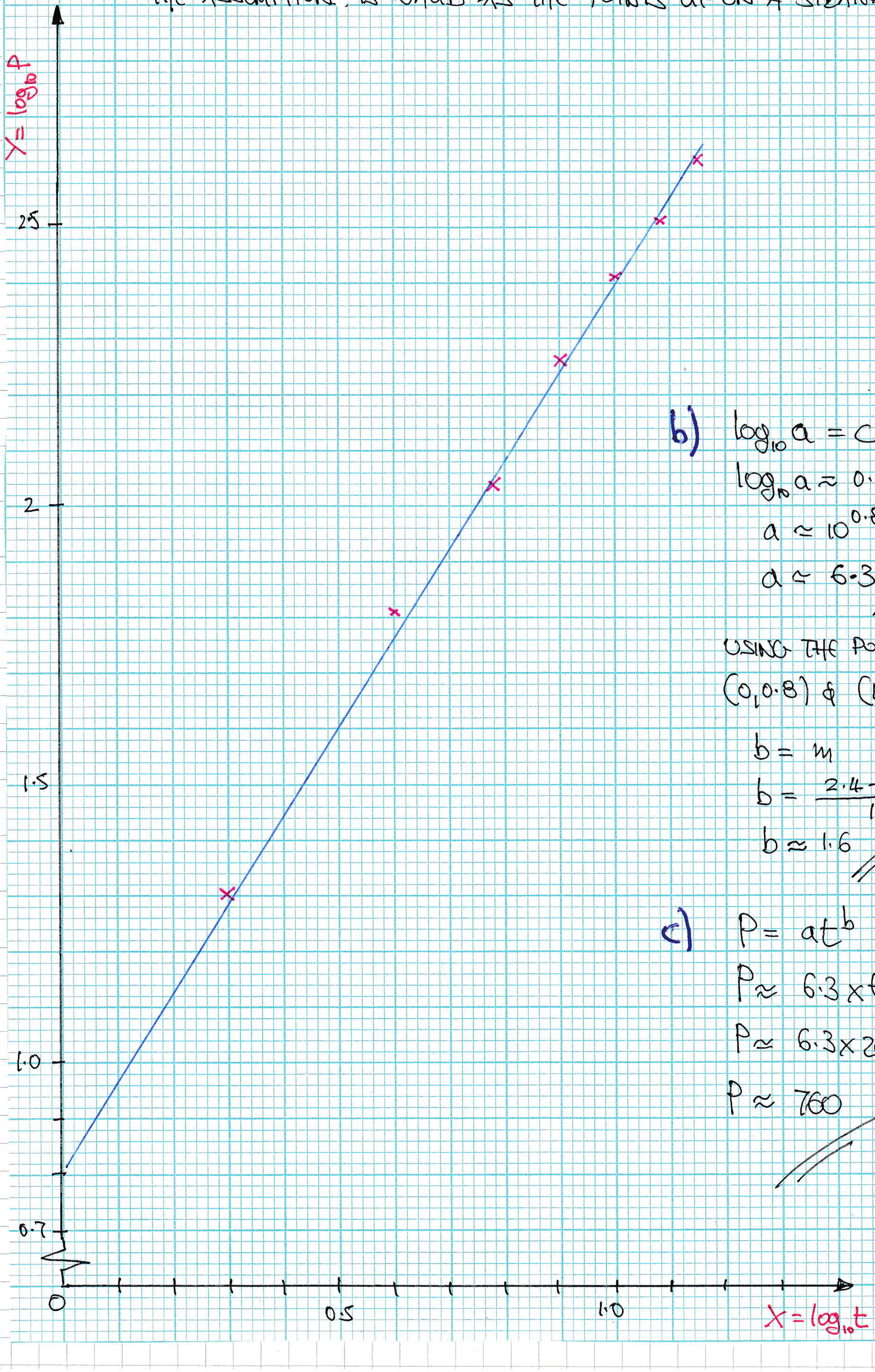
$X = \log_{10}t$	0.30	0.60	0.78	0.90	1.00	1.08	1.15
$Y = \log_{10}P$	1.30	1.81	2.04	2.26	2.41	2.51	2.62

