IYGB - MP1 PAPER E - QUESTION 1

a) SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{l} y = (x+1)^2 \\ y = 4x+9 \end{array}\right\} \Rightarrow (x+1)^2 = 4x+9$$

$$\Rightarrow x^2+2x+1 = 4x+9$$

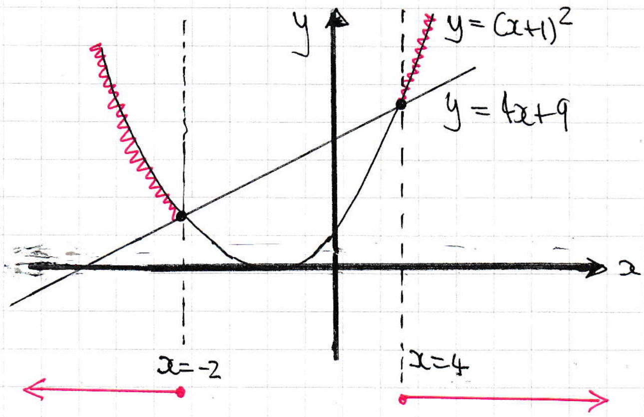
$$\Rightarrow x^2-2x-8=0$$

$$\Rightarrow (x-4)(x+2)=0$$

$$\Rightarrow x = \begin{cases} 4 \\ -2 \end{cases} \qquad y = \begin{cases} (4+1)^2 = 25 \\ (-2+1)^2 = 1 \end{cases}$$

$\therefore (-2,1)$ & $(4,25)$

b) LOOKING AT THE DIAGRAM



$\therefore x \leqslant -2$ OR $x \geqslant 4$

IYGB - MP1 PAPER E - QUESTION 2

START BY OBTAINING THE PARTICULARS OF THE GIVEN CIRCLE

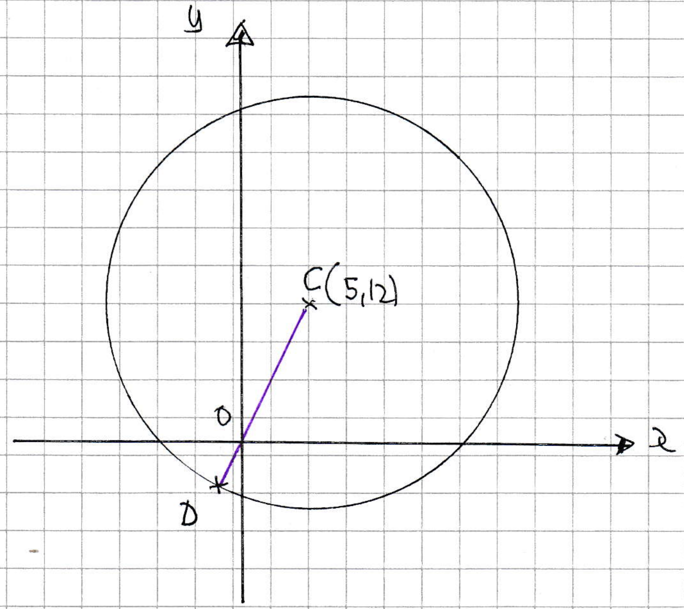
$$\Rightarrow x^2 - 10x + y^2 - 24y = 231$$

$$\Rightarrow (x-5)^2 - 25 + (y-12)^2 - 144 = 231$$

$$\Rightarrow (x-5)^2 + (y-12)^2 = 400$$

CENTRE AT $(5,12)$ AND RADIUS 20

NEXT LOOKING AT THE DIAGRAM



THE REQUIRED RADIUS R MUST BE AT MOST $|OD|$

$$R \leqslant |OD|$$

$$R \leqslant |CD| - |OC|$$

$$R \leqslant 20 - \sqrt{5^2 + 12^2}$$

$$R \leqslant 20 - \sqrt{169}$$

$$R \leqslant 20 - 13$$

$$R \leqslant 7$$

IYGB - MP1 PAPER E - QUESTION 3

a) BY THE FACTOR THEOREM

$$f(x) = x^3 - 9x^2 + 13x + 2$$

$$f(2) = 2^3 - 9\times2^2 + 13\times2 + 2$$

$$f(2) = 8 - 36 + 26 + 2 = 0 \qquad \therefore (x-2) \text{ IS A FACTOR}$$

BY ALGEBRAIC DIVISION OR MANIPULATION

$$f(x) = x^3 - 9x^2 + 13x + 2 = x^2(x-2) - 7x(x-2) - (x-2)$$

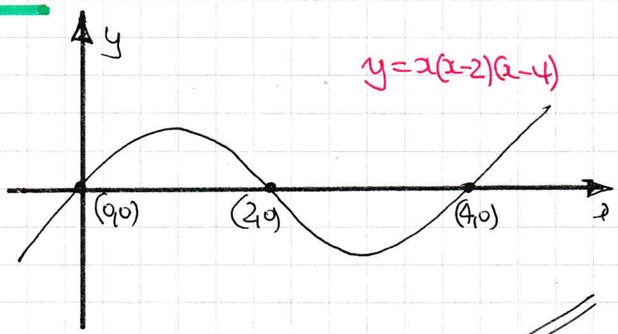
$$= (x-2)(x^2 - 7x - 1)$$

b) COLLECTING ALL THE INFORMATION

$+x^3 \Rightarrow$

$x=0 \Rightarrow y=0 \quad (0,0)$

$y=0 \Rightarrow x = 0, 2, 4$



c) SOLVING SIMULTANEOUSLY $f(x) = g(x)$

$$x(x-2)(x-4) = (x-2)(x^2 - 7x - 1)$$

$x-2$ CAN BE DIVIDED NOTING $x=2$ IS A SOLUTION

I.E $(2,0)$

$$\Rightarrow x(x-4) = x^2 - 7x - 1$$

$$\Rightarrow \cancel{x^2} - 4x = \cancel{x^2} - 7x - 1$$

$$\Rightarrow 3x = -1$$

$$\Rightarrow x = -\tfrac{1}{3}$$

$$\Rightarrow y = -\tfrac{1}{3}\left(-\tfrac{1}{3} - 2\right)\left(-\tfrac{1}{3} - 4\right)$$

$$y = -\tfrac{1}{3}\left(-\tfrac{7}{3}\right)\left(-\tfrac{13}{3}\right)$$

$$y = -\tfrac{91}{27}$$

$$\therefore (2,0) \; \& \; \left(-\tfrac{1}{3}, -\tfrac{91}{27}\right)$$

IYGB — MP1 PAPER E — QUESTION 4

a)

b) ONE APPROACH COULD BE

$$y = 3\sqrt{2x} = 3\sqrt{2}\sqrt{x} = \frac{3\sqrt{2}}{2} \times \left(2\sqrt{x}\right)$$

$\therefore$ AN ENLARGEMENT, PARALLEL TO THE y AXIS, WITH SCALE FACTOR $\frac{3}{2}\sqrt{2}$

c) A DIFFERENT APPROACH

$$y = 3\sqrt{2x} = \frac{3}{2} \times 2 \times \sqrt{2x} = 2 \times \sqrt{\frac{9}{4}} \times \sqrt{2x} = 2\sqrt{\frac{9}{4} \times 2x}$$

$$= 2\sqrt{\frac{9}{2}x}$$

$\therefore$ AN ENLARGEMENT PARALLEL TO THE x AXIS, BY A SCALE FACTOR OF $\frac{2}{9}$