P.T.O

IYGB - MP1 PAPER B - QUESTION 8

PLOTTING THE POINTS

THE ASSUMPTION IS VALID AS THE POINTS LIE ON A STRAIGHT LINE



b) $\log_{10} a = c$

$\log_{10} a \approx 0.8$

$a \approx 10^{0.8}$

$a \approx 6.3$

USING THE POINTS $(0, 0.8)$ & $(1, 2.4)$

$b = m$

$b = \dfrac{2.4 - 0.8}{1}$

$b \approx 1.6$

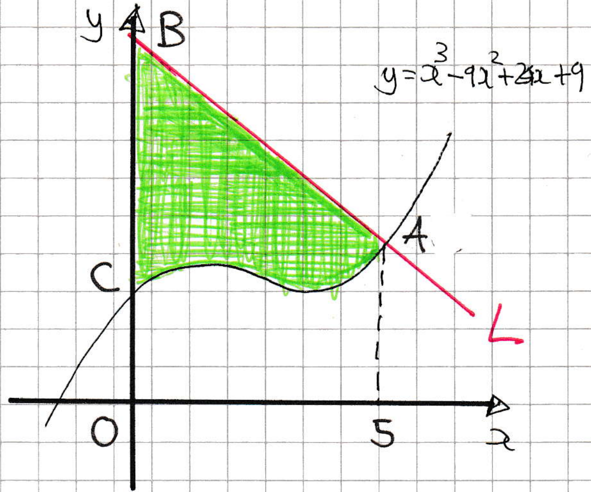
c) $P = at^b$

$P \approx 6.3 \times t^{1.6}$

$P \approx 6.3 \times 20^{1.6}$

$P \approx 760$

IYGB – MP1 PAPER B – QUESTION 9



a) $y = x^3 - 9x^2 + 24x + 9$

$\frac{dy}{dx} = 3x^2 - 18x + 24$

$\left.\frac{dy}{dx}\right|_{x=5} = 3\times5^2 - 18\times5 + 24$

$\left.\frac{dy}{dx}\right|_{A} = 9$

$\therefore$ NORMAL GRADIENT IS $-\frac{1}{9}$

WHEN $x=5$ $\quad y = 5^3 - 9\times5^2 + 24\times5 + 9 = 29$

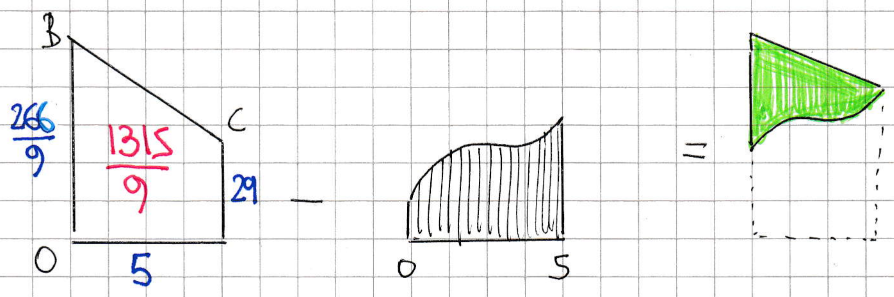
I.E $A(5,29)$

$\therefore$ EQUATION OF THE NORMAL AT A

$$y - y_0 = m(x - x_0)$$
$$y - 29 = -\frac{1}{9}(x-5)$$
$$9y - 261 = -x + 5$$
$$9y + x = 266$$

AS REQUIRED

b)



$$\boxed{A = \frac{\frac{266}{9} + 29}{2} \times 5}$$

$$\int_0^5 x^3 - 9x^2 + 24x + 9 \, dx$$

IYGB – MP1 PAPER B – QUESTION 9

$$= \left[\frac{1}{4}x^4 - 3x^3 + 12x^2 + 9x \right]_0^5$$

$$= \left(\frac{625}{4} - 375 + 300 + 45 \right) - 0$$

$$= \frac{505}{4}$$

THE REQUIRED AREA IS

$$\frac{2635}{18} - \frac{505}{4} = \frac{725}{36} \approx 20.14$$

IE APPROX 20

IYGB - MP1 PAPER B - QUESTION 10

$$\boxed{y = x^3 - 3x^2 + 3x + 5}$$

DIFFERENTIATE & SET TO ZERO

$$\Rightarrow \frac{dy}{dx} = 3x^2 - 6x + 3$$

$$\Rightarrow 0 = 3x^2 - 6x + 3$$

$$\Rightarrow 0 = x^2 - 2x + 1$$

$$\Rightarrow 0 = (x-1)^2$$

AT $x = 1$ IS THE ONLY STATIONARY POINT. $(y = 6)$

DETERMINING THE NATURE

$$\frac{d^2y}{dx^2} = 6x - 6$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=1} = 6 \times 1 - 6 = 0 \quad \longleftarrow \text{ EITHER A POINT OF INFLEXION OR THE TEST FAILS}$$

PROCEED WITH $\frac{d^3y}{dx^3}$

$$\frac{d^3y}{dx^3} = 6$$

$$\left.\frac{d^3y}{dx^3}\right|_{x=1} = 6 \neq 0 \qquad \therefore \text{ A POINT OF INFLEXION}$$

IYGB – MP1 PAPER B – QUESTION 11

a) COMPLETING THE SQUARE IN x & y

$$x^2+y^2+4x-10y+9=0$$
$$x^2+4x+y^2-10y+9=0$$
$$(x+2)^2-4+(y-5)^2-25+9=0$$
$$(x+2)^2+(y-5)^2=20$$