IYGB - MP1 PAPER E - QUESTION 5

$$\sin(3\theta + 72°) = \cos 48° \qquad 0 < \theta < 180°$$

SOLVING THE EQUATION

$$\Rightarrow \sin(3\theta + 72°) = \cos 48°$$
$$\Rightarrow \sin(3\theta + 72°) = \sin(90° - 48°)$$
$$\Rightarrow \sin(3\theta + 72°) = \sin 42°$$

$$\Rightarrow \left(\begin{array}{l} 3\theta + 72° = 42° \pm 360n \\ 3\theta + 72° = \underset{(180° - 42°)}{138°} \pm 360n \end{array} \right. \qquad n = 0, 1, 2, 3 \ldots$$

$$\Rightarrow \left(\begin{array}{l} 3\theta = -30° \pm 360n \\ 3\theta = 66° \pm 360n \end{array} \right.$$

$$\Rightarrow \left(\begin{array}{l} \theta = -10° \pm 120°n \\ \theta = 22° \pm 120n \end{array} \right.$$

- $\theta_1 = 110°$
- $\theta_2 = 22°$
- $\theta_3 = 142°$

OR

$$\sin(3\theta + 72°) = \sin 42$$
$$\sin(3\theta + 72°) = 0.6691\ldots$$
$$\arcsin(0.6691\ldots) = 42°!$$

ETC ETC

IYGB – MP1 PAPER E – QUESTION 6

a) USING STANDARD EXPANSION FORMULAE

- $f(x) = (1-2x)^6 = 1 + \frac{6}{1}(-2x)^1 + \frac{6\times5}{1\times2}(-2x)^2 + \frac{6\times5\times4}{1\times2\times3}(-2x)^3 + \dots$

$$= 1 - 12x + 60x^2 - 160x^3 + \dots$$

- $g(x) = (2+x)^7 = \binom{7}{0}(2)^7(x)^0 + \binom{7}{1}(2)^6(x)^1 + \binom{7}{2}(2)^5(x)^2 + \binom{7}{3}(2)^4(x)^3 + \dots$

$$= (1\times128\times1) + (7\times64\times x) + (21\times32\times x^2) + (35\times16\times x^3) + \dots$$

$$= 128 + 448x + 672x^2 + 560x^3 + \dots$$

b) WORK AS FOLLOWS

$h(x) = f(x)\,g(x) = (1 - 12x + 60x^2 + \dots)(128 + 448x + 672x^2 + \dots)$

$7680x^2$

$-5376x^2$

$672x^2$

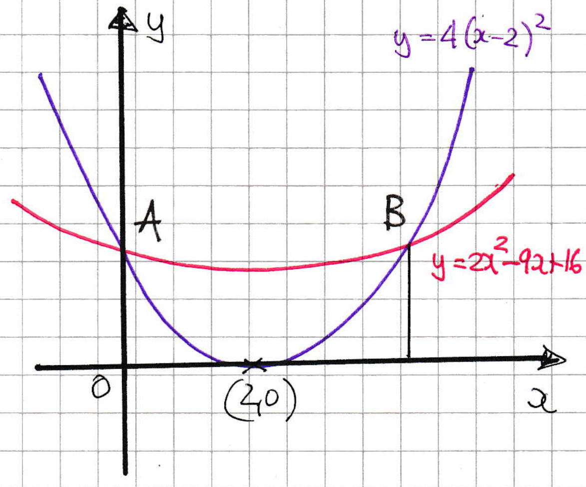
$\therefore$ REQUIRED COEFFICIENT OF x^2 IS

$$7680 - 5376 + 672 = 2976$$

IYGB – MP1 PAPER E – QUESTION 7

a) SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{l} y = 4(x-2)^2 \\ y = 2x^2 - 9x + 16 \end{array}\right\} \Rightarrow$$