$\therefore$ CENTRE AT $(-2,5)$

RADIUS $\sqrt{20}=2\sqrt{5}$

b) THE EQUATION OF THE LINE MUST BE $y=mx-1$

$$\left.\begin{array}{l} y=mx-1 \\ (x+2)^2+(y-5)^2=20 \end{array}\right\} \Rightarrow (x+2)^2+(mx-1-5)^2=20$$
$$\Rightarrow (x+2)^2+(mx-6)^2=20$$
$$\Rightarrow x^2+4x+4+m^2x^2-12mx+36=20$$
$$\Rightarrow (1+m^2)x^2+(4-12m)x+20=0$$

IF A TANGENT THIS EQUATION MUST HAVE REPEATED ROOTS

$$b^2-4ac=0 \Rightarrow (4-12m)^2-4(1+m^2)(+20)=0$$
$$\Rightarrow 4^2(1-3m)^2-80(1+m^2)=0$$
$$\Rightarrow (1-3m)^2-5(1+m^2)=0 \quad \div 16$$
$$\Rightarrow 1-6m+9m^2-5-5m^2=0$$
$$\Rightarrow 4m^2-6m-4=0$$
$$\Rightarrow 2m^2-3m-2=0$$

AS REQUIRED

c) SOLVING THE EQUATION IN m

$$\Rightarrow (2m+1)(m-2)=0$$
$$\Rightarrow m = \begin{cases} 2 \\ -\frac{1}{2} \end{cases}$$

IYGB - MP1 PAPER B - QUESTION 11

BUT IT IS GIVEN THAT $m<0 \Rightarrow m=-\frac{1}{2}$

$$\Rightarrow (1+m^2)x^2+(4-12m)x+20=0$$
$$\Rightarrow \left(1+\frac{1}{4}\right)x^2+\left[4-12\times\left(-\frac{1}{2}\right)\right]x+20=0$$
$$\Rightarrow \frac{5}{4}x^2+10x+20=0$$
$$\Rightarrow 5x^2+40x+80=0$$
$$\Rightarrow x^2+8x+16=0$$
$$\Rightarrow (x+4)^2=0$$
$$\Rightarrow x=-4$$

& USING $y=-\frac{1}{2}x-1$

$y=2-1$

$y=1$

$\therefore P(-4,1)$

IYGB - MP1 PAPER B - QUESTION 12

$$\tan^4 y = 6 + \tan^2 y \qquad 0 \le y < 360°$$

THIS IS A QUADRATIC IN $\tan^2 y$ SO WE PROCEED BY FACTORIZATION

$$\Rightarrow \tan^4 y - \tan^2 y - 6 = 0$$

$$\Rightarrow (\tan^2 y + 2)(\tan^2 y - 3) = 0$$

$$\Rightarrow \tan^2 y = \begin{cases} \cancel{-2} \\ 3 \end{cases}$$

$$\Rightarrow \tan y = \begin{cases} \sqrt{3} \\ -\sqrt{3} \end{cases}$$

SOLVING EACH OF THESE SEPARATELY

- $\tan y = \sqrt{3}$

  $\arctan\sqrt{3} = 60°$

  $y = 60° \pm 180n$

  $n = 0, 1, 2, 3, \ldots$

- $\tan y = -\sqrt{3}$

  $\arctan(-\sqrt{3}) = -60°$

  $y = -60° \pm 180n$

  $n = 0, 1, 2, 3, \ldots$

COLLECTING THE SOLUTIONS TOGETHER

$$\Rightarrow y = 60°, 240°, 120°, 300°$$

$$\Rightarrow y = 60°, 120°, 240°, 300°$$

IYGB – MP1 PAPER B – QUESTION 13

a) PROCEED AS FOLLOWS TO SOLVE $f(x)=0$

$$\Rightarrow e^{x}+10e^{-x}-7=0$$

$$\Rightarrow e^{x}+\frac{10}{e^{x}}-7=0$$

$$\Rightarrow E+\frac{10}{E}-7=0 \quad , \text{ WHERE } E=e^{x}$$

$$\Rightarrow E^{2}+10-7E=0$$

$$\Rightarrow E^{2}-7E+10=0$$

$$\Rightarrow (E-2)(E-5)=0$$

$$\Rightarrow E=e^{x}=\begin{cases}2\\5\end{cases}$$

$\therefore \ x=\ln 2$ OR $x=\ln 5$

b) REWRITE THE EQUATION AS FOLLOWS

$$\Rightarrow e^{2x-2}-7e^{x-1}+10=0$$

$$\Rightarrow e^{2(x-1)}-7e^{(x-1)}+10=0$$

COMPARE THIS EQUATION WITH

$$e^{2x}-7e^{x}+10=0 \quad \left[E^{2}-7E+10=0\right]$$

$$\Rightarrow x-1=\begin{cases}\ln 2\\ \ln 5\end{cases}$$

$$\Rightarrow x=\begin{cases}1+\ln 2\\ 1+\ln 5\end{cases}$$