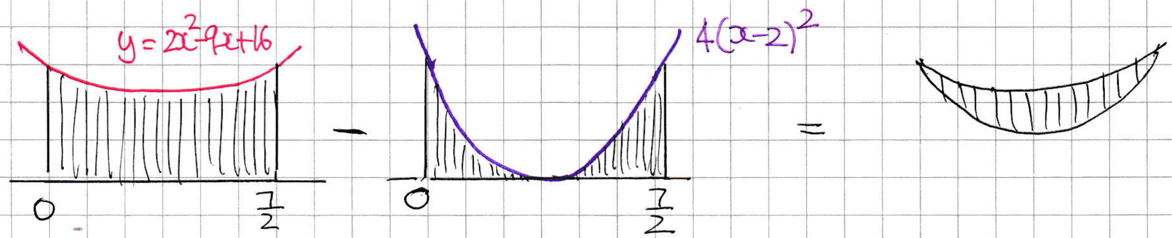


$$\Rightarrow 4(x-2)^2 = 2x^2 - 9x + 16$$
$$\Rightarrow 4(x^2 - 4x + 4) = 2x^2 - 9x + 16$$
$$\Rightarrow 4x^2 - 16x \cancel{+16} = 2x^2 - 9x \cancel{+16}$$
$$\Rightarrow 2x^2 - 7x = 0$$
$$\Rightarrow x(2x-7) = 0$$

$$x = \left\langle \begin{array}{l} 0 \\ \frac{7}{2} \end{array} \right. \qquad y = \left\langle \begin{array}{l} 16 \\ 9 \end{array} \right.$$

$\therefore A(0,16)$ & $B(\frac{7}{2}, 9)$

b)



LOOKING AT THE ABOVE DIAGRAM

REQUIRED AREA $= \displaystyle\int_0^{\frac{7}{2}} 2x^2 - 9x + 16 \, dx - \int_0^{\frac{7}{2}} 4(x-2)^2 \, dx$

$$= \int_0^{\frac{7}{2}} 2x^2 - 9x + 16 \, dx - \int_0^{\frac{7}{2}} 4x^2 - 16x + 16 \, dx$$

COMBINING INTEGRALS

$$\dots = \int_0^{\frac{7}{2}} (2x^2 - 9x + 16) - (4x^2 - 16x + 16) \, dx$$

IYGB - MP1 PAPER E - QUESTION 7

$$= \int_0^{\frac{7}{2}} -2x^2 + 7x \, dx$$

$$= \left[-\frac{2}{3}x^3 + \frac{7}{2}x^2 \right]_0^{\frac{7}{2}}$$

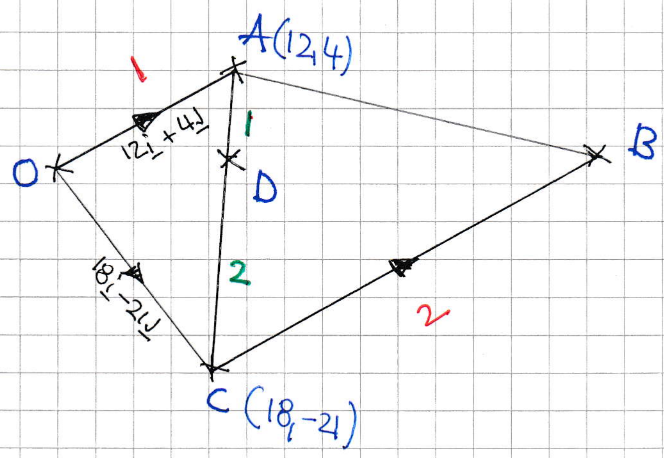
$$= \left[-\frac{2}{3} \times \left(\frac{7}{2}\right)^3 + \frac{7}{2}\left(\frac{7}{2}\right)^2 \right] - \left[0 \right]$$

$$= -\frac{343}{12} + \frac{343}{8}$$

$$= \frac{343}{24}$$

IYGB - MP1 PAPER E - QUESTION 8

a) LOOKING AT THE DIAGRAM



- $\vec{AC} = \vec{AO} + \vec{OC}$
$= -(12\underline{i}+4\underline{j}) + (18\underline{i}-21\underline{j})$
$= 6\underline{i} - 25\underline{j}$

- $\vec{AD} = \frac{1}{3}\vec{AC} = \frac{1}{3}(6\underline{i}-25\underline{j})$
$= 2\underline{i} - \frac{25}{3}\underline{j}$

- $\vec{OB} = \vec{OA} + \vec{AD}$
$= (12\underline{i}+4\underline{j}) + (2\underline{i} - \frac{25}{3}\underline{j})$
$= 14\underline{i} - \frac{13}{3}\underline{j}$

$\vec{OA} = 12\underline{i} + 4\underline{j}$
$\vec{OC} = 18\underline{i} - 21\underline{j}$

b) WE NEED TO VECTOR $\vec{DB}$ TO COMPARE IT WITH $\vec{OD}$

- $\vec{CB} = 2\vec{OA} = 2(12\underline{i}+4\underline{j}) = 24\underline{i}+8\underline{j}$

- $\vec{DC} = \frac{2}{3}\vec{AC} = \frac{2}{3}(6\underline{i}-25\underline{j}) = 4\underline{i} - \frac{50}{3}\underline{j}$

- $\vec{DB} = \vec{DC} + \vec{CB} = (4\underline{i} - \frac{50}{3}\underline{j}) + (24\underline{i}+8\underline{j}) = 28\underline{i} - \frac{26}{3}\underline{j}$

HENCE WE HAVE

$\vec{OD} = 14\underline{i} - \frac{13}{3}\underline{j} = \frac{1}{3}(42\underline{i} - 13\underline{j})$

$\vec{DB} = 28\underline{i} - \frac{26}{3}\underline{j} = \frac{2}{3}(42\underline{i} - 13\underline{j})$

AS BOTH $\vec{OD}$ & $\vec{DB}$ ARE IN THE SAME DIRECTION AND SHARE A POINT, IMPLIES THAT O, D & B ARE COLLINEAR WITH $|\vec{OD}| : |\vec{DB}| = 1:2$

+ −

IYGB − MP1 PAPER E − QUESTION 9

a) TAKES LOGS BASE 10 ON BOTH SIDES

$$\Rightarrow 4 \times 3^{x+2} = 3 \times 4^x$$
$$\Rightarrow \log(4 \times 3^{x+2}) = \log(3 \times 4^x)$$
$$\Rightarrow \log 4 + \log 3^{x+2} = \log 3 + \log 4^x$$
$$\Rightarrow \log 4 + (x+2)\log 3 = \log 3 + x\log 4$$
$$\Rightarrow \log 4 + x\log 3 + 2\log 3 = \log 3 + x\log 4$$
$$\Rightarrow \log 4 + 2\log 3 - \log 3 = x\log 4 - x\log 3$$
$$\Rightarrow \log 4 + \log 3 = x(\log 4 - \log 3)$$
$$\Rightarrow x = \frac{\log 4 + \log 3}{\log 4 - \log 3} = \frac{\log 12}{\log 4/3} \simeq 8.64$$

ALTERNATIVE METHOD

$$\Rightarrow 4 \times 3^{x+2} = 3 \times 4^x$$
$$\Rightarrow 4 \times 3^x \times 3^2 = 3 \times 4^x$$
$$\Rightarrow 36 \times 3^x = 3 \times 4^x$$
$$\Rightarrow 12 = \frac{4^x}{3^x}$$
$$\Rightarrow \left(\frac{4}{3}\right)^x = 12$$

TAKING LOGS, SAY BASE 10

$$\Rightarrow \log\left(\frac{4}{3}\right)^x = \log 12$$
$$\Rightarrow x\log\frac{4}{3} = \log 12$$
$$\Rightarrow x = \frac{\log 12}{\log 4/3}$$
$$\Rightarrow x \simeq 8.64$$

IYGB - MP1 PAPER E - QUESTION 9

b) "EXTRACT" THE LOGS AS FOLLOWS

$$\Rightarrow \log_a(1+\sqrt{x}) = \tfrac{1}{2}\log_a(9+\sqrt{16x})$$

$$\Rightarrow 2\log_a(1+\sqrt{x}) = \log_a(9+\sqrt{16x})$$

$$\Rightarrow \log_a(1+\sqrt{x})^2 = \log_a(9+\sqrt{16x})$$

$$\Rightarrow (1+\sqrt{x})^2 = 9+\sqrt{16x}$$

$$\Rightarrow 1+2\sqrt{x}+x = 9+4\sqrt{x}$$

$$\Rightarrow x-2\sqrt{x}-8=0$$

$$\Rightarrow (\sqrt{x}-4)(\sqrt{x}+2)=0$$

$$\Rightarrow \sqrt{x} = \begin{cases} 4 \\ \cancel{-2} \end{cases}$$

$$\Rightarrow x = 16$$

IYGB – MP1 PAPER E – QUESTION 10

a) STANDARD EQUATION READS

CENTRE $(2,-2)$ & RADIUS $=\sqrt{20}$

b) FIND THE INTERCEPTS

- $x=0$

$(y+2)^2+4=20$

$(y+2)^2=16$

$y+2 = \begin{cases} 4 \\ -4 \end{cases}$

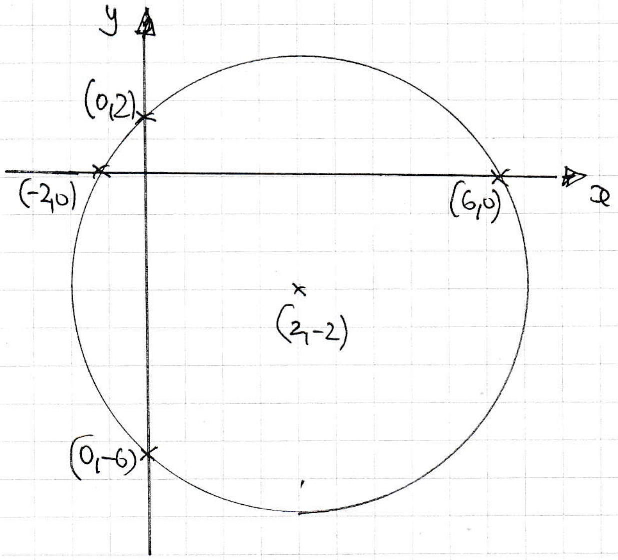
$y = \begin{cases} 2 \\ -6 \end{cases}$

- $y=0$

YIELDS IDENTICALLY

$x-2 = \begin{cases} 4 \\ -4 \end{cases}$

$x = \begin{cases} 6 \\ -2 \end{cases}$



c) SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{l}(x-2)^2+(y+2)^2=20 \\ y=2x+k\end{array}\right\} \Longrightarrow (x-2)^2+(2x+k+2)^2=20$$

TIDY UP

$(a+b+c)^2 \equiv a^2+b^2+c^2+2ab+2bc+2ca$

$$\Longrightarrow x^2-4x+4+4x^2+k^2+4+4xk+4k+8x=20$$

$$\Longrightarrow 5x^2+4x+4kx+8+4k+k^2=20$$

$$\Longrightarrow 5x^2+4x+4kx+k^2+4k-12=0$$

$$\Longrightarrow 5x^2+x(4x+4k)+k^2+4k-12=0$$

$$\Longrightarrow 5x^2+4(k+1)+k^2+4k-12=0$$

AS REQUIRED

IYGB - MP1 PAPER E - QUESTION 10

d) IF TANGENT THIS QUADRATIC IN x, MUST HAVE REPEATED ROOTS

$$\Rightarrow b^2 - 4ac = 0$$

$$\Rightarrow [4(k+1)]^2 - 4 \times 5 \times (k^2+4k-12) = 0$$

$$\Rightarrow 16(k+1)^2 - 20(k^2+4k-12) = 0 \quad \div 4$$

$$\Rightarrow 4(k+1)^2 - 5(k^2+4k-12) = 0$$

$$\Rightarrow 4(k^2+2k+1) - 5k^2 - 20k + 60 = 0$$

$$\Rightarrow 4k^2 + 8k + 4 - 5k^2 - 20k + 60 = 0$$

$$\Rightarrow -k^2 - 12k + 64 = 0$$

$$\Rightarrow k^2 + 12k - 64 = 0$$

$$\Rightarrow (k-4)(k+16) = 0$$

$$\Rightarrow k = \begin{cases} 4 \\ -16 \end{cases}$$